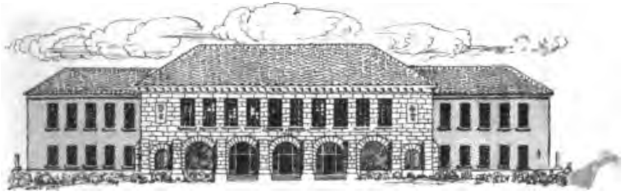

NATURAL PHILOSOPHY FOR BEGINNERS.

PART II.

NATURAL PHILOSOPHY FOR BEGINNERS

WITH NUMEROUS EXAMPLES

BY

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PART II.

SOUND, LIGHT, AND HEAT.

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PREFACE.

The present volume forms the second part of an elementary treatise on Natural Philosophy. The former part explains the Mechanical Properties of solid and fluid bodies; the present is devoted to the *Secondary* Mechanical Sciences, namely, Sound, Light, and Heat. These three subjects are closely connected; they all involve the idea of some medium by which mechanical action is transmitted, and all bring before us alike such important principles as reflection, refraction, and interference. Thus each subject illustrates the others, and the difficulties become much diminished by mutual comparison.

All the subjects here considered have been earnestly cultivated for a long time, and consequently have been developed into a large amount of interesting facts. I have endeavoured to make a profitable selection from the quantity of material thus available; taking especially those topics which afford exercise for reasoning in explaining them, and those which admit of valuable practical applications.

The examples are nearly 400 in number; they have been partly selected from published examination papers, but many of them are original. They will furnish ample exercise of the student's knowledge, and in some cases they lead by easy steps up to important general results.

The very favourable reception which has been given to the first volume, shews that the plan and execution have been widely approved, and leads to the hope that the complete work will be found well adapted for the purposes of instruction.

I. TODHUNTER.

ST JOHN'S COLLEGE,
October, 1877.

CONTENTS.

SOUND.

I. VELOCITY OF SOUND IN AIR.

1. The name *Acoustics*, from a Greek word signifying *to hear*, is given to the science which treats of Sound. This science includes the nature and production of sound, and the laws of its propagation through the various substances which convey it to our ears.

2. We need not specify the different modes in which sounds are produced, as they are sufficiently obvious. It appears on investigation that in all cases sound originates in *motion*. Suppose a string, fastened at both ends and stretched not too tightly, to be set in motion by pulling it a little out of its position of equilibrium; then the string vibrates, that is moves to and fro, in a manner obvious to the eye. Let the string be more tightly stretched, and then set in motion as before; in this case the eye may be scarcely able to discern the different positions which the string assumes, so that the general appearance resembles that of a fine web: but at the same time the reality of the motion is established by means of the sound derived from it.

3. We know that we are surrounded by the atmosphere, and thus all sounds are conveyed to us under ordinary circumstances through this medium; it does not however immediately follow that the presence of the atmosphere is essential for the conveyance of sound. But soon after the invention of the air-pump it was found that when hard bodies were struck together within an exhausted receiver no appreciable sound was produced. The experiment has been carefully repeated in more recent times when the construction of the air-pump has been improved, so that the exhaustion of the air can be carried much further than formerly. Thus we conclude that some medium is necessary for the conveyance of sound; air is that which presents itself most readily, but it is found that both solid and liquid bodies will also serve the purpose. It is necessary to take suitable precautions in order to ensure the success of the air-pump experiment: the bodies which are struck must not be allowed to be in contact with the plate on which the receiver of the air-pump stands; for, if they were, sound would pass through this plate and the contiguous parts of the machine, and finally reach the ear.

4. We are familiar with the fact that sound does not travel instantaneously; it takes *time* to pass from its origin to our ears. We see a woodman cutting a tree at a distance, and we observe that the sound of his blow follows at an appreciable interval after his axe has struck the tree. So also we believe the thunder and lightning to have their origin at the same instant, but we may see the flash some seconds before we hear the peal.

5. Many experiments have been made for the purpose of determining the velocity of sound in the atmosphere. The general principle of all these experiments is the same. Two stations are taken at a known distance apart; a sound is produced at one station, generally by the discharge of a gun, and the instant at which the sound is heard at the other station is noted. The instant at which the sound was produced may be determined by actually observing at the second station the flash which accompanies the dis-

charge of the gun, or by agreement that the discharge shall be made at a prescribed instant. If the former method is used it is assumed that the time which light takes to pass over the distance may be disregarded; and this is justified by the fact that the velocity of light is enormous, being about 186000 miles per second.

6. The early experiments on the velocity of sound were rude and did not agree very well with each other; nor is this remarkable considering the small interval of time which had to be correctly estimated. But more recently by taking abundant precautions, and by adopting the average of many trials, a result has been obtained which may be accepted with confidence. Sir John Herschel in 1830 examined all the numerical values of the velocity of sound which had been proposed up to that date, and found that the average was about 1090 feet per second; he concluded that "we may, therefore, adopt 1090 feet without hesitation (as a whole number) as no doubt within a yard of the truth, and probably within a foot." This applies to air at the temperature of the freezing point of water. Since the year 1830 the power of correctly estimating small intervals of time has been largely increased, but this has not led to any appreciable change in the determination of the velocity of sound. Some experiments lately made at the Cape of Good Hope gave 1090·6 feet per second as the velocity at the temperature of the freezing point of water.

7. The velocity of sound in the atmosphere may be said roughly to be at the rate of a mile in five seconds; this velocity falls far short of that of light, but still it is one of the greatest with which we can be said to be practically familiar. If at a place in latitude about 45 degrees, that is, about midway between the equator and one of the poles, a gun were fired just as the sun crossed the meridian, the sound of the explosion would arrive in succession at places to the West, having the same latitude, just as the sun crossed the meridian.

8. We shall hereafter consider and explain the various ways in which sounds may differ; we will here only notice as familiar truths that one sound may be louder or fainter

than another, and, in the language of music, one sound may be higher or lower than another. Now it has long been admitted as a general principle, obtained both from theory and from experiment, that sounds of all kinds travel with the same velocity in the atmosphere. The simplest illustration of this principle is the fact that when a band of music is playing the effect of the music is not impaired by considerable changes of distance on the part of the listener. If there were any difference in the velocity of the various sounds the notes would not reach a remote ear so as to preserve the exact intervals of time at which they were played; and the component notes of a harmony though sounded together by the performers would not be simultaneous to the hearer.

9. Nevertheless some exceptions to the general principle have been adduced in recent times. Direct experiment seems to assign an appreciably less velocity to a feeble sound than to a loud sound; and also a less velocity to a high sound than to a low sound, though to a scarcely appreciable amount. Also in the second Arctic expedition of Captain Parry, on the occasion of some artillery practice, it was found by persons stationed at a distance that the sound of the explosion was heard *before* the command of the officer to fire. But although it is difficult to resist the evidence of this observation, or to find a good explanation of it, we are scarcely justified at present in abandoning the principle, which theory and the majority of experiments seem to maintain, that sounds of all kinds travel with the same velocity. See Airy on *Sound.*

10. The effect of wind on the velocity of sound must be noticed. As sound is conveyed through the air, and wind consists of a movement of the mass of the air, it is obvious that sound must be quickened or retarded according as the wind is moving with the sound or against it. In fact if it were possible for wind to travel as fast as sound, then in the direction exactly contrary to that of the wind sound would be completely arrested, while it would travel with twice its usual velocity in the direction of the wind. But the most violent hurricane does not move with much more than one-tenth of the velocity of sound.

11. In accurate experiments to determine the velocity of sound, care is taken to avoid any error arising from the influence of the wind. For instance, let us denote two stations at a known distance apart by A and B respectively; then *simultaneous* observations are made of the time taken by sound in passing from A to B, and also of the time taken by sound in passing from B to A. If the wind favours the passage of sound in one direction it will retard the passage in the contrary direction; and the average of the two times will be very nearly the same as if there were no wind at all.

12. The velocity of sound has been investigated by theory as well as by experiment, and although the process is far beyond an elementary book we will just allude to it. Newton first obtained a result, though his method is now seen to have been unsatisfactory: other writers afterwards came to the same result by a different method. The result may be thus stated: to find the velocity of sound through the air in feet per second, *multiply the number which expresses the force of gravity by the number which expresses the height of the homogeneous atmosphere and take the square root of the product.* Now the first number is about 32, and the second number is about 26000: see Vol. I. Arts. 288 and 509: the product of these two numbers is 832000, of which the square root is about 912.

13. The result just assigned by theory, namely 912, differs widely from that which is obtained by experiment, namely, 1090. This discrepancy for a long time perplexed writers on the subject until the difficulty was explained by Laplace, who shewed that a circumstance had been omitted in the theoretical investigation, namely, the influence of the *heat* which is developed and absorbed by the alternate condensation and refraction of the air through which the sound is conveyed. When this circumstance is properly taken into account by mathematical calculations it is found that the product of the two numbers mentioned in Art. 12 must be multiplied by 1·42; this gives 1181440, and the square root of this is very nearly 1090.

14. We have remarked in Vol. I. Art. 509, that the height of the homogeneous atmosphere is the same at all points of the atmosphere: hence according to the theoretical expression of Art. 12, the velocity of sound is the same whatever be the part of the atmosphere through which it is moving, so that sound travels at the same rate upwards into a rarer stratum or downwards into a denser stratum. This has been verified by experiment. Two stations were taken, one on Faulhorn mountain, the other on the lake of Brienz in Switzerland; the stations were nearly six miles apart, and the height of one about a mile and a quarter above that of the other. It was found that sound took the same time in passing from the lower station to the higher as in passing from the higher to the lower.

15. We have stated that the velocity of sound through the atmosphere at the *temperature of the freezing point of water* is about 1090 feet per second. It is found by theory, and confirmed by experiment, that the velocity increases as the temperature rises, at the rate of about 2 feet for every degree of the Centigrade thermometer, that is about 1·1 feet for every degree of Fahrenheit's thermometer. This is altogether distinct from the difficult matter noticed in Art. 13, and depends on a very simple consideration: if the temperature of the air rises while the pressure to which the air is exposed remains unchanged, the air expands and so the density becomes less; hence the height of the homogeneous atmosphere is increased, and so the velocity is increased according to the result of Art. 12.

II. CHANGE OF INTENSITY.

16. Since air is in general the medium for the conveyance of sound, it is natural to consider the circumstances which affect the intensity of sound in its passage through the air. It has been found that when the air is condensed the intensity of the sound is increased, that is the sound seems louder. Thus corresponding to the experiment of ringing a bell in an exhausted receiver, we may make another in which a bell is rung in a receiver

into which air has been condensed: it is found in the latter case that the sound is stronger than in ordinary air and the intensity of the sound increases as the density increases. The same results have been obtained in diving bells in which the air is necessarily condensed. On the other hand the fact that sound becomes feebler when the air is rarefied is very familiar. Thus the sound of a pistol shot on a high mountain is said to be like that of a pop-gun at the foot of the mountain. The human voice too in such elevated regions seems to lose much of its force; though this may be partly due to the diminution of muscular energy produced by the rarefied air. The intensity of a sound depends on the density of the air at the spot where the sound originated; and it has been found that persons in a balloon when elevated considerably above the surface of the earth have heard sounds which were produced at the surface: thus the whistle of a steam engine has been heard at the height of about 20000 feet. It seems also to be clearly established that sounds reach the earth from heights at which the air must be of almost inconceivable tenuity. For instance, loud explosions have been heard connected with meteors which must have been 25 or even 50 miles above the surface of the earth.

17. If air be confined entirely as in a pipe, or partially as by walls, sound may travel far with little loss of intensity. This was tested by Biot in a pipe of more than 3000 feet long: the slightest whisper at one end was distinctly audible at the other. The tubes now much used for conveying messages through large buildings illustrate the fact; words which can be scarcely heard by a person standing close to the speaker are well heard by a listener at the other end of the tube. But sound which travels through unconfined air diminishes rapidly in intensity as the distance from the origin is increased. Thus if the distance is doubled the intensity becomes one-fourth of what it was; if the distance is tripled the intensity becomes one-ninth; and so on. The fact is expressed by saying that the *intensity varies inversely as the square of the distance:* this is the same law as holds for the diminution of the force of gravity when the distance increases: see Vol. I. Art. 301.

18. Sound is conveyed with great clearness and strength through the air which is just over the surface of still water. Thus at a quiet part of the Thames, Dr Hutton distinctly heard a person reading at twice the distance he could have heard the same person on land. Lieutenant Foster, in the third Polar Expedition of Captain Parry, found he could converse with a man across the harbour of Port Bowen, which is a distance of about a mile and a quarter. Extraordinary statements are on record as to the distance at which the sound caused by firing cannons has been heard; putting this distance in some cases at upwards of 100 miles.

19. Sound when it meets with obstacles bends round them, and thus is not completely cut off from points beyond the obstacle. Nevertheless there is a diminution of the intensity of sound by the interposition of an obstacle between the origin of sound and the ear.

20. The intensity of sound in the atmosphere is affected by the state of the weather and the wind. The subject is of great practical importance, because ships may be informed during dark hours of their dangerous vicinity to land by loud signals, provided there is some security as to the distance at which the signals can be heard. It is well known that on some rocky islands the incessant noise of sea birds serves as a warning to sailors, and that in consequence the destruction of the birds is discouraged.

21. It is however only quite recently that the facts of the subject have been accurately collected, or that any attempt at a theory has been made. The older writers assumed, for instance, that fog is "a powerful damper of sound," and that "falling rain tends powerfully to obstruct sound"; but it has been shewn by Professor Tyndall that neither of these general statements is trustworthy. He has investigated the subject very fully, and thus sums up his conclusions: "The real enemy to the transmission of sound through the atmosphere has, I think, been clearly revealed by the foregoing inquiry. That enemy has been proved to be not rain, nor hail, nor haze, nor fog, nor snow—not water in fact in either a liquid or a solid form,

but water in a vaporous form, mingled with air so as to render it acoustically turbid and flocculent. This acoustic turbidity often occurs on days of surprising optical transparency. Any system of measures, therefore, founded on the assumption that the optic and acoustic transparency of the atmosphere go hand in hand must prove delusive." *Proceedings of the Royal Society*, XXII. 68.

22. It has long been observed that a wind blowing in the direction contrary to that in which sound is proceeding diminishes the audibility of the sound. Cases are known in which against a high wind guns could not be heard at a distance of 550 yards, although on a calm day the same guns might be heard from twenty to thirty miles. At first sight it may seem quite reasonable that sound should be, as it were, blown back by a high wind, for sound is known to travel through the air, and if the whole mass of air is itself moving the sound must be carried with it; but on consideration it is found that this does not explain the effect of the wind on the intensity of sound. For the velocity of sound is about 1100 feet per second, and the velocity even of a high wind, which is from 50 to 100 miles per hour, is small compared with this; so that the diminution of the velocity of sound would be scarcely appreciable.

23. The influence of wind on the intensity of sound seems due to the fact that, owing to obstructions opposed by the ground, there is a considerable difference between the velocity of the wind close to the ground and the velocity at the height of a few feet above the ground. Thus in a meadow the velocity of the wind at one foot above the surface may be only half what it is at eight feet above the surface. The process of tracing the consequences of this fact is somewhat difficult, and may not be fully apprehended by the beginner at once. Let us take the velocity of sound at 1100 feet per second, and suppose that the velocity of a contrary wind is 10 feet per second at the surface, and 20 feet per second at the height of 8 feet above the surface. Thus, considering this circumstance alone, the wave of sound at the end of a second would be at the surface 10 feet in advance of its position at 8 feet above the

surface; so that the front of the wave instead of being a vertical plane would be inclined to the horizon. In the diagram of Vol. I. Art. 245, if we suppose *AC* to be 10 feet, and *BC* to be 8 feet, the angle *BAC* represents this inclination to the horizon. But, by the nature of sound, the direction of sound is at right angles to the front of the wave, so that the direction is not along *CA*, but is at right angles to *AB*. Thus the sound instead of proceeding horizontally becomes turned *upwards*. It only remains to add that this tilting of the front of the wave is not delayed until the end of a second, but begins at the origin of the sound and increases gradually. Hence a ray of sound, so to speak, instead of travelling horizontally is curved upwards, and thus passes over the head of a person stationed at a distance from the origin. A contrary wind then diminishes the intensity of sound by lifting the sound off the ground, and the amount of this lifting increases as the distance from the origin increases.

24. The various consequences which may be deduced from the preceding theory have been verified by experiments. Thus it follows that a listener when the wind is contrary may expect to recover a sound, which he has lost at a certain distance from its origin, by ascending to some height above the surface. Also the influence of a wind will be but small if the surface be very smooth; thus sounds are heard against the wind much farther over calm water than over land. Again, suppose the origin of the sound to be elevated above the surface: then if the listener be also raised above the surface he may hear a very loud sound made up of two parts, namely, that which has travelled horizontally, and that which has been tilted upwards from the ground by the action of the contrary wind.

25. Next suppose the wind to be *favourable* instead of *contrary*. In this case the higher part of the wave of sound moves more rapidly than the lower, and so the plane front of the wave is tilted *forwards*, and the rays of sound are bent *downwards* to the advantage of the listener on the ground.

26. Thus the influence of the wind on sound has l shewn to depend on the circumstance that when wind is blowing, the velocity of sound is different at ferent heights above the ground: similar effects will th fore follow if this difference of velocity is produced any other cause instead of by the wind. Now chang temperature affects the velocity of sound: if the temp ture rise one degree of Fahrenheit's thermometer velocity increases by about a foot per second. In gen as we ascend in the air during the day the temperat decreases, and therefore so also does the velocity of sou Thus the result is the same as in the case of a *contra* wind; the ray of sound is lifted over the head of a pers on the ground, so that the audibility of the sound is minished.

27. The presence of vapour in the atmosphere al affects the propagation of sound; the velocity increases the quantity of vapour increases. The direct effect ho ever is very slight, but indirectly the vapour is of cons quence, for it gives to the air a greater power of radiati and absorbing heat, and so promotes inequality of temp rature. The variation of temperature is greatest when th sun is shining, so that it is greater by day than by nigh and greater in summer than in winter. Hence, accordin to the theory now explained, sounds ought to be hear more plainly by night than by day, and more plainly i winter than in summer. That sounds are heard mor plainly by night than by day is a well-known fact.

28. We have supposed that the temperature *decrease* as we ascend in the atmosphere; but it may happen o some occasions that the temperature at the surface is *lowe* than it is a little above the surface. This may be the cas for instance over the surface of the sea in the day-time and over the surface of the land by night. Thus the effec on sound will be similar to that of a *favourable* wind. I is obvious that by the combined influence of wind an temperature the results produced may vary much as t degree; for instance, the operation of a contrary wind ma be neutralized by that of temperature rising as we ascen above the surface. The whole theory is given in the *Proceedings of the Royal Society*, Volumes XXII. and XXIV.

III. VELOCITY OF SOUND IN OTHER MEDIA.

29. Sound may be transmitted through the various gases. The theoretical value of the velocity would be similar to that of Art. 12; and it would require a similar correction to that of Art. 13, on account of the heat developed and absorbed. It is not easy to determine the velocity by experiment in a direct manner, as we cannot procure a column of any gas of sufficient length and purity; but theory points out a connexion between the velocity of sound and the pitch of the note produced by an organ pipe, and thus by filling an organ pipe with the gas to be examined the velocity of sound in the gas is indirectly determined. It is found that at the temperature of the freezing point of water the velocity is 1040 feet per second in oxygen, and 4164 feet per second in hydrogen.

30. Hydrogen, the lightest of the gases, is somewhat peculiar in relation to sound. The velocity of sound in this gas, as we have just stated, is about four times as great as in air. But the most remarkable circumstance is the power which hydrogen seems to possess of deadening sound and almost stifling it. The sound of a bell in a receiver charged with hydrogen is said to be scarcely more audible than that of the bell in an exhausted receiver.

31. The transmission of sound through *vapours* presents some points of interest arising from the fact that a vapour readily condenses, at least partially, into a liquid, when there is an increase of pressure, such as according to theory must occur in the course of the transmission of sound. But on the other hand if the temperature is sufficiently raised the condensation into a liquid is prevented; and it appears from experiment that there is such a rise of temperature. Thus we obtain some indirect confirmation of the truth of the remark made by Laplace: see Art. 13.

32. Sound can be transmitted through liquids. It is found by trial that a diver when under water can hear sounds addressed to him by a person near the surface, and also a sound produced under the water at a considerable distance, as the striking together of two stones half a mile off. Workmen in a diving bell make signals to those on the surface by striking the bell. Moreover fishes are provided with organs of hearing, and often have an acute sense of hearing. Some very careful experiments made at the lake of Geneva in 1826 gave 4708 feet per second as the velocity of sound in water, the temperature of the water being about 46½ degrees of Fahrenheit's thermometer. It is found that if a salt is dissolved in water the velocity of sound in the water is increased; and in water, as in air, the velocity increases as the temperature rises.

33. Theory shews that the possibility of transmitting sound through water depends on the fact that water is really elastic; that is, it may be diminished in bulk by pressure, and will regain its original bulk when the pressure is withdrawn. It is known from direct experiment that water is compressible; at the depth of about 7000 feet below the surface of water the volume of any portion would be about one hundredth less than at the surface, so that the density would be one hundredth greater. The value which theory assigns for the velocity of sound in water agrees reasonably well with that found by experiment. It appears that no sensible amount of heat is developed in liquids by compression, so that no correction is required like that of Art. 13.

34. Sound can also be transmitted through solids, as may be shewn by a simple experiment. Let a person put his ear near one end of a long piece of good timber, and let another person tap the other end of the timber slightly, or even scratch it with a pin: then the first person will hear the noise distinctly; indeed if he did not it would be taken practically as evidence that the timber was not sound throughout. In general all hard firm solids of uniform texture transmit sound well and rapidly. A blow struck against a rock in a stone quarry is heard at a distance *twice*; first, almost immediately, as transmitted through

the stone, and next as transmitted through the air. The same circumstance is noticed in mines with respect to the sound excited by blasting. So also a sound produced at one end of a long wire is heard twice at the other end; first, almost immediately, as transmitted through the wire, and next as transmitted through the air.

35. The velocity of sound in iron has been investigated by direct experiment. Biot made use of a length of about 3000 feet of iron, consisting of 376 pipes joined at their ends. A sound was produced by striking the pipe at one end, and was heard twice at the other end; the interval between the arrival of the two sounds was observed, and as the velocity of sound in air was known, that in iron could be deduced. The experiment could not be very satisfactory, for the transmitting body was not a continuous mass of iron; the connection between adjacent pipes was made by collars of lead and tarred cloth. By selecting the observations which seemed the best it was inferred that the velocity in iron is 11090 feet per second. Sir John Herschel remarks on Biot's experiment: "From this determination we may estimate the time it requires to transmit force, whether by pulling, pushing, or by a blow, to any distance, by means of iron bars or chains. For every 11090 feet of distance the pull, push, or blow, will reach its point of action one second after the moment of its first emanation from the first mover. In all moderate distances, then, the interval is utterly insensible. But were the sun and the earth connected by an iron bar, no less than 1074 days, or nearly three years, must elapse before a force applied at the sun would reach the earth. The force actually exerted by their mutual gravity may be proved to require no appreciable time for its transmission. How wonderful is this connection!"

36. The velocity of sound along bars of metals has also been investigated indirectly, by observing the musical note produced by such bars when set in vibration. Thus it has been found that for tin, silver, and gold the velocities are respectively about 7½, 8, and 6½ times as great as in air. For ordinary iron wire a velocity of more than 16000 feet per second was obtained.

37. The velocity of sound depends very much on the arrangement of the molecules of the substance through which it is conveyed. This is shewn in a striking manner by some results obtained indirectly with respect to the velocity of sound through wood. Thus, for example, in fir the velocity along the fibres is said to be 15218 feet per second; but in directions at right angles to the fibres it is much less, being 4382 feet per second *across* the rings, and 2572 *along* the rings. Sound is obstructed by any want of uniformity in the composition of the substance through which it is conveyed; it would seem that at every change of substance some portion of the sound is *reflected*, and thus the intensity of the sound transmitted is diminished: moreover the direction of the transmitted sound often undergoes a change. In fact sound is liable to changes of motion similar to those which, as we learn from optics, light experiences.

38. It is well known that sounds seem more audible by night than by day, and this is mainly owing to the circumstance that the temperature and the density of the air are more nearly uniform by night than by day; for in the day-time the heat of the sun produces perpetual currents of warm air from the surface of the ground upwards, by which the uniformity of the air is destroyed. Sir J. Herschel remarks: "There is no doubt, however, that the universal and dead silence generally prevalent at night renders our auditory nerves sensible to impressions, which would otherwise escape them. The analogy between Sound and Light is perfect in this as in so many other respects. In the general light of day the stars disappear. In the continual hum of noises which is always going on by day, and which reach us from all quarters, and never leave the ear time to attain complete tranquillity, those feeble sounds which catch our attention at night make no impression. The ear, like the eye, requires long and perfect repose to attain its utmost sensibility." It has been stated however that the greater audibility of sounds by night than by day has also been observed near the torrid zone, where the day seemed quieter than the night, which was disturbed by insects.

39. A general expression is sometimes given which applies to the velocity of sound both in gases and liquids; and this we will now briefly notice: the considerations however are somewhat beyond the range of a beginner, and so he cannot be expected to pay much attention to them.

40. We have treated of the property of Elasticity in Volume I., and have seen that it belongs to all matter. Bodies when acted on by force experience a change of volume; and they regain their original volume when the force ceases to act. Theory shews that in virtue of this property sound can be transmitted through bodies.

41. In Art. 12, we have stated the velocity of sound in air, as assigned by theory; this velocity is the square root of a certain quantity. The general expression for the velocity of sound in gases and liquids, which we have now to notice, is this: the velocity is the *square root of the quotient of the elasticity divided by the density.* The advantage of this mode of expression is that it brings the two cases of the velocity of sound in gases and liquids under one general statement; and that it presents more obviously to the mind the two elements on which the velocity depends. But, on the other hand, it must be observed that in order to apply the rule we must know the way in which elasticity is measured; and *apparently* this is not measured in the same way for both gases and liquids. In gases the elasticity is measured by the pressure to which the gas is exposed, and in liquids it is inversely as the compressibility of the liquid. These remarks would have to be much developed in order to be fully understood; but it is sufficient for our purpose to have just noticed the subject.

IV. REFLECTION OF SOUND.

42. When sound in its course meets with any obstacle of sufficient regularity and extent it is *reflected,* that is, deviated from its original direction and sent in a new direction which may be called roughly *backwards.* The process resembles in some degree what takes place when a ball strikes a hard plane surface: see Vol. I. Art. 284.

Sound thus diverted may be transmitted to a person who by reason of some intervening object may be unable to see the place from which the sound started, or even to hear the sound by a direct course. Moreover the sound may be sent back to the place from which it started, after one or more reflections, and may thus produce what is called an *echo*. It is possible that there may be several obstacles suitable for causing a reflection, and thus sound may return to its starting point by various routes, giving several repetitions of the original. In the neighbourhood of almost every place situations can be found where an echo is perceptible, and in general it is easy to recognize the obstacle which causes the reflection; as for instance a wall or a rock. There are well-known localities in which tourists are entertained with remarkable echoes; as, for instance, particular spots near the Rhine, and near the Lakes of Killarney: Sir J. Herschel notices a very fine echo beneath the suspension bridge over the Menai Straits. Sometimes on old bridges there are recesses in the form of half vaults constructed of stone, on opposite sides of the bridge, and thus words spoken in one recess may be heard by a person in the opposite recess, being brought to him by two reflections. An echo is sometimes produced by what we may call an *acoustic cloud* in the atmosphere, that is by a stratum of air which owing to temperature or to moisture differs in character from the air near it. The sails of a distant ship at sea have also been known to produce an echo.

43. As the velocity of transmission in air is known it is easy to calculate the time which ought to elapse between the origin of any sound and its echo, if we know the course which the sound has taken. Thus we may determine the distance of the obstacle which has caused the reflection; or if we know that distance we can verify the received value of the velocity of sound. The simplest case is that of *direct* reflection, namely, when the sound goes from its starting point to an obstacle, and then back again along the same straight line. This case occurs when a person stands directly in front of a large building and makes a sound which is reflected by it. Suppose a person to stand between two parallel walls, which we will denote by A and B;

if he makes a sound he will hear an echo from both walls, and these will be simultaneous if he stands midway between the two walls: if he is not midway between them the echo from the nearer wall will be heard first. The sound reflected from *A* may also be reflected again from *B* with sufficient energy to be audible; and in like manner the sound first reflected from *B* may undergo a second reflection from *A* and be audible: these sounds, which have undergone *two* reflections, will reach the listener simultaneously whatever be the position he occupies between the walls, namely, at the interval, after the origin, which sound takes in passing twice over the distance between *A* and *B*.

44. In the case of the oblique reflection of sound, as in that of the oblique reflection of a perfectly elastic ball, the angle of reflection is equal to the angle of incidence: see Vol. I. Art. 284.

45. The number of consecutive syllables which can be heard distinctly as an echo will depend on the distance at which the listener is placed from the obstacle which causes the reflection. A speaker can articulate so as to be distinctly audible at the average rate of four syllables in a second. Suppose he stands at the distance of 1100 feet from an obstacle; sound then takes two seconds to traverse this distance twice, so that if the speaker pronounces eight syllables an echo of the first arrives just as the last is completed: if the speaker goes on to use his voice without intermission the echo and the direct sound become blended and confused.

46. Two familiar applications of the principle of the reflection of sound may be noticed. The ear trumpet which persons who are rather deaf use to assist their hearing consists of a tube, one end of which is slender and is inserted in the ear, while the other end is wide and is directed towards the sound which is to be received. The sound falling on the inner surface of the tube is reflected from side to side until it arrives at the ear; thus the effect is much the same as if the receptive part of the ear were enlarged to the size of the mouth of the tube. The speaking trumpet is somewhat of the same

kind as the ear trumpet, only it is straight instead of being curved. The speaker applies the small end to his mouth; much of the sound which, if there were no trumpet, would diverge in all directions, is reflected by the sides of the trumpet and thrown off in a direction parallel to the axis of the trumpet: thus it falls with great force on the ear of the person towards whom the trumpet is pointed. Speaking trumpets are used on board ships. The case which we have noticed in Art. 42, of recesses on the two sides of a bridge, illustrates the use both of the speaking trumpet and the ear trumpet. The sound uttered by the speaker is reflected by the stone near which he stands, in one direction, in the manner of the speaking trumpet; thus it travels across the bridge with more intensity than the direct sound: and the stone near which the listener stands collects this sound and brings it with force to his ear in the manner of the ear trumpet.

47. The subject of the reflection of sound is important with respect to the construction of such buildings as Lecture Halls and Churches, in which the audibility of a speaker is to be ensured. Few persons can have considered the matter without observing cases in which there is a complete failure in this respect: indeed architects and builders in general seem to regard only the appearance of their erections, and not the end for which they are designed. A large room is constructed, and it is found on the first trial that it is difficult to hear the speaker distinctly even when near to him, and impossible when at a moderate distance. Theory at present seems unable to anticipate the results beforehand; but at least the facts of the subject might be collected and classified so as to furnish some guide for practical men. A few general remarks may be made. If a room be *small*, sound reflected from any part of it will be separated by so brief an interval from the original that the reflection strengthens without confusing the impression; in this case the object would be to secure reflection, and therefore deep recesses, hangings, and carpets should be avoided as unfavourable to audibility. But if a room be *large* the reflected sound may occur so long after the original as to produce confusion; the echo of one note or syllable may coincide with

the original of the next; thus ruining the effect of music and rendering a speaker unintelligible. Attempts have been made in churches to assist the transmission of the preacher's voice by placing behind him or above him a surface which should reflect the sound that would otherwise be lost, and send it forward to corroborate the impression produced by direct transmission. But no great success seems to have been attained by these *sounding boards* as they are called, while they have the obvious disadvantage of bringing the noise of the congregation with condensed effect to the preacher's ears.

48. We may notice an interesting application which has been made of the fact that sound can be reflected. The *pneumatic despatch tubes* are long tubes of metal which are employed for the rapid conveyance of small parcels from one point to another. The parcel is put into a small gutta-percha case covered with felt, called the *carrier;* this just fits the tube, and is blown along it by compressed air behind, or, as it were, drawn along it by making a vacuum before it. In London these tubes are used in the postal service; the longest extends from the chief office to Charing Cross: the distance is about a mile and a half, and is traversed by the *carrier* in four minutes. In France such tubes are used to convey messages between Paris and Versailles. Sometimes a carrier sticks in the tube: this is a troublesome accident because it involves the opening of the tube, and besides there is the difficulty of ascertaining the exact spot where the stoppage exists. An ingenious method has been devised for overcoming this difficulty. An elastic skin is stretched over the end of the tube, and a noise is made near it by the discharge of a pistol. The noise passes through the skin, along the air in the tube, to the point of stoppage; it is reflected back again to the skin, and causes a tremulous motion in it. The number of seconds between the explosion and the return of the sound to the skin must be carefully observed; and as the velocity of sound is known the space through which it can travel in this time is known: half this space is the distance of the point of stoppage from the end of the tube. Thus the skin takes

the place of the ear, and the operation is like that of observing the time which elapses between the production of a sound and its return in the form of an echo.

49. The student of Optics knows what is meant by that bending of the rays of light which is called *refraction.* A common example is seen in the bending of the sun's rays as they pass through a convex lens, so that they are brought to meet with powerful effect at a point; thus the lens becomes a *burning glass.* A similar thing has been observed with respect to sound, though here it is of little consequence, while the refraction of light is of extreme importance. Carbonic acid gas is enclosed in a thin india-rubber balloon, and an origin of sound, as, for example, a ticking watch, is placed at a suitable distance from this sound-lens; then the rays of sound are found to be bent by passing through the gas, and are brought together with concentrated effect at a certain point.

V. NATURE OF A WAVE.

50. We have hitherto said nothing of the nature of the motion of the particles of air which constitutes sound. These particles are themselves invisible, and so of course are their motions: we are therefore compelled to have recourse to some illustrations.

51. When a stone is thrown into a smooth pond it is seen that waves travel out in all directions from the point of the water at which the stone fell. That is to say, ripples in the form of circles are produced round this point as centre, successively at greater distances, until they become too faint to be distinguished. Now it must be observed that there is no movement of the general mass of water from the centre of the circle; each particle of water moves nearly *up and down*, never departing far from the place which it occupied before the disturbance: this can be clearly made out by watching a leaf or a piece of straw, which happens to be on the surface of the water when the disturbance reaches it.

52. The nature of the mechanical action which takes place can in some degree be understood. The stone falling on the water forces part of it down, and the surrounding part is heaped up above its natural level; this part then subsides and fills up the depression produced by the stone. The ridge, while it is thus subsiding, exerts for some time on the surrounding fluid a pressure greater than that which subsisted before the disturbance, and thus forces up another ridge: this ridge in subsiding forces up another, and so on.

53. It is easy to see what is meant by the *velocity* of the wave, and also to determine the amount of it, at least roughly. Suppose we have two stakes fixed in the ground, at a certain known distance apart, such that the two are in a straight line with the origin of the disturbance. Note the instant when the disturbance first reaches the nearer stake, and also the instant when it first reaches the further stake; then, as we know the distance between the stakes, we can find the average velocity with which the disturbance has travelled from one to the other. If we have a series of stakes we may ascertain whether the velocity remains the same, or not, at all distances from the origin: in fact it is found that the velocity diminishes as the waves travel to a greater distance from the origin.

54. The velocity of a *wave* is quite a different thing from the velocity of any *particle* of the water: it would scarcely be practicable to determine the latter by observations. It would be difficult to measure both the small space through which the particle moves, and also the small interval of time during which the motion continues. Moreover the movement would not be uniform, but would somewhat resemble that of a pendulum, being sometimes faster and sometimes slower than the average.

55. The analogy between the waves in water which we have just considered and the waves in air which produce sound is not very close. To take a simple case, suppose sound transmitted through the column of air contained in a long slender tube. Each particle of air always remains near its original position, but moves forwards and backwards *parallel to the length of the tube;* and we may sup-

pose that all the particles which were originally in any one plane at right angles to the tube, move precisely in the same way, so as always to lie in one plane. If we could see the moving particles there would not appear *ridges* and *depressions* as on the surface of the pond, but instead of them alternately a plane where one layer of particles would be *closest* to the next layer, and a plane where one layer of particles would be *furthest* from the next layer; these layers being all equidistant in the undisturbed state. The wave motion in this case consists in the transmission along the tube of the state of contiguous layers; and this state is most easily understood by adverting to its extreme forms of *greatest condensation* and *greatest rarefaction;* the former meaning the state in which contiguous layers are *closest*, and the latter the state in which they are *furthest apart.* The wave moves along the tube uniformly at the rate of about 1100 feet per second.

56. By the aid of mathematical investigation we learn the circumstances of the transmission of a wave of sound through a long tube; we will state the results thus obtained. Suppose that near one end of a long tube we have a piston which moves regularly forwards and backwards through a certain space. First while the piston moves forwards it compresses the stratum of air in contact with it; this stratum acts in the same way on the next to it; and so on. Thus a wave of *condensation* is propagated forwards through the tube. Each particle of air moves in the same manner as a point in the piston moves, that is, it moves through an equal space, and its velocity is the same as that of the piston in the corresponding position: the particle of air begins to move at the end of the interval which sound, travelling with its known velocity, takes to pass from the piston to the particle. The condensation is greatest at those points where the particles are moving with the greatest velocity. For instance, suppose the piston to move after the manner of a pendulum, with a velocity gradually increasing up to a certain value, and then diminishing in a corresponding degree. In this case the condensation will be greatest in the middle of the wave, and will diminish gradually to the ends. Next, while the piston moves backwards a wave of *rarefaction* is in like

manner propagated through the tube; and the rarefaction is greatest where the velocity of the particles is greatest. We have spoken of a wave of condensation and a wave of rarefaction; but it is more convenient to regard them as two parts together making up a single wave. The two parts correspond to the elevation and the depression which together constitute a wave in water. When the piston moves forwards the second time and backwards the second time, another whole wave, consisting partly of a condensation and partly of a rarefaction is propagated through the tube.

57. The term *length of a wave* requires careful notice. It is the distance between two consecutive particles which are in similar states of motion. Thus in Art. 51 it is the distance between the summits of two adjacent crests, or the bottoms of two adjacent hollows. In the case of Art. 56 it is the distance between one point where the condensation is greatest, and the adjacent point of the same nature; or we may say that it is the distance between two consecutive points of greatest rarefaction: we assume here that wave follows wave steadily without any interruption.

58. Now we can arrive at some knowledge respecting the *time of vibration* of each particle, that is the time which the particle takes in moving from one of its extreme positions back again to the same point. Divide the length of a wave by the velocity of propagation, and the quotient is this time. For instance, take the case of Art. 56: during the time the piston moves once forwards and once backwards the disturbance travels over a wave length; suppose this wave length to be 11 feet, the sound moving at the rate of 1100 feet per second takes $\frac{11}{1100}$ of a second, that is, $\frac{1}{100}$ of a second, to pass over this wave length: thus the piston executes a vibration in $\frac{1}{100}$ of a second, and so also does each particle of the air in the tube. Thus the velocity of sound is quite independent of the velocity of the particles of air; the former remains unchanged,

while the latter may have widely differing values, each value having its own corresponding wave length. If we know the time of vibration of the particles we may determine the wave length; it will be simply the space through which sound could travel in that time. For example, suppose the time of vibration is $\frac{1}{250}$ of a second; then the length of the wave is $\frac{1100}{250}$ feet, that is, $\frac{22}{5}$ feet.

59. Other cases present themselves to our attention in which the transmission of waves may be observed. Thus, for instance, on the surface of the sea waves may be seen to move in the form of parallel ridges and hollows. So also we may notice the ripples which the wind makes as it sweeps over a field of standing corn. The motion bears a decided resemblance to that of the air in a tube which causes sound; for the ears of corn describe arcs of curves which do not differ much from straight lines, and in some places they are brought closer together, while in others they are wider apart than when there is no wind. The mechanical circumstances, however, are very different in the two cases. For the ears of corn are all moved by the wind and not by action one upon another; moreover they are all confined near their original places by the stalks.

VI. MUSICAL SOUNDS.

60. Although every impulse communicated to the air must be propagated onwards, and so may reach an ear, yet an audible sound is not produced unless the disturbance of the air is of sufficient extent and suddenness. Thus the fall of a leaf through the air may be unheeded, while the contact of the leaf with the ground may be distinctly heard.

61. An irregular impulse communicated to the air produces what we may call a *noise*, as distinguished from a *musical sound*. The ear, like the eye, retains for a short time an impression made upon it, so that if single impulses are produced at very short intervals the impression

is that of a continuous sound. In order to ensure this the impulses must be produced at least as often as sixteen times in a second; the limit may be slightly different for different ears. When the impulses are exactly similar and occur at exactly equal intervals, they produce a sound which is continuous and uniform, and pleasing to the ear; this is called a musical sound. The sound may gradually die away, so that the only change is the diminution of loudness, without losing its claim to be called musical. The human voice can produce either kind of sound; namely, a musical sound in singing, or a non-musical sound in speaking. The distinction between a musical sound and a non-musical sound is however not very precise.

62. An extract from Mr Sedley Taylor's *Sound and Music*, bearing on this distinction, will be read with interest: "We may then define a musical sound as a *steady* sound, a non-musical sound as an *unsteady* sound. It is true we may often be puzzled to say whether a particular sound is musical or not: this arises, however, from no defect in our definition, but from the fact that such sounds consist of two elements, a musical and a non-musical, of which the latter may be the more powerful, and therefore absorb our attention, until it is specially directed to the former. For instance, a beginner on the violin often produces a sound in which the irregular scratching of the bow predominates over the regular tone of the string. In bad flute-playing, an unsteady hissing sound accompanies the naturally sweet tone of the instrument, and may easily surpass it in intensity. In the tones of the more imperfect musical instruments, such as drums and cymbals, the non-musical element is very prominent, while in such sounds as the hammering of metals, or the roar of a waterfall, we may be able to recognize only a *trace* of the musical element, all but extinguished by its boisterous companion."

63. There are three important characteristics to be noticed in connexion with a musical sound, namely, *intensity*, *pitch*, and *quality*. The *intensity* is the greater or less loudness of the sound, other things remaining the same. A change of intensity can be produced in various

ways. Thus we may approach to, or recede from, the origin of the sound; we may strike a key on the piano-forte with greater or less force; or we may take a musical sound produced by one violin, and afterwards have the same sound produced simultaneously by two or more violins. The *pitch* is that property by virtue of which some musical sounds are called *bass*, or *grave*, or *deep*, or *low*, while others are called *treble*, or *acute*, or *shrill*, or *high*. A musical sound is called a *note* with especial reference to its pitch. The difference of pitch is at once recognized by striking in succession two keys of a piano-forte, one towards the right-hand end of the key board, and the other towards the left-hand end. The *quality* is that property by virtue of which there is a difference between various voices and instruments which are all producing the same note with the same intensity. Thus a good musical ear can distinguish between a note sounded on a piano-forte made by Broadwood, and the same note sounded on a piano-forte of the same size made by Erard. Even an ordinary ear must recognise the difference between a note produced by a violin, and a note of the same intensity and pitch produced by a flute. At present we have considered *intensity*, *pitch*, and *quality* merely as facts known by observation; we shall afterwards indicate the causes on which they depend.

VII. STRETCHED STRINGS.

64. If a string, tightly stretched and fastened at its ends, be withdrawn a little from its position by pulling it or striking it and then leaving it to itself, it vibrates to and fro, and a musical sound is heard. A string of a harp is a familiar example, and all stringed musical instruments are similar cases. The motion has been investigated by theory, and an expression found for the time of an oscillation. This time depends on the length and the weight of the string, and on the tightness with which it is stretched. The tightness may be conveniently expressed by means of a length of the string which would be in weight equal to the

force with which the string is stretched, that is to what is called the tension of the string: this length of the string may be called the *tension-length*. The following is the Rule for finding the time of an oscillation in seconds, the lengths being all expressed in feet: *multiply the tension-length by* 32 *and extract the square root of the product; divide the length of the string by this square root, and the quotient is the required time of an oscillation.* By an *oscillation* here is meant the motion of the string from the extreme position on one side of the straight line which it forms when at rest to the extreme position on the other. We shall use the word *vibration* to denote the motion from one extreme position back again to the same position, so that a vibration is a double oscillation. There is some variety among writers as to the meaning of the words *oscillation* and *vibration.*

65. The vibrations of the string here considered are what are called *transverse* vibrations; each particle of the string moves forwards and backwards nearly in a straight line at *right angles to the length* of the string. It is possible to produce in a string what are called *longitudinal* vibrations, where each particle of the string moves forwards and backwards *in the line* of the string; these are like the vibrations of air in a tube which produce sound. Longitudinal vibrations may be produced in a string by rubbing the string in the direction of its length with a piece of cloth sprinkled with resin: we shall not consider such vibrations here.

66. For an example of the Rule in Art. 64 suppose that the tension-length is 200 feet. The product of 32 and 200 is 6400; the square root of this is 80. Hence if the length of the string is 2 feet the time of oscillation is $\frac{2}{80}$ of a second, that is $\frac{1}{40}$ of a second; therefore the time of vibration is $\frac{1}{20}$ of a second, and consequently 20 vibrations will occur in a second.

67. Various inferences may be deduced from the Rule of Art. 64. If the tension be quadrupled the time of vibration is halved; if the tension be made nine fold its

original value the time of vibration becomes a third of its original value; and so on. We may also observe the consequence of making the vibrating string finer or stouter while keeping the same length and tension. Suppose the diameter of the string to be halved; then the area of a section becomes one-fourth of its original value: thus the tension-length is quadrupled, and therefore the time of vibration is halved. The Rule shews that the *shorter*, the *tighter*, the *finer*, and the *lighter* in weight the string is, the briefer will be the time of vibration, and consequently, as we shall presently see, the higher the note produced.

68. The laws of the transversal vibrations of stretched strings are all involved in the Rule given in Art. 64; but those who are familiar with technical language may perhaps find it convenient to have these laws explicitly stated. The *rate* of vibration, that is the number of vibrations in a given interval, is directly proportional to the square root of the tension, inversely proportional to the length of the string, inversely proportional to the diameter of the string, and inversely proportional to the square root of the density of the string. These statements follow from the fact that the number of vibrations in a given interval is obtained by dividing the number of seconds in that interval by the time of a vibration in seconds.

69. The amount of motion which a fine string can communicate directly to the air is too small to produce a very vigorous sound; so that for musical instruments the assistance of a *sound-board* is secured. For instance, in a violin the vibrations of the strings are conveyed to the wood of the instrument, and thence to the air; the excellence of the violin depends on the sonorous character of the wood, and it is found that this improves by age, and by the constant use of the instrument.

70. The Rule of Art. 64 can be experimentally verified by means of an instrument called the *Sonometer;* this is essentially a wire of string stretched over a *sounding-box*, which is of the nature of the body of a violin. The tension may be changed at pleasure by changing the stretching weight. The effective length may be changed at

pleasure by putting a bridge, like that of a violin, between the ends of the wire: for then the wire is practically separated into two shorter wires, and either of them singly can be used. The Rule of Art. 64 can also be well illustrated by playing on a violin; the length of a string can be altered by fingering; the tension is increased by using the screws provided for tightening the strings. Also the consequences which result from changing the thickness or the density of a string, while the length remains the same, can be traced by the aid of the four different strings used in a violin.

71. We can now explain the origin of two of the three characteristics mentioned in Art. 63; leaving the last for consideration in the next Chapter.

72. The *intensity* of sound depends on the distance through which each particle of air moves in its vibration. The distance between the two extreme points of its path is called the *amplitude;* and it is found that the intensity of sound is proportional to the *square* of the amplitude. Thus if the amplitude is doubled the intensity is quadrupled; if the amplitude is tripled the intensity becomes nine times as great, and so on. This supposes that the hearer remains at the same distance.

73. The *pitch* of a sound depends on the time of vibration; or, to express the same thing in other words, it depends on the number of vibrations in a second; the greater the number of vibrations in a second the higher the note is. The note which is called *middle C* on a piano-forte corresponds to 264 vibrations in a second. The lowest note heard in an orchestra is one in which there are about 41 vibrations in a second. Modern piano-fortes range from a note with about 27 vibrations in a second to a note with about 3520. A good ear can distinguish between notes which differ but little as to the number of vibrations in a second. It is said that trained violinists can distinguish about seven hundred sounds lying between one note and another which corresponds to twice the number of vibrations. Thus a note differing from *middle C* to the extent of less than half a vibration in a second would be distinguishable from *middle C.*

74. Many persons when near a railway must have noticed that if a train passes while the driver is sounding his whistle the pitch of the note *rises* as the train approaches, and *falls* as the train withdraws. In fact if the source of sound and the ear approach each other, by the motion of one or both, the waves fall on the ear with increased frequency; and the contrary if the source of sound and the ear separate from each other. It is found that a speed of about 40 miles an hour will sharpen the note of the whistle of an approaching railway train by what is called a *semitone;* and flatten the note as much when the train is receding.

75. Instruments are made by the aid of which a note of a definite pitch can be obtained accurately. The *tuning-fork* is an example; a piece of steel is bent so as to form two prongs, and is adjusted so that when struck it will vibrate a certain number of times in a second. Thus a tuning-fork may be made to vibrate 440 times in a second, giving a note which corresponds to the *A* which follows the *middle C* of a piano-forte. Or it may be made to vibrate 528 times in a second, giving a note which corresponds to the *C* that follows this *A*. The same number of vibrations is made by a tuning-fork whether it vibrates feebly or strongly.

76. The *Siren* is an instrument by which we can produce any assigned note. The wind of a bellows issues through a small tube; a disc is made to revolve very near to the tube, and so in fact to close it except that through a hole or holes in the disc the air can escape when the hole is just over the end of the tube. It is obvious that by making the disc revolve at a certain uniform rate the emissions of the air will occur with regularity and as frequently as we please. The sound produced is clear and sweet like the human voice. The siren will sound under water, if water be forced through it instead of air, and this gave rise to its name. A *steam-siren* is found to be an advantageous instrument in connection with the object mentioned in Art. 20. The construction of one used by Professor Tyndall in the investigations noticed in Art. 21 is thus described. "A boiler had its steam raised to a

pressure of 70 pounds to the square inch; on opening a valve this steam would issue forcibly in a continuous stream, and the sole function of the syren was to convert this stream into a series of separate strong puffs. This was done by causing a disk with 12 radial slits to rotate behind a fixed disk with the same number of slits. When the slits coincided a puff escaped; when they did not coincide the outflow of steam was interrupted. Each puff of steam at this high pressure generated a sonorous wave of great intensity; the successive waves linking themselves together to a musical sound so intense as to be best described as a continuous explosion." It was found that taking the fluctuations of the atmosphere into account the best development of sound for the purpose of a signal was obtained by making the siren perform from 2000 to 2400 revolutions in a minute; so that from 400 to 480 waves were produced in one second.

VIII. QUALITY.

77. The origin of that characteristic of a musical sound which is called its quality, is the most difficult to explain of the three which are mentioned in Art. 63. Although the nature of the explanation had been previously conjectured by mathematicians, it has only recently been completely worked out by Helmholtz; and it is justly regarded as one of the most important contributions ever made to the science of Acoustics.

78. Suppose that a stretched string is vibrating, and that the vibrations are maintained by the use of a fiddle bow, or in any other way. It is possible to touch the string lightly at a certain point, so as to reduce that point to rest, while yet the string continues vibrating. The string resolves itself, as it were, into shorter strings, so as to bring one point of division at the stopped point. Thus if the middle point be stopped the string vibrates as if it were made up of two equal strings. The time of vibration is thus reduced to half what it was before, so that there are now twice as many vibrations as before in one second. The note obtained when the string vibrates as a whole is

called the *fundamental* note; the note obtained when the string vibrates as if it were divided into two equal strings is called the *octave* of the fundamental note. In like manner a stretched string might, as it were, resolve itself into *three* equal strings, and then there would be *three* times as many vibrations in a second as for the fundamental note; or the stretched string might, as it were, resolve itself into *four* equal strings, and then there would be four times as many vibrations as for the fundamental note; and so on.

79. When a string in vibration resolves itself, as it were, into separate portions, the points of rest are called *nodes;* and the intermediate portion between two successive nodes is called a *ventral segment.*

80. In the next place theory shews that two or more modes of vibration may *exist together*. For instance the different particles of a string may have given to them simultaneously two sets of velocities, one set by virtue of which the string would vibrate as a whole, and another set by virtue of which it would vibrate as if resolved into two equal segments. This may be somewhat difficult for a beginner to understand completely; but it is fully made out by theory and confirmed by experiments. It has long been known that besides the fundamental note of a string an experienced ear could detect other notes, as for instance the *octave,* and even some of those which correspond to still more rapid vibrations, as mentioned in Art. 78. In bells and other sounding bodies these *harmonic sounds* as they are called are detected more easily than in strings.

81. That a string can thus vibrate, according to two modes at once, may be rendered visibly evident. Suppose a string vibrating in such a manner that its fundamental note is the distinctly predominant sound. Touch the string lightly at its middle point, that is at the point of greatest excursion of the string from its mean position; then the vibration of the string as a whole, which gives rise to the fundamental note, is destroyed. But if the vibration corresponding to the octave above has also coexisted with that of the string as a whole, then since the middle point of the string is a *node* with respect to this system, the corresponding vibration is not in any way

checked, and therefore will continue. This is frequently the case; the string is observed to continue in motion still; and if a small piece of paper be put on the string at the middle point it will remain there, while if it be put on the string at any other point it will be thrown off.

82. When a string is thus put into vibration, it depends on various circumstances how many of the harmonic sounds are produced at the same time. One simple fact may be noticed. Suppose the string to be set in vibration by being struck or pulled aside at a certain point; then no harmonic can be produced which would require that point to be a node. For instance, if the string is thus treated at its *middle* point, the *first* harmonic cannot be produced; for that requires a node at the middle point. If the string is thus treated at a distance of a *third* from one end, the *second* harmonic cannot be produced; for that requires a node at the specified point. And so on. Makers of pianofortes had found that the most pleasing effects are produced on the middle strings of the instrument, when the wire is struck by the hammer somewhere between one-seventh and one-ninth of the length from one end; and Helmholtz has accounted by theory for this fact.

83. We can now give the explanation of the *quality* of a note, according to Helmholtz. What is called a *note* of a musical instrument is in general a *mixed* sound. In this mixture we have strongly predominant the sound corresponding to the number of vibrations which the note is held to represent. But mixed with this we may have one, two, or more, of the harmonics; and each of the harmonics is susceptible of various degrees of intensity compared with that of the fundamental sound. Hence we may have the fundamental sound modified in an enormous number of ways, giving rise to all the varieties of quality. For instance, suppose we consider the fundamental sound and five harmonics. First, if the fundamental sound be combined with *one* harmonic we can thus produce *five* different qualities, even without any regard to variations of intensity: if each harmonic be supposed susceptible of two degrees of intensity we may produce ten different qualities. Next, the fundamental sound may be combined with any *pair* of harmonics; and as *ten* different

pairs can be formed we may thus produce *ten* different qualities, even without any regard to variations of intensity: and, as before, by allowing for variations of intensity the number of different qualities becomes largely increased. Again, we may combine the fundamental sound with any *three* of the harmonics; and so on. It is said that in one musical note there may enter as many as 15 or 20 of the harmonics: in notes formed by the violin the first 8 harmonics occur.

84. "The quality of pianoforte notes varies greatly in different parts of the scale. In the lower and middle region it is full and rich, the first six partial-tones being audibly present, though 4, 5, 6 are much weaker than 1, 2, 3. Towards the upper part of the instrument the higher partial-tones disappear, until in the uppermost octave the notes are actually simple-tones, which accounts for their tame and uninteresting character." Sedley Taylor on *Sound and Music.*

85. The sound of a tuning-fork when mounted on a sounding-box is very nearly an unmixed note. Let us then take a series of six or eight tuning-forks, such as will yield a certain fundamental sound and its harmonics. Then sound that which gives the lowest note, simultaneously with one or more of its harmonics; and give to the harmonics successively various degrees of intensity: we can thus manufacture the quality which belongs to the fundamental note on any instrument we please. This imitation of any proposed quality is found to be very successful.

86. We must now advert to the nomenclature of the subject. The word *note* had always been applied to a sound corresponding to a definite number of vibrations; since it has been made out that a note produced by an instrument is in general a mixed sound, it has been proposed that in this case instead of note the word *clang* should be used, which corresponds to the German *klang.* For quality the French use the word *timbre*, which signifies literally a *stamp;* the Germans use the word *klangfarbe* which signifies *sound-tint.* Instead of *quality* it has been proposed to use *character*, or *clangtint* in imitation of the German word. The *harmonics* are sometimes called *partial-tones*, and sometimes *over-tones.* See Professor Tyndall on *Sound.*

IX. NOTES FROM TUBES.

87. Musical sounds can be produced by the vibrations of air in tubes; the theory of these sounds is like that of the sounds produced by the vibrations of strings. Suppose a tube to be closed at one end and open at the other; let a disturbance of the air be excited at the open end, consisting of a condensation and a rarefaction, each occupying the same time, namely the time which sound would take to travel twice the length of the tube, so that the two together occupy the time sound takes to travel four times the length of the tube. Let such a disturbance be maintained at the open end; then the air in the tube is found to continue in regular vibration, and this gives rise to a musical sound. The closed end of the tube is in fact a *node* in the sense of Art. 79; and the open end corresponds to a point midway between two consecutive nodes. The state of vibration is maintained by the continual production of waves at the open end, and the continual reflection of them at the closed end.

88. There are various ways of producing the disturbance at the open end of the tube which the preceding Article supposes. One way is by blowing into, or rather over a tube, as in the case of a flute. Another way is by introducing a current of air through an opening of peculiar construction called a *reed*, provided with a *tongue* or flexible plate of metal which is nearly of the same size as the opening; the tongue is alternately forced away from the opening, and then brought back to the opening by its own elasticity, so that there is a regular succession of impulses such as the case requires. There are two kinds of reeds, called *free* reeds and *beating* reeds. In the former the tongue moves to and fro through the opening, just grazing its edges; in the latter the tongue is longer than the opening, and strikes against it at one end of each vibration.

89. The note which is produced in this way from a tube can be compared by the ear with that produced by some standard instrument, as the *siren;* and hence the number of vibrations performed in a second can be determined, and so the time of a vibration. Thus we could

deduce the velocity of sound in air, from the principle that in the time of one vibration sound would traverse four times the length of the tube. Practically we should not use this method for determining the velocity of sound in air, because that has been found accurately by other methods; but it has been applied to find the velocity of sound in gases: we have only to fill the tube with the gas about which we are concerned, and determine by the aid of the siren the note produced.

90. Next, suppose we take a tube open at *both* ends. Let a regular disturbance be maintained at one end such that the time occupied in the condensation and rarefaction is the time in which sound can traverse *twice* the length of the tube. Then it is found that the air in the tube will continue in vibration. If we suppose the tube to be of the same length as that which was stopped at one end, then as the vibration is performed in half the time the note is an *octave* above the former. The state of vibration is maintained by the continual production of waves at one open end, and the continual reflection of them at the other. The point midway between the two ends is found to be a node.

91. Finally, suppose a tube stopped at both ends. Then theory shews that vibrations may be maintained which are performed in the time in which sound can traverse *twice* the length of the tube. Thus the note is the same as for the tube open at both ends. The state of vibration when once produced will be maintained for a brief period by perpetual reflections at the two ends, but will pass away unless means are found to support it perpetually. The two ends of the tube are nodes.

92. The notes which may be thus obtained from tubes are called the *fundamental* notes, being the lowest which the tubes will furnish. But we have seen that a stretched string will vibrate as if it were fixed at various intermediate points as well as at the ends; and in like manner a tube may resolve itself, as it were, into shorter tubes. This indeed we may say that the tube open at both ends does in order to produce its fundamental sound; it resolves itself into two tubes of the nature of a tube open at one end: for at the middle of the tube open at both

ends we have a node, so that each half resembles a tube closed at one end and open at the other. We will proceed to consider separately the *harmonics* or *overtones* which can be produced from tubes.

93. The simplest case is that of the tube closed at *both ends*. The tube can, as it were, resolve itself into 2, 3, 4, 5,...or any number of equal parts; and thus the complete series of harmonics can be obtained which a stretched string will furnish. Each of the parts of the tube resembles the whole, having a node at each end, so that it may be considered as a tube closed at both ends.

94. Next take the tube closed at *one* end. The simplest mode of resolution is into two parts, one extending from the open end to *one-third* of the length, and the other occupying the remaining two-thirds of the length. The former portion constitutes a tube open at one end, and as its length is one-third of the length of the original tube, the time of vibration is *one-third* of the time for the fundamental note. The latter portion constitutes a tube closed at both ends. The next mode of resolution is into three portions, one extending from the open end to *one-fifth* of the length, and the other two each occupying two-fifths of the length. The first portion constitutes a tube open at one end; the other two portions constitute tubes closed at both ends: the time of vibration is *one-fifth* of the time for the fundamental note. Proceeding in this way we see that we cannot get *all* the harmonics, but only those in which the *rate* of vibration is an *odd* multiple of the rate for the fundamental note.

95. Finally, take the tube open at both ends. The simplest mode of resolution may be said to be that which actually occurs when the fundamental note is sounded; there is then a node at the middle of the tube, and the tube is practically resolved into two, each of half the length of the real tube, and each open at one end. The next mode of resolution is when there are *two* nodes, one at the distance of a quarter of the length from each open end; thus we obtain, as it were, two tubes, each open at one end and equal in length to a quarter of the original tube, together with a tube closed at both ends equal in length to half the original tube: the time of vibration is

then half of that for the fundamental note of the original tube open at both ends. The next mode of resolution is when there are *three* nodes, one at the distance of a sixth of the length from each open end, and one at the middle; thus we obtain, as it were, two tubes each open at one end, together with two closed at both ends: the time of vibration is then a third of that for the fundamental note of the original tube open at both ends. Proceeding in this way we see that we can get all the harmonics corresponding to the fundamental note.

96. "Precisely, too, as in the vibrations of strings, any number of these modes of vibration may go on simultaneously. Such combined modes may be produced by an expert flute-player, by a nice adjustment of the force of his breath; at least the octave of any note may be obtained without difficulty, and distinctly heard with the fundamental tone." Herschel on *Sound.*

97. The laws which we have stated with respect to the vibration of air in tubes are obtained by theory; they are called *Bernoulli's Laws*, as they were first investigated by Daniel Bernoulli. They are not quite exactly confirmed by experiment; for the sounds obtained in practice are deeper than those suggested by theory. The existence of nodes may be verified by inserting a moveable piston in the tube at different points; the sound is not altered when the piston is at the place of a node. Or a tube may be constructed of slight thickness so that its parts will vibrate with the internal column of air; if the tube be sprinkled with fine sand and placed horizontally the sand will leave the ventral segments and accumulate on the nodal lines: thus the existence of the nodes is experimentally verified.

X. RESONANCE.

98. The term *resonance* is applied to the cases in which one sounding body excites another to give forth sound: the following are examples. Let a string be stretched and its ends fastened; then if it be pulled a little aside it will give forth a note. But instead of pulling

the string aside let that note be played on some instrument, or sung, near it; then the string will vibrate in unison as if it had itself been pulled or struck. The vibrations produced in the air by playing or singing the note are in this case communicated to the stretched string. Similarly if one tuning-fork be set in vibration, and held near a second tuning-fork similar to the former, the latter will be set in vibration and sound the same note. If we make a tuning-fork vibrate near a tube the air in the tube is set in motion, and we have a powerful resonance, provided the length of the tube is properly adjusted so as to correspond to the same note as the tuning-fork. Thus for a tuning-fork which makes 1056 vibrations in a second, the tube, if open at both ends, should be slightly more than 6 inches long; for then the time which sound takes in traversing twice the length of the tube is equal to the time of a vibration, as it should be by Art. 90. If the tube is closed at one end, but of the same length, the tuning-fork must be an octave lower to suit it; that is the tuning-fork must make 528 vibrations in a second: see Art. 87. Another example of resonance is furnished by the known fact that if we sing near the aperture of a wide-mouthed vessel, some special note, corresponding to that which the vessel is capable of producing, will be augmented, and sometimes to a great degree.

99. The *resonance box* consists of a tuning-fork screwed into one side of an empty wooden box, which is so adjusted that the air-column in the box will be resonant to the note of the tuning-fork; thus the sound of the tuning-fork is much intensified.

100. The *resonator* is an instrument invented by Helmholtz. It consists of a hollow ball of brass, with two apertures at the opposite ends of a diameter. The larger aperture is for the entrance of sound vibrations, and the ear is applied to the smaller aperture. If a sound composed of various notes is produced the resonator selects that which is in unison with itself, and by reproducing it very powerfully renders it sensible to the ear.

101. It must be observed that what is gained with respect to the *intensity* of sound by the aid of a resonant

body is lost with respect to *duration.* Thus a tuning-fork when placed on a resonance box loses its note much sooner than if left to itself. It is obvious that a resonant body is useful or necessary for those musical instruments which produce notes by the vibrations of strings; while it is not necessary for those which are called *wind instruments*, which put a considerable body of air in motion in a direct manner. Resonance does not alter the pitch of a pure tone, but it has an important influence on the mixed sound of which a note usually consists; for resonance does not strengthen the various partial tones in the same degree: in fact it may strengthen only one or some of them, excluding all the others.

102. A tuning-fork is used to enable a singer to obtain the exact pitch of a note which is to be reproduced. It is however well known that the number of vibrations corresponding to a specified name of note has varied considerably in modern musical history. The number of vibrations corresponding to a certain note called C was about 466, according to Dr Smith in 1720; the number has been gradually increasing since: Handel's value in 1740 was nearly 500, and at the Italian Opera in 1859 it was 546. The Society of Arts recommended 528 for permanent adoption, and tuning-forks are sold which are guaranteed to give that number. There seems however to be some uncertainty as to the accuracy of tuning-forks; for an eminent writer on music found that a C fork which professed to give 528 vibrations really gave 538, and that an A fork which ought to have given 444 really gave 452. "It seems nearly impossible to prevent the continued rise of pitch. Among other causes the desire of an ambitious singer to exhibit a voice higher in tone than can be exhibited by any other, leads to the occasional raising of the pitch of all instruments: then in a short time music is written for the use of ordinary singers with reference to that raised pitch, and then the rise is established for ever." Airy on *Sound.*

XI. INTERFERENCE. BEATS.

103. Suppose that one unmixed note is produced by an instrument, and that we simultaneously produce a second of the same character; it is not necessarily an intensified sound which will be heard; on the contrary the sound *may* be extinguished. Let a tuning-fork be set in vibration and held upright at about a foot from the ear; then turn the tuning-fork slowly round and it will be found that the sound is alternately clearly audible and very faint. The situations of the ear favourable to audibility are those in planes parallel or perpendicular to the flat faces of the tuning-fork; and the unfavourable situations are those in planes midway between the former. Thus in turning the tuning-fork once round, the ear is four times in a position in which the sound is well heard, and four times in a position in which the sound is very faint. Each prong of the tuning-fork conveys to the air vibrations corresponding to the same definite note; and it may happen that at a certain point the disturbance proceeding from one prong would give to the particles of air a motion almost exactly contrary to that proceeding from the other prong; thus practically the air is brought nearly to rest at the point, and the sound becomes very faint.

104. Let a tuning-fork be set in vibration and placed in front of a tube, which then divides into two channels afterwards again united. Thus, denoting by A the place where the tuning-fork is placed, one branch of the tube may pass along a curved path ABC to a point D, and the other branch along a curved path AEF to the same point D. Then if the difference of the lengths of the two paths from A to D is zero, or any multiple of the length of the wave which corresponds to the sound, the note is well heard at D. But if the difference is any odd multiple of *half* the length of the wave the note is extinguished at D.

105. Thus when two waves of sound arrive at the same point by different courses the resulting effect may

vary between two extremes; the waves may be quite in concert and so, as it were, double the motion of any particle which each would produce singly, or they may be quite in opposition so as just to destroy the motion. The name *interference* is given to the effects produced by the meeting of two or more waves. Similar effects are seen with respect to waves of water. Thus if two stones are thrown into a pond at different points, each gives rise to a series of circular waves, which may meet at various points. Where the crest of one wave meets the crest of another the water rises to an elevation equal to the sum of the two elevations; and where the trough of one wave meets the trough of another the water sinks to a depth equal to the sum of the two depths. Effects of an intermediate character take place at other points. The same phenomena are seen on a larger scale in connexion with the tides. It may happen that a tide-wave will arrive at a certain spot by two courses of different lengths, so adjusted as to result in leaving the water without any perceptible rise or fall. There is such a spot in the North Sea about midway between Lowestoft and the coast of Holland; its position was discovered from theory by Dr Whewell, and verified from observation by Captain Hewett.

106. Suppose that we have two strings, one vibrating 100 times, and the other vibrating 101 times in a second. Let the strings be placed side by side near the ear, and sounded together. At the beginning the two vibrations combine to produce a vibration which causes double the sound of either. At the fiftieth vibration the effect of one string is just neutralised by that of the other, and there is a momentary silence. At the hundredth vibration there is again a double sound. Hence the impression on the ear is that of an intermittent sound, alternately loud and faint, and this intermittent sound receives the name of *beats*. When two strings are in perfect unison there are no beats; when the strings are nearly but not perfectly in unison the beats are heard, constituting a kind of unpleasant rattle.

107. Beats may also occur when two notes are sounded together, one of which is nearly but not quite the *octave*

of the other. Suppose, for instance, that one string vibrates 50 times in a second, and that another string vibrates 101 times in a second; then the combination of every alternate vibration of the latter string with every vibration of the former string will give rise to beats in the manner already explained. The same thing also occurs whenever the numbers of vibrations of two notes in a second are nearly but not exactly in the proportion of two small numbers. Thus if there are 100 vibrations of one note in a second and 150 vibrations of another there will be no beats; but if there are 100 vibrations of one and 151 vibrations of another there will be beats.

103. "Beats not too fast to be readily counted arise between adjacent notes in the lower octaves of the harmonium, or, still more conspicuously, in those of large organs. They are also frequently to be heard in the sounds of church-bells, or in those emitted by the telegraph-wires when vibrating powerfully in a strong wind. In order to observe them in the last instance, it is best to press one ear against a telegraph-post and close the other: the beats then come out with remarkable distinctness." Sedley Taylor on *Sound and Music.*

109. Beats may occur so rapidly that they can no longer be separately perceived; but still they produce a harsh and grating effect on the ear; the sensation is in fact identical with that which we commonly call *discord.*

110. Suppose that we have two notes simultaneously produced, and that *two* vibrations in the one are performed in exactly the same time as *three* vibrations in the other. Then every second vibration of the lower note coincides exactly with every third vibration of the higher note. The ear is sufficiently acute to take notice of this conjunction; so that besides the note actually sounded the ear perceives one which corresponds to the conjunction, and is therefore an octave lower than the lower of the two notes which are simultaneously produced. Such sounds have been called *resultant-tones*, or *combination-tones*, or *sub-harmonics;* they have also been called *Tartini's tones*, from the name of an Italian violinist who drew attention to them.

111. The older writers connected these *resultant-tones* with *beats*, and so they were often called *Tartini's beats*. But Helmholtz, who has recently investigated the subject, has come to the conclusion that they are quite different from beats. He divides resultant-tones into two classes, namely, *difference-tones* and *summation-tones*. Only the former had been previously noticed; the number of vibrations in a second is the *difference* of the numbers of vibrations in the two combined notes. The latter were first discovered by theory and their existence confirmed by experiment; the number of vibrations in a second is the *sum* of the numbers of vibrations in the two combined notes. Thus if for one note there are 200 vibrations in a second, and for another 300 vibrations in a second, the *difference-tone* will have 100 vibrations in a second, and the *summation-tone* 500.

XII. MUSIC.

112. In this Chapter we shall be concerned solely with the *pitch* of notes, and it will be necessary to have some short mode of speaking distinctly on the subject. The pitch of a note depends on the number of vibrations which the particles of air make in a given time, say one second. If for a certain note there are 100 vibrations in a second we shall say that 100 is the *vibration number* of that note; if for another note there are 200 vibrations in a second we shall say that 200 is the *vibration number* of that note. So when we say that the *vibration numbers* of two notes *are as* 1 *and* 2, or *are in the proportion of* 1 *and* 2, we mean that in the first note 1, or 10, or 100 vibrations are made respectively in the same time as 2, or 20, or 200 vibrations are made in the second. So also we may speak of two notes whose vibration numbers *are as* 1 and $\frac{3}{2}$: here 100 vibrations in the first note are made in the same time as 150 in the second.

113. When two notes of the same pitch are sounded simultaneously they blend together and affect the ear with a sensation of accordance which is called a *unison*. When two notes, not of the same pitch, are sounded

simultaneously, the ear is able to perceive both notes, and to attend to either of them singly. Moreover most persons who have been brought up in civilized countries experience a feeling of satisfaction or of the reverse; the combined sound that is pleasing to one person is also pleasing to others, and that which is displeasing to one is also displeasing to others. There is however a wide difference between people as to the *degree* of sensitiveness to these pleasing or displeasing sounds; and in common language some are said to possess a good ear for music and some to have no ear for music.

114. Now it is found that the union of two notes is pleasing to the ear when the vibration numbers are in a *proportion* which can be expressed by *very low* numbers, as 1 to 2, or 1 to 3, or 1 to 4, or 2 to 3. But the union of two notes is displeasing to the ear when the vibration numbers are in a *proportion* which requires high numbers to express it, as 8 to 15. The absolute numbers of the vibrations are not of any consequence; the *proportion* is the essential thing. These facts are the foundation of all harmony. The union of two notes when the sound is pleasing is called a *concord*, and when the sound is displeasing is called a *discord*. A combination of notes of different pitch is called a *chord;* if *two* notes are combined it is called a *binary* chord; if *three* notes are combined it is called for brevity a *triad*. In a triad there are three ways of forming a pair of notes, and each pair must form a concord, if the effect of the triad is to be pleasing.

115. The most satisfactory concord is the *unison*, in which the vibration numbers are as 1 and 1, that is are equal. Next to this is the case in which the vibration numbers are as 1 and 2; the word *octave* is used for this case: the higher note is called the octave of the lower, or an octave is said to be the interval between the notes. Sir J. Herschel says: "The octave approaches in its character to a unison, and indeed two notes so related when played together can hardly be separted in idea; and when singly, appear rather as the same note differently modified, than as independent Sounds." Again, the union of two notes whose vibration numbers are as 1

and 4 produces a very agreeable and perfect concord; this is called the *octave of the octave*, or the *fifteenth*. Also the result is pleasing when there is a union of notes whose vibration numbers are as 1 and 8; and so on.

116. The union of notes whose vibration numbers are as 1 and 3 is pleasing; it is called the *twelfth*. Still more pleasing is the union of two notes whose vibration numbers are as 2 and 3, that is as 1 and $\frac{3}{2}$; this is called the *fifth*. The union of two notes whose vibration numbers are as 1 and $\frac{5}{4}$ is called the *major third;* of two whose vibration numbers are as 1 and $\frac{4}{3}$ is called the *fourth;* and of two whose vibration numbers are as 1 and $\frac{5}{3}$ is called the *major sixth:* all these are concords. We may also mention the proportions of 1 and $\frac{6}{5}$, of 1 and $\frac{8}{5}$, and of 1 and $\frac{16}{9}$; these are called respectively the *minor third*, the *minor sixth*, and the *minor seventh:* the first is a concord and the others discords.

117. As an example of a triad take three notes whose vibration numbers are as 1, $\frac{4}{3}$, and $\frac{5}{3}$. Here, by Art. 116 the first and second notes form a concord, and so also do the first and third. Take then the second and third notes; the vibration numbers here are in the proportion of $\frac{4}{3}$ and $\frac{5}{3}$, that is in the proportion of 4 and 5; and this is a concord by Art. 116. Hence the effect of the whole triad will be pleasing.

118. The *natural or diatonic scale* consists of a series of notes which may be represented thus:

1st	2nd	3rd	4th	5th	6th	7th	8th
1,	$\frac{9}{8}$,	$\frac{5}{4}$,	$\frac{4}{3}$,	$\frac{3}{2}$,	$\frac{5}{3}$,	$\frac{15}{8}$,	2,
24,	27,	30,	32,	36,	40,	45,	48.

In the first row the situation of the notes in order is given. In the second row we have the proportions of their vibration numbers, taking unity for the lowest note, so that fractions are used to express the other notes. In the third row the same proportions are expressed by whole numbers. There are said to be seven distinct notes in this series, for the last note being the octave of the first is regarded as a mere repetition of it. When these notes are sounded in succession, either upwards or downwards, the effect is universally allowed to be pleasing.

119. The series of notes may be continued indefinitely by adding above it the octave of every note, and then again the octave of every new note, and so on; and the series may be continued in a similar way in the direction below. But no human ear can appreciate the notes indefinitely upwards or downwards. When the vibrations are less frequent than 16 in a second the ear does not receive the sensation of a continuous sound, but that of a succession of sounds which may be counted. On the other hand when the frequency of the vibrations exceeds a certain limit all sense of pitch is lost, and a shrill squeak is heard. It is also remarkable that some persons who are quite free from deafness as to ordinary sounds, are insensible to certain sounds which affect other people to whom they seem peculiarly acute. Thus one person's range of audible notes may extend two octaves higher than the range of another. Sir J. Herschel says: "The whole range of human hearing comprised between the lowest notes of the organ and the highest known cry of insects, seems to include about nine octaves."

120. It must however be observed that very *high* notes are also in general very *feeble*. We have no mechanical means of producing heavy blows at the rate of an enormous number per second. Thus the want of intensity rather than the extreme pitch may render very high notes inaudible to most ears. The limits of perceptible sounds are now known to be wider than those assigned by Sir J. Herschel. According to Helmholtz the lowest limit corresponds to 16 vibrations and the highest to 38000 vibrations in a second.

121. The notes in the greater part of the tunes sung by persons not acquainted with technical music are included within the compass of an octave. The average compass of the human voice is about two octaves. The lowest note of a bass voice corresponds to about 90 vibrations per second, and the highest of a soprano to about 800; but in exceptional cases these limits may be as low as 50 vibrations, and as high as 1500.

122. *Harmony* means the agreeable *consonance* of simultaneous notes, and this we have already noticed. A few words from Sir G. B. Airy may be quoted respecting *Melody* or the agreeable *succession* of notes. "This must depend on some peculiar properties of our nervous physiology. It would almost seem that there is something in our Sensorium which is put into vibration by vibrations of air, and that these vibrations subsist after the cessation of the atmospheric cause, through a time sufficiently long to be mingled with the vibrations produced by the next atmospheric disturbance, and thus to produce the effects of genuine Harmony. The expression of 'sound continuing to ring in our ears' may not be so purely poetical as is usually thought."

XIII. INTERVAL. TEMPERAMENT.

123. In Art. 118 we have the following proportions for the vibration numbers of the notes in the diatonic scale:

1st	2nd	3rd	4th	5th	6th	7th	8th
1,	$\frac{9}{8}$,	$\frac{5}{4}$,	$\frac{4}{3}$,	$\frac{3}{2}$,	$\frac{5}{3}$,	$\frac{15}{8}$,	2.

Thus under the fifth note we have $\frac{3}{2}$; this tells us that 150 vibrations are made for the fifth note in the same time as 100 vibrations are made for the first. This number $\frac{3}{2}$ is said to indicate the *interval* between the two notes. In like manner $\frac{5}{3}$ indicates the interval between the first note and the sixth. Similarly the interval between any two notes is expressed by the proportion of their vibration numbers: thus the interval between the third note and the fourth is the proportion of $\frac{4}{3}$ to $\frac{5}{4}$, that is of $\frac{16}{12}$ to $\frac{15}{12}$, that is of 16 to 15; this is written as $\frac{16}{15}$. Thus from the numbers given above we obtain the following to express the interval between each note of the diatonic scale and *that immediately before it:*

1st	2nd	3rd	4th	5th	6th	7th	8th
	$\frac{9}{8}$,	$\frac{10}{9}$,	$\frac{16}{15}$,	$\frac{9}{8}$,	$\frac{10}{9}$,	$\frac{9}{8}$,	$\frac{16}{15}$.

124. In the series just given it will be perceived that only three different intervals are obtained, namely $\frac{9}{8}$, $\frac{10}{9}$, and $\frac{16}{15}$; of these $\frac{9}{8}$ is slightly greater than $\frac{10}{9}$, and $\frac{16}{15}$ is decidedly less than $\frac{10}{9}$. The intervals $\frac{9}{8}$ and $\frac{10}{9}$ are called *Whole Tones*, and $\frac{16}{15}$ is called a *Half Tone* or *Semi Tone*. It will be observed that the word *Tone* thus used to denote an interval has a different sense from that which it elsewhere implies, in which it is nearly identical with *note*. As some aid to the reader we shall always employ a capital letter when the word is used to denote an *interval*.

125. We see from Art. 123 that the interval corresponding to a fourth is $\frac{4}{3}$, and the interval corresponding to a fifth is $\frac{3}{2}$. Now $\frac{4}{3} \times \frac{3}{2} = 2$: this is expressed by saying that the successive intervals of a fourth and a fifth make up an octave. The successive intervals of two Semi Tones do not however make up exactly a Tone; for $\frac{16}{15} \times \frac{16}{15} = \frac{256}{225}$,

which is greater than $\frac{9}{8}$, and of course therefore greater than $\frac{10}{9}$. The proportion of $\frac{256}{225}$ to $\frac{9}{8}$ is the same as that of 2048 to 2025.

126. It is found convenient to enlarge the diatonic scale by inserting *five* other notes; namely one between every consecutive pair of notes, except between the third and fourth and between the seventh and eighth. The scale thus enlarged is called the *chromatic scale;* it consists of twelve notes, not counting the octave. The intervals between the successive notes of the *diatonic* scale are, as we have seen, not all equal; nor will the intervals between the successive notes of the *chromatic* scale be all equal, though the intervals here may all be called roughly Semi Tones. Musicians are not agreed as to the most advantageous method of constructing the chromatic scale; some have proposed to make all the twelve intervals equal, and for this purpose they alter slightly some of the notes of the diatonic scale. Such arrangements are called *systems of temperament,* but the discussion of them belongs rather to practical music than to natural philosophy.

127. A simple example will shew the necessity of some system of temperament. The note called the fifth, corresponding to $\frac{3}{2}$ in the diatonic scale, occupies the eighth place in the chromatic scale, so that there are seven intervals between it and the first note. If the chromatic scale were perfect twelve fifths should correspond exactly to seven octaves. This however is not the case. For twelve fifths is denoted by what is called the twelfth power of $\frac{3}{2}$, that is by the product of twelve numbers each equal to $\frac{3}{2}$; this gives $\frac{531441}{4096}$: while seven octaves is denoted by the seventh power of 2, that is by $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$, that is by 128, that is by $\frac{524288}{4096}$. These numbers are not equal;

the proportion of the former to the latter is $\frac{531441}{524288}$, that is about $\frac{129\cdot74}{128}$.

128. The system of temperament which is now almost universally adopted is that mentioned in Art. 126, which is called the system of *equal temperament*. To determine in this case the interval between successive notes we must seek for the number which has 2 for its twelfth power; such a number cannot be found exactly, but mathematicians can get a solution as near to exactness as they please: to five places of decimals the number is 1·05946. For an approximation, which is a little too great, we may take 1·06, that is $\frac{106}{100}$. Thus, in the chromatic scale of equal temperament, 100 vibrations are made for any note in about the same time as 106 vibrations for the next note. Taking then for the lowest note that of which the vibration number is 264, the following are the vibration numbers, to the nearest integer, for the thirteen notes between this and the octave, both inclusive; for the sake of comparison the corresponding numbers for the eight notes of the diatonic scale are placed below:

264,	280,	296,	314,	333,	352,	373,	395,	419,	444,	470,	498,	528,
264,		297,		330,	352,		396,		440		495,	528.

129. The pianoforte being the musical instrument from which our ideas of pitch are usually obtained, persons are liable to the mistake of supposing that pitch changes *discontinuously*, by a series of twelve jumps as it were, between a note and its octave. But this is an obvious error; a better instrument, as a violin, can produce notes varying by almost insensible gradations of pitch. By an instrument like the Siren, also, the continuity of pitch is fully manifested. Those who wish to prosecute the study of Music should consult the excellent work on *Sound and Music* by Mr Sedley Taylor.

XIV. SONOROUS VIBRATIONS OF VARIOUS BODIES.

130. The vibrations of any body will produce sound if they are of sufficient force to be communicated to the ear through the air or any other medium. Experiments have been made on the vibrations of rods, and of narrow plates of wood, glass, and steel.

131. Rods and narrow plates may vibrate either transversely or longitudinally. The former vibrations resemble those which we have already considered in the case of strings. It is found with respect to these transversal vibrations that in rods of the same kind, and also in narrow plates of the same kind, the number of vibrations in a given time varies *directly as the thickness and inversely as the square of the length.*

132. Examples of such transversal vibrations are supplied by a tuning-fork, and by a musical box. In a musical box small plates of steel of various sizes are fixed on a rod, so as to resemble the teeth of a comb. A cylinder having its axis parallel to the rod is placed near the rod and turned round on its axis. The surface of the cylinder is studded with hard prominences arranged in a suitable manner; and as the cylinder is turned these strike the steel plates and set them in vibration. Thus a tune can be played, which corresponds to the arrangement of the prominences on the cylinder.

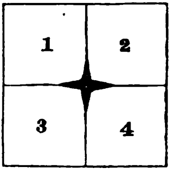
133. We pass to the *longitudinal* vibrations of rods and plates. If a straight rod of glass or metal be struck at the end in the direction of its length it will yield a musical sound, which if the rod be only of moderate length will be of very high pitch. The vibration is analogous to that of air in a tube, but the pitch is higher because sound in general is propagated with much greater speed in solids than in air. For instance the velocity of sound in iron is, on the lowest estimate, ten times that in air; thus the note of an iron rod will be the same as that of a tube of air stopped at both ends of less than one-tenth of the

length. It is found with respect to these longitudinal vibrations, that in rods of the same kind the number of vibrations in a given time *varies inversely as the length*, and does not depend on the diameter or the form of the section of the rod.

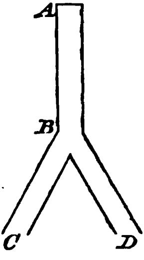
134. The longitudinal vibrations are best produced by rubbing the rods longitudinally; a glass rod is rubbed with a wetted towel, and a rod of wood or metal with a piece of leather on which powdered resin has been strewed. The relative velocities of sound in different substances may thus be determined. For example, it is found that a rod of deal 6 feet long yields the same note as a rod of mahogany 4 feet long; hence we infer that the velocities of sound in deal and mahogany are in the proportion of 6 to 4. A musical instrument has been constructed, something like a harp in appearance, but instead of *strings* which are set in vibration by *pulling*, it has *rods* which are set in vibration by *rubbing*.

135. We may next advert to the transversal vibrations of *plates* of glass or metals, or of *membranes*, as for example of those which form the ends of a drum. A series of very interesting experiments is made by setting a plate of glass in vibration. The plate should be held in a horizontal position by the points of a clamp screw, which are covered with leather to prevent any jar; then the plate may be set in vibration by drawing a fiddle-bow across an edge. Light sand is scattered on the surface, and thus the nature of the vibration is rendered visible; for the sand is thrown into a definite figure, being heaped up at the points where the plate is at rest, and so indicating the *nodal lines*. By varying the position of the point of support, and the position of the point of application of the fiddle-bow, an immense number of interesting figures can be obtained. These have been described by Chladni; and some of his diagrams have been reproduced by Sir J. Herschel, and by other writers on Acoustics. With respect to these vibrations it has been found that in plates of the same kind and shape, adjusted to give the same figure of nodal lines, the number of vibrations in a given time *varies directly as the thickness of the plate and inversely as the surface.*

136. We will take a very simple case of these figures. Suppose a square plate is held by a suitable clamp at the centre, and let the bow be drawn across the edge near a corner, while the middle point of an edge is gently touched so as to keep it at rest. Then the sand is heaped up along two straight lines, parallel to the edges, and passing through the centre of the figure; the plate now gives forth the deepest note of which it is susceptible. Of the four squares into which the figure is divided by the nodal lines it is found that 1 and 4 are always in the same stage of vibration, and that 2 and 3 are always in the same stage of vibration which is contrary to that of 1 and 4; so that when 1 and 4 are *above* their original position 2 and 4 are *below* theirs.



137. A simple experiment illustrating interference may be made with the aid of the glass plate vibrating in the manner just explained. *AB* is a tube which divides at *B* into two branches ending at *C* and *D* respectively. The end *A* is closed with a membrane over which fine sand is scattered. Let the part *AB* be held in a vertical position, so that *C* and *D* may come over the middle points of the squares 1 and 2 respectively of Art. 136; then the sand on the membrane remains still: the waves of air proceeding up the channels *CBA* and *DBA* are of *opposite* character, the condensation in one coinciding with the rarefaction in the other, and so they *neutralise* each other. But suppose *C* and *D* to be placed over the middle points of the squares 1 and 4 respectively of Art. 136; then the waves are of the same character, and strengthen each other, so that the sand is forcibly agitated. In the former case, where the waves neutralise each other, if we stop up *one* of the two channels *C* and *D*, the sand at *A* is



thrown into agitation: that is, by cutting off one out of two sources of sound we, as it were, largely increase the resulting effect.

138. The vibration of a bell when it sounds its deepest note bears some resemblance to that of the glass plate as described in Art. 136. The bell divides itself, as it were, into four parts by lines extending from the mouth to the crown; these four lines are *nodal* lines, and the two pieces adjacent to a nodal line are in *opposite* stages of vibration. It is possible for a person to place himself in such a position that, owing to *interference*, the waves of sound will almost neutralise each other, and thus the noise even of a large bell will seem very moderate: this will be the case if he is just under the centre of the bell, or exactly opposite to a nodal line.

139. The transmission of vibrations is exemplified in a curious manner by an experiment devised by Sir C. Wheatstone: by the aid of four long wooden rods a concert played in a cellar was transferred to an upper room of a house. One of these rods was put in contact with the sounding-board of a pianoforte, another with the bridge of a violin, the third with that of a violoncello, and the last with a clarionet. The four rods were carried from the cellar upwards, and at the other end of each was attached a wooden tray to serve as a resounding board. When the instruments were played in the cellar the rods vibrated in unison, and the resounding boards at their other ends dispersed the music through the upper room: the effect was very striking as the music seemed to issue from these boards without any exciting cause.

140. Professor Tyndall has made some curious experiments with respect to singing flames. When a gas flame is surrounded by a tube the air in the tube is thrown into a state of vibration, in the manner usual for a tube open at both ends; while the vibrations continue the flame goes through a rapid series of extreme diminutions, or even of extinctions, between every two of which the brightness recurs. Flames unprotected by tubes exhibit, under certain circumstances, extreme sensitiveness as to sounds.

For instance a certain flame when undisturbed was 24 inches high; and the slightest tap on a distant anvil reduced the height to 7 inches. The flame was sensibly affected by the tick of a watch held near it, falling at every tick; the twitter of a distant sparrow or the note of a cricket produced the same effect. See Professor Tyndall on *Sound*.

XV. MISCELLANEOUS REMARKS.

141. *The Ear*. The form of the external organ of hearing is obvious from inspection; it is also well known that the only part which is of great use for the conveyance of sound is a certain recess in it, for if this be stuffed with wool the sense of hearing seems almost lost. The skin which forms the boundary of this recess is called the *tympanic membrane;* and behind this membrane is a cavity containing air, which is called the *drum* of the ear. At the other end of the drum is a bony partition, pierced with two holes, one round and the other oval: these holes are covered by membranes. The drum of the ear contains four little bones; the first, called the *hammer*, has its smaller end in contact with the tympanic membrane, and its larger end in contact with the second bone; the second bone, called the *anvil*, has one end in contact with the *hammer*, and the other end in contact with the third bone; this third bone, which is very little and round, is also in contact with the fourth bone; the fourth bone, called the *stirrup*, has its base resting on the membrane of the oval hole above mentioned. Behind the drum is a chamber called the *Labyrinth*, excavated in the bony substance of the skull, and filled with fluid; in this are situated the ends of the nerves which convey the sensation of sound to the brain. If the membrane which bounds the labyrinth is pierced, so that the fluid escapes, incurable deafness is the result. Thus when sound is excited in the air the waves strike on the tympanic membrane and set it in vibration; these vibrations are conveyed through the drum, partly by the medium of the air in it, and partly by the chain of the four small bones; they pass through

the membranes which close the round and oval holes, and so enter the tympanum: from this they are conveyed by the nerves to the brain.

142. But although we have thus a general notion of the way in which we perceive sounds, yet our knowledge of the subject is very vague and imperfect. It seems that the drum with its contents is not indispensably necessary, for it is said that when the tympanum is destroyed, so that the chain of small bones hangs loose, deafness does not follow. Moreover in the labyrinth anatomists have discovered various things of which they can only conjecture the use. Thus there are small hard bodies, called *otoconia* or *otolithes*, which are supposed to vibrate and knock against the extremities of the auditory nerves. There is also a structure called from its discoverer *Corti's fibres;* the part where this is found has been compared to a "keyboard, in function as well as appearance, the fibres of Corti being the keys, and the ends of the nerves representing the strings which the keys strike." Huxley's *Elementary Physiology.*

143. *The Voice.* The human voice is produced by the aid of air which is forced from the lungs up a channel called the *trachea* or *wind-pipe.* Near the top of this channel are two elastic bands like the parchment of a drum slit in the middle; they are called *vocal chords:* the slit between them in adults is about four-fifths of an inch long, and a twelfth of an inch wide. By changes in the tension of the vocal chords the *pitch* of the sound can be changed. The following are stated by Professor Huxley to be the essential conditions of the production of the human voice: "The existence of the so-called *vocal chords.* The parallelism of the edges of these chords, without which they will not vibrate in such a manner as to give out sound. A certain degree of tightness of the vocal chords, without which they will not vibrate quickly enough to produce sound. The passage of a current of air between the parallel edges of the vocal chords sufficiently powerful to set the chords vibrating."

144. Although the essential principles on which Sound depends have been long familiar to men of science, yet a student of the Chapters on the subject here given will

observe that various important parts have been explained only in comparatively recent times. For examples we may notice the account of the action of the wind in changing the intensity of sound, and the explanation of the cause to which the quality of musical notes is due: see Arts. 23 and 83.

145. Moreover there still remain some matters, though in general of subordinate interest, which have not yet been thoroughly settled, perhaps mainly because they have not been sufficiently examined. For example we may refer to the constant failures which occur in the attempts to construct buildings adapted for a large number of listeners: see Art. 47. The theory of the speaking trumpet has not yet been made clear; experience shews that the funnel-shaped portion at the end farthest from the mouth is of great service, but why it should be so has not been ascertained. Various forms have been suggested for ear-trumpets, but no convincing reasons for the choice of a particular form have been assigned. The causes of the remarkable echoes described by travellers have not always been clearly exhibited; though this may be owing to the insufficient accounts of the locality and the circumstances: on the other hand, places seem sometimes suited for the production of echoes, while yet none are heard. Such a well known and familiar instance of the transmission and corroboration of sound as is supplied by the Whispering Gallery of St Paul's Cathedral in London cannot be said to be fully understood.

146. We will finish the subject of Sound by briefly noticing four cases of historical interest in which remarkable effects have been observed.

147. *The Vocal Memnon.* On the banks of the Nile, near Thebes, are two statues in a sitting position, each 47 feet in height above the level of the soil, and extending 7 feet below it. Both the statues represent king Amunoph the Third, who began his reign about 1400 years B.C. Each statue was originally formed from a single block, but one of them, namely that to the east, has been restored in five pieces from the waist upwards. When the Greeks came

to settle in Egypt they found this statue shattered from the waist upwards, probably owing to an earthquake; they called it Memnon. Ear witnesses affirmed that this statue would sometimes in the first hour after sunrise send forth a musical voice, which they likened to the sound of a harp-string when breaking. After the date 194 A.D. there is no record of the sound being heard. The sound is now attributed to the transmission of rarefied air through crevices in the stone; and other examples have been adduced of the action of the morning sun on the air in the hollows of rocks: especially some effects of the same kind were observed by Humboldt among granite rocks on the banks of the Oronooko. The damaged statue was repaired, it is supposed about in the time of the emperor Severus; and it is probable that the crevices were then filled up which had given rise to the sound, so that it was heard no more. *Quarterly Review*, April 1875.

148. *The Ear of Dionysius.* Near Syracuse there is a cavern in a rock, extending in a winding form for about 200 feet; it is 70 feet high, and varies in width from 15 to 35 feet. The whisper of a person at the further end is heard distinctly at the entrance, provided the speaker puts his mouth to the wall; otherwise an indistinct murmur is all that is heard. The general effect has been compared with that produced in the Whispering Gallery of St Paul's Cathedral in London. The following is the origin of the name given to the cavern. It is known that Dionysius, the tyrant of Syracuse, was in the habit of using stone quarries for dungeons; and in the sixteenth century the painter Caravaggio suggested that this cavern, which is close to a stone quarry, might have been used for a dungeon, and that the tyrant by stationing himself near the entrance might have been able to hear the conversation of the prisoners within. The suggestion has been handed down by tradition, and has been sometimes accepted as a fact. Murray's *Handbook for Sicily*.

149. *Sounding Sand.* At a place called *El Nakous* in the neighbourhood of Mount Sinai in Arabia some remarkable sounds are heard connected with the motion of loose sand down a declivity. Herschel in his Essay on *Sound* quotes the narrative given by a traveller in 1812,

and calls the phenomenon very surprising and utterly inexplicable. An interesting account of the locality, and of the circumstances under which the sounds are heard, is given in Professor Palmer's work on *The Desert of the Exodus*, published in 1871; we will extract a small part of this account. "The mountain itself is composed of white friable sandstone, and filling a large gully in the side facing west-southwest, is a slope of fine drift sand about 380 feet in height, 80 yards wide at the base, and tapering towards the top, where it branches off into three or four narrow gullies. The sand lies at so high an angle to the horizon, namely 30°, and is so fine and dry, as to be easily set in motion from any point in the slope, or even by scraping away a portion from its base. When this is done, the sand rolls down with a sluggish viscous motion, and it is then that the sound begins, at first a low vibrating moan, but gradually swelling out to a roar like thunder, and as gradually dying away again, until the sand has ceased to roar...... We found that the sand on the cool shaded portions, at a temperature of 62° produced but a very faint sound when set in motion; while that on the more exposed parts, at a temperature of 103° gave forth a loud and often even startling noise. Other sand-slopes in the vicinity were also experimented upon, but these which were composed of coarser grains and inclined at a lower angle produced no acoustic phenomena whatever."

150. *Arago's Observation.* Some elaborate experiments on the velocity of sound were made in June 1822 by Arago and other eminent French philosophers. Two stations were taken, Villejuif and Montlhéry, both lying south of Paris, and distant about 11·6 miles apart. Guns were fired at each place, and the velocity of sound was deduced from the observed interval between the flash as seen and the report as heard at the other place. Now in the course of these experiments the following singular fact was noticed: every report of guns fired at Montlhéry was heard distinctly at Villejuif, while few reports of guns fired at Villejuif were heard at Montlhéry. There was very little wind at the time, but such as there was blew from Villejuif to Montlhéry. This difficulty has remained

unexplained up to recent times. Professor Tyndall has investigated it and arrived at the following conclusion. Villejuif is close to Paris; consequently all the warm air from an immense number of chimneys was wafted to Villejuif and formed round it a non-homogeneous atmosphere, which acted with respect to sound proceeding from Villejuif like a screen with respect to light. In this case the sound starting from Villejuif was intercepted by the neighbouring screen, and so prevented from passing on to Montlhéry; while on the other hand sound starting from Montlhéry was yet able to penetrate this screen, remote from the origin of the sound, and reach Villejuif. Direct experiment shewed results of the same kind. For details we must refer to Professor Tyndall's valuable work on *Sound*.

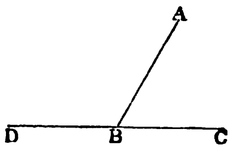
LIGHT.

XVI. MATHEMATICAL PRELIMINARIES.

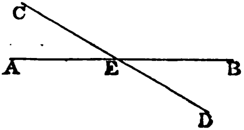
151. A little more mathematical knowledge than we have hitherto assumed will be useful, though not absolutely necessary, for the study of the subject of Optics; we will collect here the things which will be required. It will be sufficient for the beginner to read them once carefully, and to return to them afterwards as occasion arises. In Art. 152 we exemplify an Arithmetical definition; in Arts. 153...156 we enunciate some propositions in Geometry, and supply references to the places where the demonstrations can be found; and in Arts. 157...164 we treat of angles and of what are called the *sines* of angles.

152. The *reciprocal* of any fraction is obtained by making the numerator and the denominator change places: thus the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$, and the reciprocal of $\frac{3}{2}$ is $\frac{2}{3}$; so also, since 5 is equal to $\frac{5}{1}$, the reciprocal of 5 is $\frac{1}{5}$, and the reciprocal of $\frac{1}{5}$ is $\frac{5}{1}$, that is 5.

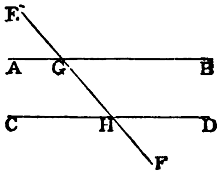
153. Let the straight line *AB* make with the straight line *CD* on one side of it the angles *ABC* and *ABD*: these angles will be together equal to two right angles. See *Euclid* I. 13, or *Mensuration* page 9.



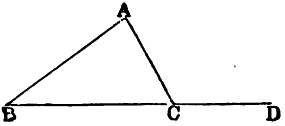
154. Let two straight lines *AB* and *CD* cut one another at *E*: the angle *AEC* will be equal to the angle *BED*, and the angle *AED* will be equal to the angle *BEC*. See *Euclid* I. 15, or *Mensuration* page 9.



155. Let the straight line *EF* cut the parallel straight lines *AB* and *CD*: the angles *AGH* and *GHD* will be equal. These angles are called *alternate* angles. See *Euclid* I. 29, or *Mensuration* page 9.



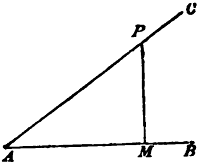
156. Let *BC* a side of the triangle *ABC* be produced to *D*; the exterior angle *ACD* will be equal to the two interior and opposite angles *ABC* and *BAC*; and the three interior angles of the triangle are together equal to two right angles. See *Euclid* I. 32, or *Mensuration* page 10.



157. A right angle is divided into 90 equal parts called *degrees;* and any angle is measured by ascertaining how many degrees it contains. A degree is subdivided into sixty equal parts called *minutes.* Symbols are used as abbreviations for the words degrees and minutes; thus $6^0\ 42'$ is used to denote an angle of 6 degrees 42 minutes.

158. The *complement* of an angle is the difference between the angle and a right angle; thus an angle of $83^0\ 18'$ is the complement of an angle of $6^0\ 42'$, for the sum of the two is 90^0.

159. Let *CAB* be any angle. From any point *P* in *AC* draw *PM* perpendicular to *AB*. Then the proportion which *PM* bears to *AP* is called the *sine of the angle A.* The sine of an angle is a very important and useful thing in mathematics; various facts respecting it are demonstrated in books on Trigonometry. Thus, for example, it is shewn that if the perpendicular is drawn from any other point in *AC*, so long as the angle *CAB* is not changed the sine is not changed: if *PM* in the diagram is ·4 of *AP*, then if from a second point in *AC* a perpendicular were drawn on *AB*, the new perpendicular would be ·4 of the distance of the second point from *A*.



160. In books on Trigonometry we learn how we must proceed if we wish to calculate the sine of any angle. It is found that if the angle is expressed exactly in degrees and minutes the sine is scarcely ever a terminating decimal; but the value can always be obtained to any extent of accuracy we please. The values of the sines of angles have been calculated and published in Tables; some of these give, to seven places of decimals, the sines of angles increasing by single minutes from one minute up to a right angle.

161. We shall find it sufficient to have a Table which gives, to three places of decimals, the sines of angles increasing by single degrees from one degree up to a right angle; and accordingly we place the Table here.

Degrees	Sine	Degrees	Sine	Degrees	Sine
1	·017	31	·515	61	·875
2	·035	32	·530	62	·883
3	·052	33	·545	63	·891
4	·070	34	·559	64	·899
5	·087	35	·574	65	·906
6	·105	36	·588	66	·914
7	·122	37	·602	67	·921
8	·139	38	·616	68	·927
9	·156	39	·629	69	·934
10	·174	40	·643	70	·940
11	·191	41	·656	71	·946
12	·208	42	·669	72	·951
13	·225	43	·682	73	·956
14	·242	44	·695	74	·961
15	·259	45	·707	75	·966
16	·276	46	·719	76	·970
17	·292	47	·731	77	·974
18	·309	48	·743	78	·978
19	·326	49	·755	79	·982
20	·342	50	·766	80	·985
21	·358	51	·777	81	·988
22	·375	52	·788	82	·990
23	·391	53	·799	83	·993
24	·407	54	·809	84	·995
25	·423	55	·819	85	·996
26	·438	56	·829	86	·998
27	·454	57	·839	87	·999
28	·469	58	·848	88	·999
29	·485	59	·857	89	1·000
30	·500	60	·866	90	1·000

The sines are given to the *nearest* figure; so that in some cases they are a little too small and in others a little too great. When the angles are near 90° the sines cannot be distinguished without going to more than three places of decimals; so we will give them to five places:

86°, ·99756; 87°, ·99863; 88°, ·99939; 89°, ·99985.

162. In looking at the Table of sines the following facts are apparent: The sine of an angle is never so great as unity except in the extreme case of a right angle; in all other cases the sine is less than unity. As the angle increases so also does the sine, but not quite so rapidly. Thus the sine of an angle of 20^0 is not quite so much as twice the sine of an angle of 10^0: the sine of an angle of 10^0 is by our Table just twice the sine of an angle of 5^0; but if our Table extended to 7 places of decimals the sine of an angle of 10^0 would not be quite so much as twice the sine of an angle of 5^0.

163. The Table can be used either to find the sine when the angle is known or to find the angle when the sine is known. Thus if we require the sine of an angle of 8^0 we find it is ·139; if we require the angle whose sine is ·707 we find it is 45^0.

164. If we wish to find the sine of an angle which is not contained exactly in our Table we must estimate it as well as we can from the sines of the two angles in the Table between which it lies. Thus suppose, for example, we require the sine of an angle of $9\frac{1}{2}^0$; the sine of an angle of 9^0 is ·156, and the sine of an angle of 10^0 is ·174; now the number ·165 is midway between ·156 and ·174, and we may therefore take it for the sine of $9\frac{1}{2}^0$. In like manner if we have ·165 given as the sine of some angle we may infer that the angle is $9\frac{1}{2}^0$. The results thus obtained, though not absolutely correct, will be sufficiently exact for our purposes.

XVII. RECTILINEAR PROPAGATION.

165. The name *Optics*, from a Greek word signifying *to see*, is given to the science which treats of light and vision. We observe various natural bodies, as the sun and the stars, and various artificial bodies, as candles and

lamps, which send forth light to our eyes, by means of which we see these bodies themselves and others which transmit in various ways the light received from them. Bodies are called *self-luminous* when they are themselves the origin of the light they afford to us; most bodies however are not self-luminous but derive from some external source the light by which we see them.

166. We shall hereafter say something as to the nature of light, but at present we shall confine ourselves to the laws of its transmission, and the first thing we have to remark is that light *proceeds in straight lines so long as it continues in the same medium.* Let the shutters of a room be carefully closed, so that the room is in darkness; let a very small hole be made in a shutter opposite to the sun: then a bright spot of light is seen on the floor, or on the wall, so situated that the sun, the hole, and the bright spot appear to be, at least roughly, in a straight line. Also the fact that the path of the light from the hole to the bright spot is a straight line appears from the shining track which the beams of the sun mark out as they illuminate all the specks of dust they meet between the hole and the bright spot. This shining track of the sun's light will enable us to form an idea of what is meant by a *ray* of light. A *ray* of light is the slenderest conceivable beam of light, and is to be considered as a geometrical line; it is in fact what we can imagine as presented by the experiment of the hole in the shutter, if the hole were only a geometrical point, and the sun, instead of being a vast though remote body, were only a brilliant geometrical point.

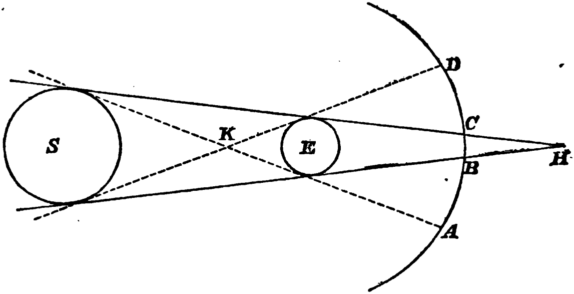
167. A collection of rays of light is called a *pencil of rays*, or briefly a *pencil.* We shall have to consider only *cylindrical* pencils and *conical* pencils. A cylindrical pencil consists of rays which are all parallel to each other. A conical pencil consists of rays the directions of which all pass through one common point: if the rays are considered as coming from the point the pencil is called *divergent*, and if they are considered as going towards the point the pencil is called *convergent.*

168. The rectilinear path of light is also established by the phenomena of *shadows*. Suppose that a bright point is sending out light in all directions, and we put an object before the point. If the object is *transparent*, as glass, the rays of light pass through it; but if the object is *opaque*, as a book or a coin, the rays of light which fall on it are stopped. Let a large screen be placed to receive light from the point, so that the opaque object is between the bright point and the screen; then there is a dark space on the screen corresponding to the rays which are stopped by the object, and the dark space is the *shadow* of the object. The form of the shadow will be determined in the following manner: draw *straight lines* from the bright point to the boundary of the object, and produce them to the screen; then the points at which they meet the screen constitute the outline of the shadow. Thus if the object be a round coin, and the screen be parallel to it, the shadow will also be round; but if the screen be not parallel to the coin the shadow will in general not be round but oval; it will in fact be the curve which mathematicians call an *ellipse*.

169. We have hitherto spoken of a shadow as it would appear if it were formed by stopping the light from a bright *point;* but in reality an origin of light is always a body of finite size, and not a mere point, though it may be supposed for convenience to consist of a vast number of points. Suppose that we hold up a coin before a common candle, and receive the light on a screen behind the coin. Then corresponding to *each point* of the candle-flame there will now be a shadow on the screen. The shadow corresponding to one point of the candle-flame does not fall *exactly* over the shadow corresponding to another point; the consequence is that the shadow as a whole, instead of being sharp and well defined, consists of a darker central part surrounded by a band which is less obscure, and which changes gradually to the bright part where there is no shadow.

170. We may as an example of a shadow consider the case of the sun and the earth. Let the circles of which

the centres are S and E represent the sun and the earth respectively. Let a pair of straight lines be drawn to touch these circles, meeting at the point H beyond the earth. Let another pair of straight lines be drawn to touch the circles, meeting at the point K between the earth and the sun; these straight lines are the dotted lines in the diagram. Let $ABCD$ be a portion of the circle described round the earth as centre, with the moon's distance for radius. If the moon were at any point of the arc between B and C not a ray from the sun could reach it; for a straight line drawn from any point in BC to any point



of the sun would pass through the earth. If the moon were at any point of the arc between A and B some of the rays of the sun would reach it; for instance, if the moon were about midway between A and B, a straight line drawn from it to touch the lower part of the circle representing the earth would, when produced, pass nearly through the sun's centre: the rays from the lower half of the sun would reach the moon in this case, and the rays from the upper half would not. With respect to the arc between C and D remarks may be made similar to those with respect to the arc between A and B. The whole shadow is thus divided into two parts: the part corresponding to BC, in which all the sun's rays are cut off, is called the *umbra;* and the part corresponding to AB and

CD, in which some of the sun's rays are still received, is called the *penumbra*. The obscurity of the shadow increases gradually from *A* to *B*, is total from *B* to *C*, and diminishes gradually from *C* to *D*. The diagram does not profess to represent the various parts in their correct proportions, for this could not be done without a loss of distinctness.

171. We will now give a little further attention to the experiment of passing light into a dark room through a small hole, which was noticed in Art. 166. Suppose a bright *point* of light outside a room which is entirely closed and dark; let a hole be made in the side opposite the point, so that light may enter the room, and form a bright patch on a screen opposite the hole. If the hole is triangular so also will the patch be; if the hole is four-sided so also will the patch be; if the hole is round the patch will be round or oval. If instead of a bright *point* we have a bright *body*, as the sun, the appearances will be different, and they may be summed up in the following statement: *let the light of the sun be admitted through a small hole of any form, and received on a screen; then if the screen is very near the hole the bright patch resembles the hole, but if the screen is very remote from the hole the bright patch resembles the sun.* For the sake of definiteness consider the hole as *triangular*. Every visible point of the sun's surface will produce on the screen an illuminated patch of a triangular form, and if the screen is close to the hole these triangular patches, although not occupying *exactly* the same position, do *nearly* coincide: thus on the whole we have a central part, triangular in form, of uniform brightness, surrounded by a narrow band in which the brightness gradually fades away. But if the screen is very remote from the hole the illuminated patches, instead of nearly coinciding, are diffused over a space which is large compared with the size of each patch. To determine the form of the bright space we need consider only the bright points which constitute the *boundary* of the sun's visible surface; corresponding to these points there will be a set of illuminated patches arranged as we may say close to each other round a circle, so that there will be a circular boundary to the bright space.

172. Common observation shews that the intensity of the light received on an assigned surface diminishes when the distance of that surface from the origin of the light is increased. Thus if we are reading at night, by the assistance of a lamp, we bring the lamp nearer to the book when we wish to increase the quantity of light which falls on a page. The law respecting the diminution of intensity owing to the increased distance from the origin of light, is the same as holds for the diminution of sound, and for the diminution of the force of gravity; namely, the intensity diminishes in the same proportion as the square of the distance increases: see Art. 17. Thus if light falls from a bright point on a screen, and we move the screen to a second position, similar to the former but at *twice* the distance from the point, the intensity of the light on the screen will be *one fourth* of what it was before. In experimental lectures means are devised for confirming this statement; for instance, four equal candles are placed side by side, and it is shewn that they together produce the same illumination on an object at a certain distance, as a single candle would produce on the object at half the distance. But with the aid of a little geometry the fact appeals more forcibly to the mind. Suppose a piece of wood or metal, in the form of a square, held up before a bright point, and let a screen be placed parallel to the square, and so that the square is midway between the bright point and the screen. Then the shadow on the screen is a square, four times as large as the square object, being twice as long and twice as broad. Thus the same amount of light which falls on a certain area would be diffused over four times that area if placed at twice the distance from the origin; and therefore the illumination at a point becomes one fourth of what it was before at the corresponding point.

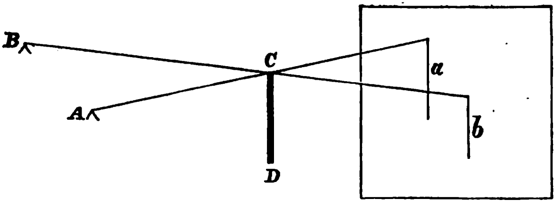
173. The illumination received by a surface from an origin of light depends on the distance of the surface from that origin, in the manner just explained; the illumination depends also on the *inclination* of the surface to the rays which fall upon it. Return to the experiment of admitting light from the sun through a very small hole into a

room. The rays appear to spread out in a conical form from the hole where they enter. Hold a screen at a fixed distance from the hole, but place it successively at different inclinations to the rays. When the rays fall on the screen as nearly as possible at *right angles* the patch is smallest and brightest; but when the rays fall so as to make a very *small angle* with the screen the patch is large and much less bright: the *same* amount of light is in this case spread over a larger area, and so the brightness is diminished to a corresponding degree. It is found by theory and by experiment that when a small plane surface is kept at the same distance from the origin of light, but placed successively at different angles of inclination to the rays falling on it, then the brightness is proportional to the *sine of the angle* between the surface and the rays: see Art. 159.

174. It is obvious that sources of light may differ as to the amount of light which they send forth; thus a candle may be inferior to a lamp, or one lamp to another, in the power of illumination. It is often a matter of interest to compare the light-giving powers of two sources of light, and simple instruments are constructed to assist in this comparison; they are called *photometers*. Suppose, for instance, that we wish to compare a certain candle with a certain lamp. Two pieces of paper are placed, side by side; on one piece rays from the candle alone are allowed to fall, and on the other piece rays from the lamp alone: the rays in the two cases should fall at the same angle, say a right angle. The two pieces of paper will in general be, at first, unequally illuminated; we must then move one of the sources of light nearer to, or further from, the piece which it illuminates, until the two pieces appear to the eye to be equally bright. Suppose that the distance of the candle from the piece of paper which it illuminates is 1 foot, and that the distance of the lamp from the piece which it illuminates is 3 feet; then the light of the lamp is 9 times as powerful as that of the candle: for if the lamp were put at the same distance as the candle, that is at the distance of 1 foot, the illumination which it produces would be 9 times as great as at present, that is 9 times as great as the illumination from

the candle. In like manner if the lamp at the distance of 3½ feet, that is at the distance of $\frac{7}{2}$ feet, is equivalent to the candle at the distance of 1 foot, the lamp is $\frac{7}{2} \times \frac{7}{2}$ times as powerful as the candle, that is $\frac{49}{4}$ times, that is more than 12 times.

175. A method of comparing the powers of two lights by the aid of shadows may be noticed. *A* and *B* denote



two lights, as candles or lamps; *CD* is a rod. A white screen is disposed so as to receive, as nearly as possible at right angles, rays from the two lights. The screen then will appear of nearly uniform brightness, except at *a* and *b* where the shadows corresponding respectively to *A* and *B* are formed; and by suitably adjusting the distances of the two lights from the screen these shadows can be made to appear of the *same degree of obscurity:* suppose this adjustment made. Now the shadow *a* is illuminated only by the light *B*, and the shadow *b* is illuminated only by the light *A*; hence the power of *A* is in the same proportion to the power of *B* as the square of the distance between *A* and *b* is to the square of the distance between *B* and *a*. Thus suppose the distance between *A* and *b* is 7 feet, and the distance between *B* and *a* is 10 feet; then the power of *A* is to the power of *B* in the proportion of 49 to 100: that is, the power of *A* is not quite half the power of *B*.

XVIII. VELOCITY OF LIGHT. ABERRATION.

176. Light is known to be transmitted through an empty space with enormous velocity, namely at the rate of about 185500 miles per second. This fact was originally obtained from two different Astronomical investigations, and in recent times has been confirmed by direct experiment; we will here explain the principles of the Astronomical investigations, and hereafter notice the direct experiment.

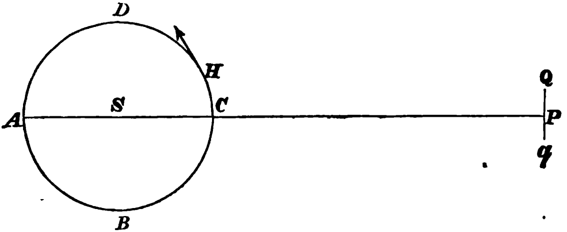
177. The first knowledge on this point was obtained by Roemer a Danish astronomer in 1676. The planet Jupiter is accompanied by four satellites which revolve round him as the moon revolves round the earth; of these satellites we shall now be concerned only with that which is nearest to Jupiter: this passes through the shadow of Jupiter, and so becomes eclipsed, once in every revolution which it makes round Jupiter. Now the motion of this satellite has been observed for so many years that the average time between the ends of two successive eclipses is well known, namely about 42½ hours. Suppose the earth at that point of its orbit where it is nearest to Jupiter, and let the instant be noted when the satellite reappears after having been eclipsed, which is called an *emersion*. Let the instant of the next following emersion be noted; then between these two emersions the distance of the earth from Jupiter has somewhat increased, owing to the fact that the earth and Jupiter describe their orbits round the sun in very different periods: the result is that the interval between the two emersions is somewhat greater than if the earth and Jupiter had remained fixed. The

first ray of light which comes from the satellite after its eclipse is the messenger of the emersion; and this has a longer distance to travel than the messenger of the previous emersion, and so takes longer time. Similarly the interval between the second and third emersions will be greater than if the earth and Jupiter were fixed; and the same will be true with respect to successive emersions during a period of rather more than six months, as the distance of the earth from Jupiter will be continually increasing. Now the interval between two successive emersions could scarcely be determined with sufficient accuracy to shew that it was greater than the average, yet at the end of the six months, by the accumulation of small excesses the amount became perceptible; it was found by Roemer that the last emersion appeared to take place about 15 minutes later than it ought to have done. But the earth was then more distant from Jupiter than at first by the whole diameter of the earth's orbit; and consequently the inference was that light moves with such velocity as to pass in 15 minutes over the diameter of the earth's orbit. During the next six months the earth would be continually drawing *nearer* to Jupiter, and consequently the interval between two successive emersions would always be *less* than the average. The *emersion* could not be observed during these six months, as the satellite would be hidden behind Jupiter at the instant of leaving the shadow; but the *immersion*, that is the instant of the beginning of an eclipse, can be observed, which answers the same purpose.

178. A modern student on first being informed of the velocity of light is generally astonished at the almost inconceivable swiftness; but at an earlier date the wonder rather was that light should require any time at all to pass from one point to another: the belief was that it flashed instantaneously. Bacon has recorded that some cases produced in him "a suspicion altogether surprising; namely, whether the face of the serene and starry heavens be seen at the very time it exists, or not till some time later": but he proceeds to dismiss the notion as untenable. The explanation given by Roemer was not admitted by all his contemporaries; it was condemned by D. Cassini the emi-

nent astronomer, and by Fontenelle the attractive writer: it was however accepted by Huygens and by Newton, and confirmed in 1729 in an unexpected manner by Bradley.

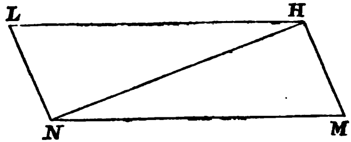
179. We shall not go into detail with respect to Bradley's discovery of what is called the *Aberration of Light*, but merely give a sufficient account of it for optical pur-



poses. Let S represent the centre of the sun; $ABCD$ the curve traced out by the earth round the sun; this curve is nearly a circle. Let P be a star in the same plane as this curve; the distance of P from S must be supposed to be immensely greater than our diagram would suggest; for SP would be, at the least, two hundred thousand times SC. Now Bradley observed that the star P did not remain fixed in the heavens, but appeared to move between two extreme positions Q and q, passing over the space Qq twice in the course of a year. When the earth is at A the star appears at q; as the earth moves through ABC the star appears to move from q to Q; and as the earth moves through CDA the star appears to move from Q to q: the star occupies its mean or average position, that is P, when the earth is at B or D.

180. To shew how the apparent changes in the position of the star are produced let us suppose that the earth is at the point H of its orbit, and moving in the direction indicated by the arrow. Light arrives at the earth from the star; and on account of the enormous distance of the star the direction of the light may be considered to be parallel to CS. Let HL be taken to represent the velocity of light in magnitude and

direction; let *HM*, on the same scale, represent the velocity of the earth in its orbit, drawn in the direction *contrary* to that of the motion of the earth when at *H*. To an observer on the earth the *relative velocity* of light will be the same as if a velocity



equal and opposite to that of the earth were given both to the light and to the earth; then the earth would be brought to rest. Hence, if the parallelogram *LHMN* be completed, *HN* will represent the velocity of light with respect to the earth, both in magnitude and direction: so that if *NH* be produced through *H* it will represent the straight line on which the star seems to be situated. If the earth had been fixed at *H*, the straight line *LH* produced would have represented that on which the star would have appeared to be situated; and hence the angle *LHN* is the apparent displacement of the star in direction, produced by the combination of the motion of the earth with the motion of light. The greatest value of this angle is found to be when the angle *LNH* is a right angle; and as *LHN* is even then only a small angle, the angle *LHM* is very nearly a right angle: see Art. 155. Thus the angle *LHN* has its greatest value when the earth is very nearly at *A* or *C*. Now observation shews that the greatest angular distance between the apparent position of a star and its mean or average position is about $\frac{1}{3}$ of a minute, that is about $\frac{1}{180}$ of a degree; and this is found to agree with the diagram when we take *LH* to *HM* in the proper proportion, that is in the proportion of 185500 to 19. When the earth is at *B* or *D*, since *HM* and *HL* are then in one straight line, *HN* is also in that straight line; so that the angle *LHN* vanishes, and the star appears in its mean or average position.

181. The name *aberration of light* is given to the apparent changes of position of a star produced, in the manner we have explained, by a combination of the motion of the earth with the motion of light. We may illustrate

this combination by comparing the rays of light to falling drops of rain. Suppose a person standing on the deck of a steamer while a shower is falling vertically. If the steamer is at rest the rain-drops will fall on the person's head, so that their apparent direction is vertical, and this agrees with their real direction. But if the steamer is in motion the rain-drops will strike the person's face, and their direction will be apparently, not vertical, but oblique to the deck of the steamer.

182. In Art. 179, we explained the *principle* on which the discovery of the aberration of light depended, but, as may be readily supposed, the actual details of the process were more complicated. The stars specially observed were not in the plane of the curve which the earth describes, but considerably above it. Each star then, instead of moving apparently backwards and forwards, describes an oval curve round the position which it would have occupied if there had been no aberration. The observations however were restricted to finding the changes in what astronomers call the *declination* of the stars. Since the numerical result obtained is the same for all the stars it follows that light travels with the same velocity whatever be the star from which it proceeds.

183. It will assist in understanding the aberration of light if we advert to the case of the moon; this body moves *with the earth* round the sun, and consequently the motion of the lunar rays *with respect to the earth* is the same as if the earth and moon were at rest instead of revolving round the sun. Hence aberration as depending on the *motion of the earth* does not occur with respect to the lunar rays; since however light takes about a second and a quarter to pass from the moon to the earth, we never see the moon in its actual position, but in the position which it occupied a second and a quarter before.

184. The velocity of light is so great that writers use various illustrations in order to assist the mind in conceiving it. Thus we have said that in a second and a quarter light passes from the moon to the earth; this distance could not be traversed in less than 6 months by a fast railway train. Light passes from the sun to the earth

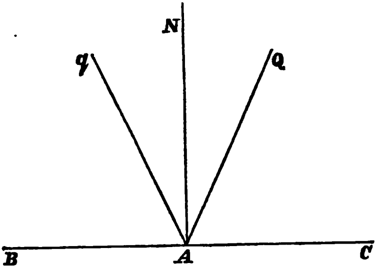
in 7½ minutes; a cannon-ball would require 17 years to perform such a journey, supposing its velocity to remain always the same as at the moment of its discharge. The swiftest bird would require three weeks of continual flight to pass round the circumference of the earth; light would flash through an equal distance in about one-eighth of a second, that is in less time than the bird would occupy for one flap of his wings. And yet, great as this velocity is, light seems but a slow and weary messenger in crossing the vast celestial spaces. It is certain that light must take at least three years to reach the earth from the nearest fixed star; and there can be little doubt that rays from some of the fainter objects in the sky must have taken hundreds of years in their journey to our eyes. Hence the supposition thrown out by Dr Whewell in his *Plurality of Worlds*, though extravagant is not impossible, "that the distant stars were sparks or fragments struck off in the formation of the Solar System, which are really long since extinct; and survive in appearance, only by the light which they at first emitted."

XIX. REFLECTION AT PLANE SURFACES.

185. We have said in Art. 166, that light proceeds in straight lines so long as it continues in the *same medium:* we must now consider what takes place when light travelling in one medium arrives at another, as for instance when light travelling in air arrives at a body formed of wood, or metal, or glass, or water. The light in this case is divided: part enters the body, and of this we shall treat hereafter; and part is thrown back by the body, as we shall now explain. In the majority of cases the light thrown back by the body is scattered in various directions, just as if the body were *self-luminous*. Thus we may be sitting by day in a room which the *direct* rays of the sun do not enter, and yet every object in the room may be distinctly visible. The sun's rays fall on the clouds, the ground, the trees, and the houses around us, and being thrown off from these in every direction they stream in at the window; then they fall on the tables, chairs, and books, and being again thrown off by these objects they

reach our eyes whatever place in the room we may occupy. Thus by daylight rays originally derived from the sun are diffused from all the things around us, and so render these things visible. By night our candles or lamps take the place of the sun; they send forth rays which are thrown back in all directions by the things on which they fall, and so render those things visible. This *diffusion* or *radiation* of light in all directions around, by the objects on which it falls, is sometimes called *irregular reflection*, in distinction from that *regular reflection* to which we now proceed, and which is usually called simply *reflection*.

186. We are now about to consider the case in which light falls on the surfaces of *smooth polished bodies* such as glass or bright metals. The surfaces may be plane or curved, but in the present Chapter we take only plane surfaces. Let BAC represent a flat polished surface, QA the direction of a ray of light which falls on the surface



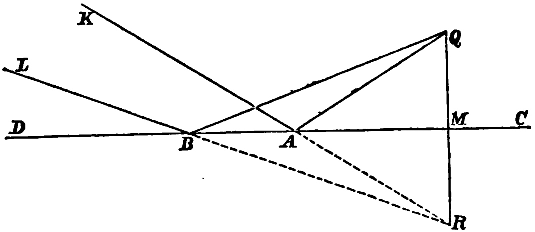
at A, and Aq the direction in which the ray is reflected. From A draw AN at right angles to the surface. Then QA is called the *incident ray*, Aq the *reflected ray*, and AN the *normal to the surface at the point of incidence;* the angle QAN is called the *angle of incidence*, and the angle qAN *the angle of reflection*. The direction Aq is determined by the following laws: Aq is in the same plane with AQ and AN; and the angle qAN is equal to the angle QAN. With the aid of the preceding definitions the laws may be stated verbally thus: *the incident ray and the reflected ray are in the same plane with the normal to the surface at the point of incidence, and on* *opposite sides of the normal; and the angle of reflection is equal to the angle of incidence.* The reader will observe that these laws coincide with those which hold with respect to the impact of a ball on a perfectly elastic plane in Mechanics: see Vol. I. Art. 284.

187. The angle QAC is the *complement* of the angle QAN, and the angle qAB is the *complement* of the angle qAN, see Art. 158: hence the angle QAC is equal to the angle qAB.

188. The laws of reflection may be verified, at least roughly, by a very simple experiment. Let light be admitted from the sun through a very small hole into a room, and allowed to fall on a mirror placed on the floor; then the track of the ray before and after reflection will be revealed to the eye by the bright particles of dust which float in the air: and it will be seen that the laws do hold with sensible accuracy. But, as in other points of Natural Philosophy, the best evidence of the truth of the laws is the fact that many and various results deduced from the laws are found to agree with observation. The laws of reflection are stated with respect to *all* smooth polished bodies, and there can be no reasonable doubt that they do hold in all such cases; but for one special case, namely that of reflection from the *surface of mercury*, the indirect evidence for the truth of the laws is overwhelming. Certain astronomical observations consist partly in noticing the image of a star reflected in a trough of mercury; the use made of such observations depends on the truth of the laws of reflection for mercury, and the accuracy and consistency of modern astronomy attest the truth of the laws. In the whole range of Natural Philosophy there is probably nothing more certainly known than the truth of the laws of reflection of light from the surface of mercury.

189. After considering the reflection of a single *ray* from a plane surface we pass to the reflection of a *pencil* of rays; and with respect to a *conical* pencil we have the following proposition: *if rays diverge from a point and fall on a plane reflecting surface, then after reflection they appear to diverge from a point as far behind the reflecting surface as the origin of light was*

before it. Let QA represent a ray incident at A on the plane reflecting surface CD; draw QM perpendicular to CD, and produce it to R, so that MR may be equal to QM. Join RA and produce it to K; then AK will be the direction in which the incident ray is reflected. For the triangles AMR and AMQ are equal in all respects, as is almost immediately obvious; and thus the angle RAM is equal to the angle QAM. But the angle RAM



is equal to the angle KAD, by Art. 154. Hence the angle KAD is equal to the angle QAM; and therefore AK is the direction in which the ray QA is reflected. Thus the direction of the ray after reflection is the same as if it had come from R. In like manner if QB represent any other ray coming from Q then after reflection it proceeds as if it came from R; and this is the case whether the plane containing QB and QM is or is not the same as the plane containing QA and QM. Thus if there be any conical pencil of rays, such as we may denote by QA and QB and intermediate rays, then after reflection it proceeds as if it were just such a similar pencil with its vertex at R instead of at Q. In the diagram the part RA of the straight line RK is drawn *dotted* while the part AK is drawn *full;* the former part represents only the direction of a ray, while the latter part represents a real ray; and a similar remark holds for the straight line RBL: it is often convenient to observe this distinction in diagrams.

190. Suppose in the preceding Article that Q is at a great distance from the reflecting surface, and that this surface is of small extent: for example Q might be a point on the sun, and the reflecting surface an ordinary mirror.

Then the rays QA and QB would be practically *parallel* and the rays AK and BL would be practically *parallel.* Hence we obtain in an indirect manner the following result with respect to a cylindrical pencil of rays: *a pencil of parallel rays consists of parallel rays after reflection.* This can be established directly by the aid of a little more geometry than we assume to be known; but the indirect way in which we have deduced it is important on account of its frequent use in mathematics.

191. In the diagram of Art. 189 let KA and LB denote two rays which are proceeding to a point R, and fall on a plane reflecting surface CD; then after reflection they both proceed to the point Q. Thus we obtain the following proposition: *if rays converging to a point fall on a plane reflecting surface they converge after reflection to a second point as far in front of the reflecting surface as the first point was behind it.*

192. A pencil of rays after being reflected by one plane surface may fall on another and be reflected again. The form of the pencil after the second reflection will be obvious after what has been said. If the pencil consisted originally of parallel rays it will consist of parallel rays after the first reflection, and again of parallel rays after the second reflection. If the pencil is originally a conical pencil such it will be after the first reflection, and such again after the second reflection; the vertices of the cones of rays before and after the second reflection will be at equal distances from the second reflecting surface and on opposite sides of it, in the same manner as we have seen with respect to the first reflecting surface in Arts. 189 and 191.

193. As we have already stated in Art. 185 when light falls on a reflecting surface only a *part* of it is regularly reflected. With respect to the *quantity* thus reflected it is usual to quote some results obtained from experiment by Bouguer long since. The quantity reflected increases with the angle of incidence; it is found that water, when the incidence is perpendicular reflects 18 rays out of 1000; when the angle of incidence is 40^0 it reflects 22 rays; when the angle is 60^0 it reflects 65 rays; when the angle is 80^0 it reflects 333 rays; and when the angle is $89\frac{1}{2}^0$ it

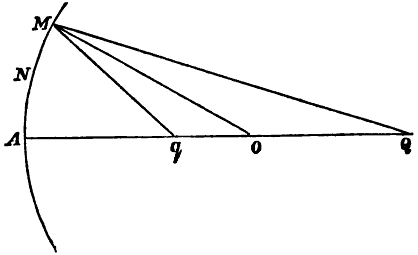
reflects 721 rays. Mercury when the incidence is perpendicular reflects 666 rays out of 1000; when the angle of incidence is $89\frac{1}{2}^0$ it reflects 721 rays.

XX. REFLECTION AT SPHERICAL SURFACES.

194. We have hitherto confined ourselves to *plane* reflecting surfaces, but *curved* surfaces of various kinds may be made smooth and polished, and so fitted to reflect light; for instance we may have spherical, cylindrical, or conical surfaces thus constructed. In Elementary Optics however it is usual to consider only spherical surfaces.

195. Suppose a piece cut off by a plane from a spherical shell; this piece will then resemble a bowl or a watch-glass. It may be polished, and so be made capable of reflecting light, either on the inside or the outside; in the former case it is called a *concave* reflector, and in the latter a *convex* reflector. The *axis* of the reflector is a straight line drawn through the centre of the sphere perpendicular to the plane which cuts off the piece from the sphere, so that it constitutes a central line round which the reflector is symmetrical: the point at which the axis meets the reflecting surface may be called the *vertex* of the reflector. Any very small portion of a curved reflecting surface may be considered as coinciding with a corresponding portion of a plane which would touch the surface there. Thus let *M* denote a small portion of a spherical reflector; then this portion will reflect a ray of light in the same manner as a portion of a plane reflector at *M* which touches the spherical reflector there: and mathematicians shew that a plane surface touching the spherical surface at *M* will be at *right angles to the radius of the sphere there.* Hence finally the laws of reflection with respect to a spherical surface are the same as for a plane surface, provided that by the *normal to the surface at the point of incidence* we here understand the *radius of the sphere at the point of incidence.* We shall now consider the case of a concave spherical reflector.

196. *Rays of light fall on a concave spherical reflector from a point on the axis: it is required to determine their course after reflection.* Let O represent the centre of the



sphere, OA the axis of the reflector, A the vertex. We will suppose the origin of the light to be further from the vertex than O is; let it be at Q in AO produced. Let QM represent one of the rays, incident at M on the reflector; join MO and make the angle qMO equal to the angle QMO: then Mq represents the direction of the ray reflected at M. Now besides the ray QM, which is drawn in the plane of the paper, there will be a number of other rays the directions of which before reflection all make with the axis QA an angle equal to that which QM makes; namely all the rays which would form a certain conical surface having Q for vertex, QA for axis, and for base a circle with its centre on QA, its plane perpendicular to QA, and its circumference passing through M. The directions of all these rays after reflection will pass through the same point q of the axis. Next suppose that we take an incident ray not lying on the conical surface just mentioned; for instance take a ray falling on a point N of the reflecting surface which is nearer to the vertex than M is: and draw the corresponding reflected ray. Then if the diagram be carefully constructed it will be found that the reflected ray *does not cross the axis at the same point* q as the ray reflected from M, but at a point slightly nearer to O. Hence rays which fall from Q on the concave reflector do not after reflection converge *accurately* to a single point. Nevertheless it is found that if the rays from Q constitute a very slender pencil, so that the greatest value of the angle MQA does not exceed a few degrees, then the rays after reflection do converge *very nearly* to a single point; this point is called the *focus* of the reflected rays.

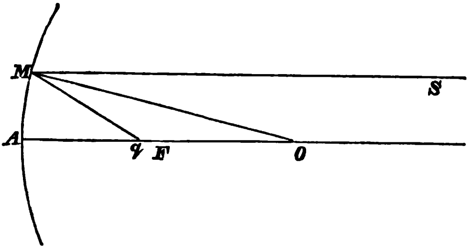
197. It is impossible to secure minute accuracy combined with clearness in the diagrams which must be employed in a work like the present; the reader should construct the diagram for himself on a much larger scale. It will then be easy, without producing any confusion, to draw the rays reflected from three or four different points of the arc which represents the spherical reflector; and thus a satisfactory notion can be gained of the course of the pencil after reflection.

198. We know that OM divides the angle QMq into two equal parts; and hence it follows from Geometry that QM bears the same proportion to Mq that QO bears to Oq: see Euclid VI. 3. This fact will serve as a test of the accuracy with which a diagram is drawn; and it is the foundation of the Rule, hereafter to be given, by which the position of q can be assigned by Arithmetic when the incident pencil is very slender.

199. Article 196 should be carefully compared with Art. 189. In the case of a plane reflector the directions of the rays proceeding from a single point pass accurately through a *single point* after reflection; in the case of a spherical reflector they do not, but they may be said to pass *approximately* through a single point. Cases such as that of Art. 196 will frequently occur in the rest of this work; and it must be borne in mind, even when it is not formally repeated, that the results when stated concisely are *nearly* true but not *strictly* true. In order that they may be reasonably trustworthy in the present case the angle which the ray most remote from the axis makes with the axis should not exceed 8^0 or 10^0. In the case of Art. 189 the point R may be called the *focus* of the reflected rays, so that we may define a focus generally as a point at which the directions of rays meet, accurately or very nearly. The focus is called *real* when, as in Art. 196, the rays themselves

pass through the point, and it is called *virtual* when, as in Art. 189, not the rays themselves but their directions produced pass through the point. Thus, according to this definition, the point Q of Art. 189 or Art. 196 may be called a focus. Also Q and R in Art. 189 are called *conjugate foci;* the name is justified by Art. 191 taken in conjunction with Art. 189. In like manner in Art. 196 a ray proceeding from q to M would be reflected to Q; and so Q and q are called *conjugate foci.*

200. We will now examine in some detail the various cases which may occur in connection with Art. 196, corresponding to various positions of Q. Suppose in the first place that the rays which fall on the reflector are all



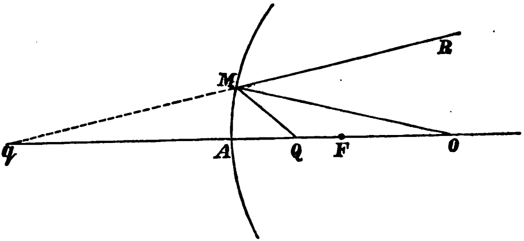
parallel to the axis. Let O be the centre, OA the axis, and SM one of the incident rays parallel to OA. Make the angle qMO equal to the angle SMO, so that Mq is the direction of the reflected ray. The angle SMO is equal to the angle MOA by the nature of parallel lines: see Art. 155. Thus the angles MOq and OMq are equal; and hence it follows by Geometry that Oq and Mq are equal. And, as two sides of a triangle are greater than the third side, Oq and Mq are together greater than OM; and therefore Oq is greater than half the radius OM, that is greater than half OA. But if the angle MOA is small Oq will be very little greater than half OA; so that finally the focus of the reflected rays may be said to be midway between the centre O and the vertex A.

201. The point midway between the centre and the vertex of the concave reflector is called the *principal focus* of the reflector; we denote this point by F: the length FA is called the *focal length* of the reflector, so that the focal length is half the radius. Rays parallel to the axis pass after reflection approximately through this point F, as we have seen, supposing that the angle MOA does not exceed a few degrees at most. The angle between two radii drawn from O, one to the highest point of the reflector and one to the lowest, may be called the *angular aperture* of the mirror; and if this is small the angle MOA, even at its greatest value, is small.

202. Next suppose that the point Q from which the rays proceed is at a very great distance from A; then the ray incident at M makes with OM an angle slightly less than if the ray were parallel to the axis, and therefore the reflected ray makes with OM an angle slightly less than for such a case: thus the point q falls slightly to the right of its position in the diagram of Art. 200. In this way we can see that if Q moves gradually from a remote distance up to O, the conjugate focus q moves from a position close to F up to O. In like manner if Q moves from O to F the conjugate focus moves from O to a very remote position on the right, so that the same relative positions of the two foci occur as when Q moves from the very remote distance up to O. The following is the Rule which connects the distances of Q and q from A: *the sum of the reciprocals of these distances is the reciprocal of the focal length.* For example, suppose the radius to be 12 inches, and the distance of Q from A to be 15 inches; find the distance of q. The focal length is 6 inches by Art. 201, and the reciprocal of this is $\frac{1}{6}$; the reciprocal of the distance of Q from A is $\frac{1}{15}$; therefore the reciprocal of the distance of q is $\frac{1}{6}-\frac{1}{15}$, that is $\frac{5}{30}-\frac{2}{30}$, that is $\frac{3}{30}$ or $\frac{1}{10}$: therefore the distance of q from A is 10 inches. Again, with the same value of the radius, suppose the distance of

Q from A to be extremely great; then the reciprocal of this quantity is so small that it may be considered to be nothing. Hence the reciprocal of the distance of q is $\frac{1}{6}$; and therefore the distance of q from A is 6 inches, so that q is at the *principal focus*. Thus the case in which Q is at an enormous distance coincides practically with the case in which the incident rays are parallel to the axis, as might have been expected: see Art. 190.

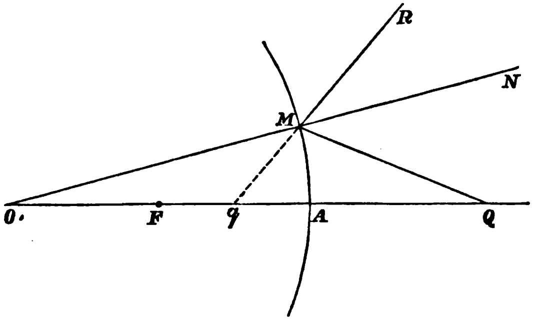
203. Finally, suppose Q to be between F and A. Then the rays after reflection proceed as if they came from a virtual focus q behind the reflector. When Q is very near to F this focus q is at a very remote distance behind A,



and as Q moves up to A from the right hand q moves up to A from the left hand, being always more distant from A than Q is, until the two meet at A. The following is the Rule which connects the distances of Q and q from A: *the difference of the reciprocals of these distances is the reciprocal of the focal length.* For example, suppose the radius to be 12 inches, and the distance of Q from A to be 4 inches. The focal length is 6 inches, and the reciprocal of this is $\frac{1}{6}$; the reciprocal of the distance of Q from A is $\frac{1}{4}$; therefore the reciprocal of the distance of q from A is $\frac{1}{4}-\frac{1}{6}$, that is $\frac{3}{12}-\frac{2}{12}$, that is $\frac{1}{12}$: therefore the distance of q from A is 12 inches.

204. The results of Arts. 202 and 203 may be su up thus. The conjugate foci Q and q always move . *posite* directions. As Q moves from a very remote tance on the right through O up to the vertex q moves first from F to O, then from O to a very remote distance on the right, and then from a very remote distance on the left up to the vertex. The reader should draw diagrams for himself with great care and on an enlarged scale, corresponding to the various cases which can occur: see Art. 197.

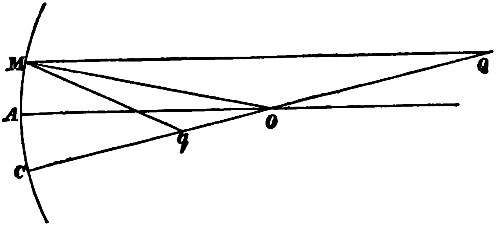
205. *Rays of light fall on a convex spherical reflector from a point on the axis: it is required to determine their course after reflection.* Let O represent the centre of the



sphere, OA the axis of the reflector, A the vertex, Q the origin of light on OA produced. Let QM represent one of the rays, incident at M on the reflector; join OM and produce it to N; make the angle RMN equal to the angle QMN; then MR represents the direction of the ray reflected at M: produce RM to meet the axis at q. We will state briefly results similar to those which have been worked out for the case of the *concave* spherical reflector. If the angle MOA does not exceed 8^0 or 10^0 for the extreme ray from Q, then all the rays after reflection proceed very nearly as if they came from a single point q, so that q is a *virtual* focus. The point Q is more distant than the point q from the vertex, and the following is the Rule which connects these distances: *the difference of the reciprocals of these distances is the reciprocal of the focal length.* When the incident rays are *parallel* to the axis they appear after reflection to proceed from a point F which is midway between O and A; and as in Art. 201 the point F is called the *principal focus* and FA the *focal length* of the reflector. The conjugate foci Q and q always move in opposite directions; as Q moves from a very remote distance on the right up to the vertex of the reflector, q moves from the principal focus to the vertex.

206. In considering the case of a concave reflector we supposed the rays of light to be *diverging* from a point in the *front* when they fall on the reflector; but it is possible that instead of this they may be *converging* to a point *behind* when they fall on the reflector. This case can be completely treated by the aid of Art. 203; we have only to suppose that a ray, instead of proceeding along QM and then being reflected along MR, proceeds along RM and is then reflected along MQ: in fact q must now be considered the focus of the incident rays, and Q the focus of the reflected rays. In like manner, in Art. 205 we supposed the rays of light to diverge from a point in front when they fall on the reflector; but instead of this they may be converging to a point behind when they fall on the reflector. If this point be between A and F then the diagram and the statement of Art. 205 may be easily applied; we must suppose q to be the focus of the incident rays, and Q to be the focus of the reflected rays. But if the rays are converging to a point to the left of F then the rays after reflection appear to diverge from a virtual focus also to the left of the reflector: the nature of the diagram may be inferred from that in Art. 196, supposing that the convex side of the sphere is polished, and that the ray falls on it from the left-hand side so that Mq is the direction of the ray before reflection and QM after reflection, or that MQ is the direction before reflection and qM after reflection. With the diagram of Art. 205, as the focus of the incident rays moves from F through O to a very remote distance on the left the focus of the reflected rays moves from a very remote distance on the left through O up to F; both foci being *virtual.*

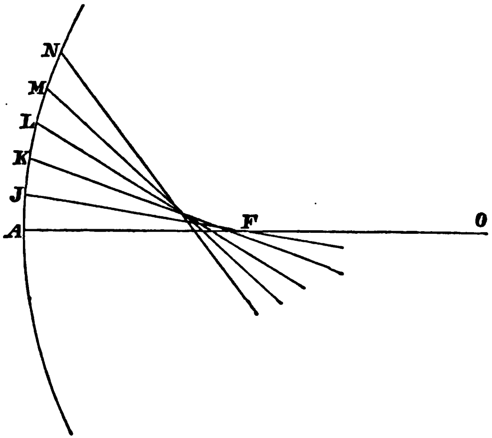
207. Hitherto we have considered the origin of light to be *on the axis* of the reflector; we will now advert to the case when the origin is *not* on the axis: take for example a *concave* spherical reflector. Let O be the centre,



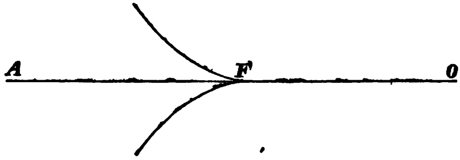
OA the axis, A the vertex, Q the origin of light. Join QO and produce it to meet the reflector at C. Then it is found by mathematical investigation that provided the angle AOC is small we may proceed as if QOC were the *axis of the reflector*. The rays from Q after reflection will have approximately a focus q on the straight line QC, in the same manner as in Art. 196 the reflected rays had a focus on the axis. When the focus of the incident rays is not on the axis the pencil is often called a *secondary* pencil, and the straight line passing through the focus of incident rays and the centre of the sphere is called the *secondary* axis: thus QC is the secondary axis of the pencil proceeding from Q as focus.

208. When rays are reflected by a spherical surface they are brought not *exactly* but only *nearly* to a focus; and even this is not the case unless the reflector, so far as the rays fall on it, is only a *small* portion of a sphere. If by careful drawing, or by calculation, we determine the exact course of the rays after reflection, some interesting results are exhibited; we will take one case as an example. Suppose that rays parallel to the axis fall on a concave reflector. For the sake of distinctness in the diagram the incident rays are not drawn, but only the rays reflected from five points J, K, L, M, N. Let O be the centre, A the vertex and F the principal focus. The reflected rays

all cross the axis between F and A; the nearer an incident ray is to the axis, the nearer to F is the intersection of the



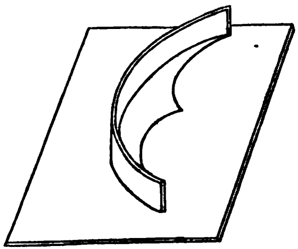
reflected ray with the axis. The intersections with each other of successive reflected rays in the plane of the paper give an assemblage of points which may be considered to form a curve; this curve is called a *caustic*. The reflected rays in the diagram are all drawn from the part of the reflector which is *above* the axis, and the corresponding part of the caustic is *above* the axis. If we suppose the



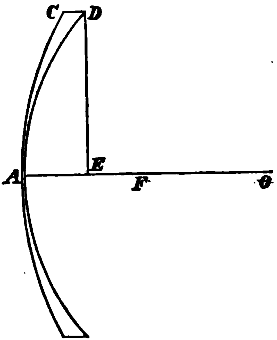
rest of the rays drawn in the plane of the paper we shall obtain the other part of the curve; and the whole then exhibits the form shewn in the diagram, having at F a c[...] that is a sharp point. Since the reflector is spherical [...] shall have precisely the same circumstances in every plane section made through the axis; and thus there will be a caustic *surface* corresponding to the caustic *curve* of the diagram. This surface would bear some resemblance to that of a convolvulus flower, or to that of the part of a horn near the tip. The fact that rays reflected from a spherical surface do not pass *accurately* through a point is expressed by saying that there is *spherical aberration*.

209. As the rays of light intersect on a caustic curve or caustic surface, there is, as it were, a *condensation of light* there; and the consequent brightness can be exhibited by experiment. Thus, for instance, take the spherical reflector of the preceding Article, and cover up all the surface except a circular strip corresponding to points the distances of which from A lie between those of M and N. Thus the incident rays practically reduce themselves to such as form a *cylindrical shell*, and the reflected rays form another shell, which has no simple name. Then by holding a small screen at right angles to AO and moving it to and fro a little we shall easily find a position in which the reflected rays trace on the screen a narrow bright circular ring: this represents the portion of the caustic surface which corresponds to the cylindrical shell of incident rays.

210. Caustics may be formed by other reflecting surfaces as well as by spherical surfaces. Thus let a bright piece of thin metal, as a watch-spring, be bent and placed on a card; let the card be held so that its plane passes nearly through the sun, or through a bright light. Then the reflected rays are not *quite* parallel to the plane of the card, and trace out on the card the form of a caustic curve.



211. We see by the diagram of Art. 208 that parallel rays after reflection by a *spherical* surface are not brought to a focus *exactly;* but it is possible by using another form of reflecting surface to effect this result. The reflecting surface must be such that a section of it by a plane through the axis must be, not a part of a circle, but a part of the curve called a *parabola.* The axis of the reflector must be what mathematicians call the axis of the parabola; the rays must fall on the reflector parallel to this axis, and then after reflection they all pass exactly through the point which is called the *focus* of the parabola. Thus in the diagram the left-hand curve may represent the parabola, and the right-hand curve that circle which of all circles is closest to the parabola at the vertex *A*. If *O* is the centre of the circle, and *F* is midway between *O* and *A*, so that *F* would be the *principal focus* of the spherical reflector, then *F* is also the focus of the parabola; and it is the point to which rays incident parallel to *OA* are *accurately* reflected by the parabolic reflector. And the course



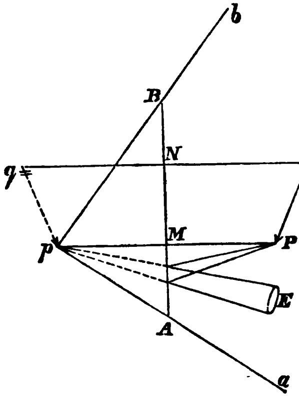
of the rays may be reversed, so that if a bright point be placed at *F* the rays from it, after reflection by the parabolic surface, will proceed in directions strictly parallel to *AO*. Thus a parabolic reflector is useful when it is necessary to throw a beam of light in one direction without loss by diffusion, as for instance if it be required to illuminate a long straight passage. Such a contrivance is found of great service in some light-houses; the light is placed at the focus of the parabolic reflector, and so if the reflector is fixed the light is thrown off in one fixed direction: the reflector is however sometimes made to move about a vertical axis which passes through the focus, and then the beam of reflected light is made to sweep round the horizon in turn, at regular intervals, as for instance once in every two minutes.

212. It is curious to observe how nearly the parabolic reflector will coincide with a spherical reflector of the same focal length when this focal length is large, and the breadth of the reflector not very large compared with the focal length. Let us take for an example the dimensions of the parabolic reflector made by the late Earl of Rosse for his great telescope. Here AF is 54 feet, and the half breadth DE is 3 feet; it is found by calculation that AE is about $\frac{1}{2}$ of an inch, and CD is about $\frac{1}{10000}$ of an inch. And yet slight as is the deviation of the parabolic reflector from the spherical reflector, it is sufficient to render the former much more advantageous for the purpose of the telescope.

XXI. IMAGES FORMED BY REFLECTION.

213. In the two preceding Chapters we have considered various cases in which a pencil of rays going from or towards a point falls on a reflecting surface; corresponding to the focus of the incident rays we obtained always, at least approximately, a focus of the reflected rays. We are now about to consider pencils of rays issuing from the various points of a body, and to shew that the assemblage of the corresponding foci after reflection constitutes an *image* of such a body. We begin with reflection at *plane* surfaces; with this we are familiar from our experience of common looking-glasses: the process here however is a little complicated from the fact that the rays go through the plate of glass before they fall on the metallic surface at the back. In order to avoid for the present this complication the reader may suppose that our plane mirrors are, like those of the ancients, simply polished metallic surfaces; or that we have looking-glasses in which the glass itself is of quite inappreciable thickness.

214. *An object is placed before a plane reflecting surface: it is required to determine the image of it.* Let PQ denote an object placed before a plane reflecting surface



AB. Consider any point of the object as P; draw PM perpendicular to AB, and produce it to p, so that Mp may be equal to MP. The rays which fall from P on the reflector will after reflection proceed as if they came from p, so that p is the image of P. Take any other point of the object as Q, draw QN perpendicular to AB, and produce it to q, so that Nq may be equal to NQ; then in like manner q is the image of Q. Similarly to every other point of the object there is its corresponding image; and thus we find that the image of the whole object is precisely equal to the object itself, and is as far behind the reflecting surface as the object is before it. The image is *virtual*, for the rays of light do not actually pass through it, but only their directions.

215. Suppose in the diagram that AB is vertical; then Q is the top of the object and q is the top of the image: so that the image is upright with respect to the object. Next suppose that AB is horizontal; then a

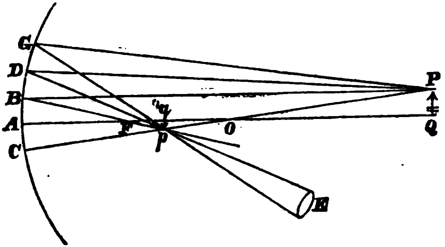
person placed between the mirror and the object, and looking at the object, sees the tip of the arrow towards his right-hand side, but if he looks at the image he sees the tip of the arrow towards his left-hand side. Thus the image as far as regards the direction of right and left may be said to be *reversed* with respect to the object. This circumstance can be recognised at once if a printed page be held upright before a mirror, and the image noticed. A person by looking at his own face in a glass becomes familiar with what we may call its reversed image, and consequently is not a very good judge of a portrait which exhibits him as he appears to others.

216. The image differs from the object which it represents in one respect which must be carefully noticed. The object, which is luminous by itself or by derivation from the sun or some other source, sends forth rays in *all directions*, and so may be viewed by an eye at *any* point whatever, provided no opaque obstacle be interposed: the image however does not send forth rays in all directions, but only in directions within a certain range which depends on the extent of the reflector. For instance, from the point p of the image rays proceed virtually in the plane of the paper in directions comprised between pAa and pBb; so that if an eye in the plane of the paper is to see the point p of the image, the eye must be placed at some point on the right-hand side of AB, between the directions Aa and Bb, both produced indefinitely.

217. Suppose we wish to trace to the eye of a spectator the pencil of rays by which an assigned point of the image is seen; as for instance the point p. Let E denote the eye; join the extreme points of the eye, say the highest and lowest points, with p; and join with P the points where the former straight lines cut the reflecting surface. Thus we determine the extreme rays of the pencil which proceeds from the point P and after reflection reaches the eye in the assigned position. It will be observed that only a small portion of the reflecting surface is used by the eye in its vision of a definite point

of the image; and that various portions of the reflecting surface are used for the vision of various points of the image. In joining the extreme points of the eye with p the parts of the straight lines on the right-hand side of the reflector are drawn *full;* the parts on the left-hand side are drawn *dotted* as representing not *real* rays, but only their *directions:* see Art. 189.

218. *An object is placed before a concave spherical reflector: it is required to determine the image of it.* Let PQ denote an object placed before a concave spherical



reflector; we suppose it more distant from the surface than the centre is. Let O be the centre, OA the axis, A the vertex of the reflector. Consider any point of the object, as P; the rays which fall from P on the reflector will after reflection proceed very nearly to a single point p on the straight line POC: see Art. 207. The position of the point p may be determined by the Rule of Art. 202; or it may be determined thus. *All* the rays from P after reflection pass very nearly through some single point on the straight line POC; consider the ray PB which is parallel to O; then this ray after reflection must take the direction BF, through F the *principal focus* of the reflector: see Art. 200. Hence, the intersection of BF produced with PC determines the position of p, the image of P. Similarly to every other point of the object there is its corresponding image; and in this way we obtain pq the image of the whole object. It is obvious that the image is *real* and *inverted*. We have not however in this case the perfect accuracy which we had in Art. 214. The rays proceeding from any point P do not after reflection pass strictly through a single point. Moreover the image is not absolutely similar to the object; it is found, for example, by mathematical investigation, that if PQ is a straight line the image pq is not straight but slightly curved. Nevertheless if the angle POQ does not exceed a few degrees pq may be taken to be similar to PQ, so as to be an image of it in the ordinary sense of the word *image*.

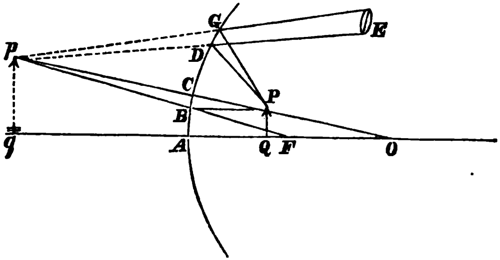
219. The reader may exercise himself in drawing the diagram carefully on a large scale, and then he will find that the image pq is smaller than the object PQ. The same fact is still more conclusively established by the application of a little geometrical knowledge; for regarding PQ and pq as straight lines, the triangles POQ and pOq are *similar*, and pq bears the same proportion to PQ as Oq bears to OQ: then pq is less than PQ by Art. 198.

220. *To trace the course of a pencil by which an assigned point of the image is seen.* Let p denote the point of the image. The position of the eye must lie somewhere within the angle formed at p by two straight lines drawn from the extreme points of the reflector to p, and produced through p. Let E be the position of the eye; join the extreme points of the eye, say the highest and lowest points, with p, and produce these straight lines to meet the reflector at G and D respectively; draw PG and PD. Then PG and PD are the extreme rays of the pencil which proceeds from the point P, and after reflection reaches the eye in the assigned position.

221. We find from the preceding construction that the eye may use only a small portion of the reflector in its vision of a single point of the image. Now, although rays falling from any point on a large portion of the reflector will not all be brought to a single focus, yet rays falling on a small portion of the reflector will be so without sensible error; and thus an image is formed more distinctly than the theory might at first lead us to expect.

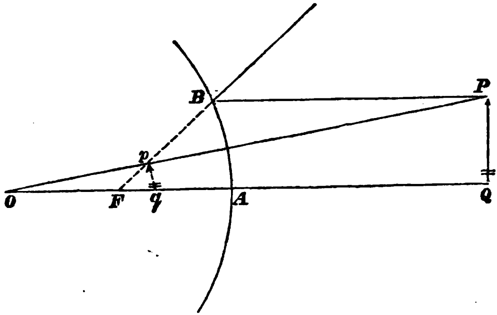
Thus if the diagram be carefully drawn, the rays proceeding from any point P of the object, and actually entering the eye of a spectator will be found to come very nearly from a single point p. This point also will not be far from the position which our somewhat rough process assigns for it. The assemblage of all such points may however be by no means *similar* to the object PQ; for instance, it may be decidedly curved when the object is a straight line. It appears however that in such cases the mind will to some extent control the eye. Suppose, for instance, the object to be an arrow: the mind knows that an arrow is in general straight, and the eye is not a very rigid judge of the distance of the point from which a pencil of rays seems to come, so that the image, though really curved, may seem to the spectator reasonably straight.

222. In Art. 218 we supposed the object to be further from the reflector than the centre is: two other cases have to be noticed; the object may be between the centre and the principal focus, or it may be between the principal focus and the reflector. If the object is between the centre and the principal focus the image is real, in front of the reflector, more distant than the centre; it is inverted and larger than the object. The diagram of Art. 218 will serve for this case; we have only to suppose that pq now denotes the object and PQ the image. There



remains then only the case in which the object is between the principal focus and the reflector. The image is in this case *virtual*, behind the reflector, erect, and larger than the object. In the diagram PQ denotes the object, and pq the image. The ray PB parallel to the axis OA after reflection must pass through F; so that the position of p is determined by the intersection of FB and OP both produced: see Art. 218. Also PG and PD denote the extreme rays of the pencil by which the point p of the image is seen by an eye at E.

223. *An object is placed before a convex spherical reflector: it is required to determine the image of it.* Let PQ denote an object placed before a convex spherical



reflector; let O be the centre, OA the axis, A the vertex of the reflector. Consider any point of the object as P; the rays which fall from P on the reflector will after reflection appear to proceed very nearly from a point p on the straight line PO. The position of p is determined as in Art. 218; draw PB parallel to the axis, then this ray after reflection appears to have come from the *principal focus*, so that p is the point of intersection of PO and BF. Similarly to every other point of the object there is a corresponding image; and in this way we obtain pq the image of the whole object. It is obvious from the diagram that the image is virtual, behind the reflector, erect, and smaller than the object.

224. The relative sizes of the object and the image may be compared in Arts. 222 and 223, as they were in Art. 218. The most obvious proportion is that given at once from the diagram, namely PQ bears the same proportion to pq as OQ bears to Oq. Then, as follows from what we have stated in Art. 198, this proportion is nearly the same as AQ bears to Aq. Again, take the diagram of Art. 218; the straight lines OF and PB are parallel: hence it follows by Geometry that PO bears the same proportion to Op, as BF bears to Fp. But the proportion of PO to Op is the same as that of PQ to pq; so that the object is to the image in the same proportion as BF to Fp. When the angle BFA is small we may take this proportion to be the same as that of AF to Fq. Finally, we observe that if pq be regarded as an object PQ will be its image; and then by what we have just seen pq will bear the same proportion to PQ as AF bears to FQ: therefore PQ will bear the same proportion to pq as FQ bears to AF. These remarks hold also with respect to the cases of Arts. 222 and 223.

225. We have shewn in the preceding Article that the proportion which the object bears to the image can be expressed in *four* different ways: the reader should verify that these four ways give the same result, by numerical examples, such as those of Arts. 202 and 203. In particular from the last two of the four ways we infer that FQ bears the same proportion to FA that FA bears to Fq; hence from the nature of proportion we know that the product of FQ and Fq must be equal to the square of FA.

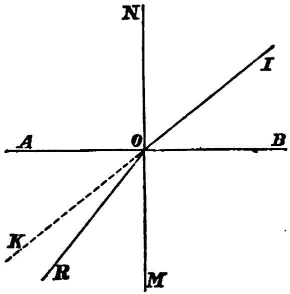
226. The formation of images by spherical reflectors can be well examined by a person if he looks at the image of his face formed by such a reflector, and notices the changes in the image as the position of his face changes. The case of a convex reflector presents itself most familiarly, as such reflectors are occasionally to be found among the ornamental furniture of a house. In this case the image is virtual, erect, and behind the reflector; it is smaller than the object and gradually increases in size

as the spectator approaches the reflector from a distance. In the case of a concave reflector the image is real, inverted, in front of the reflector, smaller than the object and gradually increasing in size as the spectator moves from a distance up to the centre of the reflector. As he moves from the centre up to the principal focus the image is formed behind him, and so is invisible to him. As he moves from the principal focus up to the reflector the image is virtual, erect, behind the reflector, larger than the object, and gradually diminishing in size. It is found in practice that the eye is sometimes unable at first to recognise an image as being in *front* of the reflector when the theory shews that it is there: it seems that by long familiarity with the operation of *plane* reflectors the eye is naturally disposed to refer an image to some position *behind* the reflector.

XXII. LAWS OF REFRACTION.

227. We have said in Art. 185 that when a ray of light moving in one medium arrives at another medium part of the light is reflected and part enters the new medium. Of that which enters the new medium more or less is lost in the medium; to use the technical word it is *absorbed.* In the case of opaque bodies, such as plates of metal or beams of wood, the whole entering light is thus absorbed; but it is known that films of metal or shavings of wood can be taken so thin that rays of light will pass through them. In the case of transparent bodies a large part of the light which falls on them is transmitted through them, and we are now about to examine the change of course which a ray of light experiences when it passes from one medium to another. The direction of the ray seems to be *bent* through some angle; so that the ray is said to be *refracted* or to suffer *refraction.*

228. Let IO represent the course of a ray of light



moving in a vacuum, which is incident at O on a plane surface of any transparent medium: let AOB represent this surface. Let OR represent the course of the ray within the medium. Draw NOM at right angles to the surface. Then IO and OR are straight lines; for, as we have already said, light moves in a straight line so long as it keeps to one medium. IO is called the *incident* ray, OR the *refracted* ray, and NOM the *normal to the surface at the point of incidence;* the angle ION is called the *angle of incidence,* and the angle ROM the *angle of refraction.* The direction OR is determined by the following laws: OR is in the same plane with OI and ON; and the sine of the angle ION bears a fixed proportion to the sine of the angle ROM. With the aid of the preceding definitions the laws may be stated verbally thus: *the incident and refracted rays are in the same plane with the normal to the surface at the point of incidence, and on opposite sides of the normal; and the sine of the angle of incidence bears a fixed proportion to the sine of the angle of refraction.* The latter part of the law is called for brevity the *law of sines.*

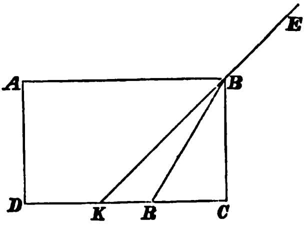
229. The number which expresses the fixed proportion of the sine of the angle of incidence to the sine of the angle of refraction is different in different media.

For water it is about $\frac{4}{3}$; so that the sine of the angle *ION* is $\frac{4}{3}$ times the sine of the angle *ROM*, or the sine of the angle *ROM* is $\frac{3}{4}$ of the sine of the angle *ION*. For glass this number is $\frac{3}{2}$; for diamond about $\frac{5}{2}$. The number for any medium is called the *index of refraction* for that medium. In passing from vacuum to any medium the ray is bent as in the diagram so as to be brought *nearer* to the normal.

230. Let us suppose that light is passing from vacuum to water, and consider in some detail the various cases which may occur. If the direction of the incident ray is *NO* then the direction of the refracted ray is *OM*; it is convenient to speak of *OM* as the refracted ray still, though in this case there is no *bending*, for the ray goes straight on. In every other case the ray undergoes some deviation; for instance if *IO* represents an incident ray then the ray in water does not go along *OK*, which is the production of *IO*, but along *OR*, which is nearer to the normal *OM*. The angle *KOM* is equal to the angle *ION* by Art. 154: the angle *KOR* is the deviation which the ray undergoes. The greater the angle of incidence is, the greater will the deviation be. Suppose the angle *NOI* to be almost equal to a right angle; by the Table of Art. 161 the sine of this angle will be 1; and by Art. 229 the sine of the angle *ROM* will be $\frac{3}{4}$, that is ·75: hence by the same Table the angle *ROM* will then be about 49°.

231. It is an experimental law in Optics, which is thoroughly established, that if a ray can go by any path from one point to another, then a ray can also go back along the same path from the second point to the first. Thus if *IO* and *OR* in Art. 228 constitute the path of a ray between *I* and *R* in passing from vacuum into water, then a ray can go along *RO* and *OI* from water to a vacuum. This principle enables us to give an interesting experi-

mental verification of the laws of refraction. Let *ABCD* be a vessel, and suppose an eye at *E* looking exactly over

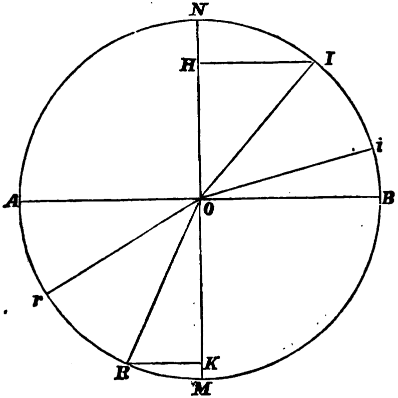


the edge of the vessel in the direction *EBK*. Then an object at the bottom of the vessel, such as a stone or coin, placed at *R*, would be invisible to the eye. But let the vessel be now filled with water, then it may be possible for the eye to see the object; all that is necessary is that *R* should be at a certain distance from *C*. Let the proper position of *R* be assigned by trial; and let the angles *KBC* and *RBC* be measured: then it will be found that the sine of the angle *RBC* is $\frac{3}{4}$ of the sine of the angle *KBC*.

232. By such contrivances as that of the preceding Article we may verify the truth of the laws of refraction. By putting the eye in different positions we can make the angle *KBC*, which is equal to the angle between *BE* and *CB* produced, take any value we please; and it will be found that for water the *law of sines* always holds, and that the *index of refraction* is $\frac{4}{3}$. In like manner we may experiment on glass. Suppose *ABCD* in Art. 231 to be a thick plate of glass; then the end *BC* may be blackened or covered with paper to prevent any light from passing through it: let a wafer be placed on some part of *KC* so as to be seen by an eye at *E*, and results will be obtained similar to those for water.

233. The reader will observe that he can draw with tolerable accuracy the course of the refracted ray corresponding to a given incident ray, when the index of refrac-

tion is known. Let AOB denote the refracting surface, and suppose that the angle of incidence is 40°. Draw NOM at right angles to AB; and set off the angle NOI equal to 40°, which may be done with the aid of a graduated



brass semi-circle, such as is usually to be found in a box of mathematical instruments. By the Table of Art. 161 we see that the sine of an angle of 40° is ·643. Suppose the medium into which the ray is proceeding to be glass; then the sine of the angle of refraction is $\frac{2}{3} \times {\cdot}643$, that is ·429: by the Table this corresponds to an angle of $25\frac{1}{2}°$. Hence if we take the angle ROM equal to $25\frac{1}{2}°$ then OR is the direction of the refracted ray. If we had not the Table to consult we must proceed thus. Make the angle ION equal to 40°, and from I draw IH perpendicular to ON. Then measure off IH on any finely divided scale of equal parts, as one of tenths or twelfths of an inch; suppose IH to be equal to 9 such divisions. Then $\frac{2}{3} \times 9 = 6$; and we must find a point R such that its perpendicular distance RK from OM is equal to 6 of the divisions on the scale: then OR is the direction of the refracted ray.

234. The *law of sines* seems extremely simple, but the discovery of it did not take place until long after the origin of science; the ancient Greeks wrote on Optics, but they were ignorant of this law. It could not fail to be observed that if the angle of incidence were increased the angle of refraction would also be increased; but the precise connection between these two angles was not obvious. The *law of sines* was discovered about 1621 by Snell; but it was first published by Descartes who had seen Snell's papers.

235. Tables of Indexes of Refraction are to be found in several works; the values given by various experimenters do not quite agree, and the principal reason of the diversity will appear hereafter when we treat of *colour*. The following list is selected from such Tables.

Substance.	Index of Refraction.
Diamond	2·439
Sapphire	1·794
Rock Salt	1·557
Alcohol	1·372
Sea Water	1·343
Pure Water	1·336
Ice	1·310

Two kinds of glass, called respectively *crown* glass and *flint* glass, are much used in the construction of optical instruments; the composition of each kind, as furnished by different manufacturers, varies slightly. The index of refraction for crown glass lies between 1·530 and 1·563, and for flint glass between 1·576 and 1·732. We shall continue however to use for the purpose of illustration $\frac{4}{3}$ as the index of refraction for water, and $\frac{3}{2}$ as the index of refraction for glass.

236. Suppose we take the index of refraction of a substance, square it, and subtract unity from the result: the remainder is a quantity which has some importance in theory, and has received various names, as *refractive power*, *refractive action*, *refractive force*, *absolute refrac-*

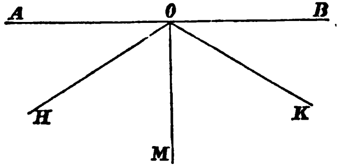
tive power. For example, taking the index for glass as $\frac{3}{2}$, we have $\frac{9}{4}-1$, that is $\frac{5}{4}$, for this remainder. Newton found that *the refractive power* was very nearly proportional to the density of bodies in many cases. The large index of refraction of diamond attracted his attention, and led him to conjecture that diamond "probably is an unctuous substance coagulated." Modern chemistry has ascertained that the substance of diamond is the same as that of carbon; and the circumstance has often been noticed as an example of Newton's sagacity. Dr Young considers it as still more singular that he also imagined water to consist of a "combination of oily or inflammable particles with others earthy or not inflammable." See Newton's *Optics*, 1721, page 249, Young's *Lectures on Natural Philosophy*, Lecture xxxv.

XXIII. VARIOUS CASES OF REFRACTION.

237. If a ray of light proceed from vacuum into any medium the *angle of incidence* may be as great as we please, up to a right angle as the extreme case; but the *angle of refraction* cannot exceed a certain limit, which is different for different media, but is always less than a right angle. We have in fact found in Art. 230 that this limit in the case of water is about 49°. As another example consider the case of glass; suppose the angle of incidence to be almost a right angle, and represent it by the angle *NOi* in the diagram of Art. 234, where it is drawn for distinctness a few degrees less than a right angle. The sine of *NOi* may then be taken as 1, and consequently the sine of the corresponding angle of refraction will be $\frac{2}{3}$. Now $\frac{2}{3}=\cdot667$, and the Table of Art. 161 shews that this

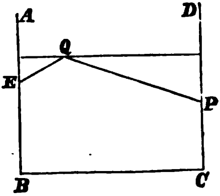
corresponds to an angle of about 42°; let this be represented by the angle *MOr*. Then if any ray be incident on the surface of the glass at *O*, between the directions *NO* and *BO*, the corresponding refracted ray must lie between *OM* and *Or*. The extreme angle for any medium, which is here denoted by *MOr*, is called the *critical angle*. Thus we may say that the *critical angle* for any medium is the angle of refraction when a ray passing from vacuum into that medium has its angle of incidence equal to a right angle. Or we may say that the *critical angle* for any medium is that angle the sine of which is equal to the reciprocal of the index of refraction for the medium.

238. Let *AB* be a plane boundary, separating a medium below it from a vacuum above it. Let *HO* represent the course of a ray in the medium, incident at *O*, so



as to make the angle *HOM* between the ray and the normal greater than the critical angle for the medium. As no ray can come from the vacuum and pass along *OH*, we may naturally expect that the ray proceeding along *HO* cannot get out of the medium at *O* into the vacuum. Experience confirms this, and shews that the ray is reflected at *O*, according to the laws of reflection; that is, the ray takes a course *OK* in the plane containing *OH* and *OM*, and the angle *MOK* is equal to the angle *MOH*. Moreover it is found that in this case the reflected ray of light is almost as bright as the incident ray, scarcely any of the ray being lost: hence this is called *total reflection*. The least value of the angle *AOH* for which the ray will pass out occurs when the angle *HOM* has the greatest admissible value; so that it is the complement of the critical angle.

239. The brightness of light which has been totally reflected may be seen by taking a tumbler formed of clear

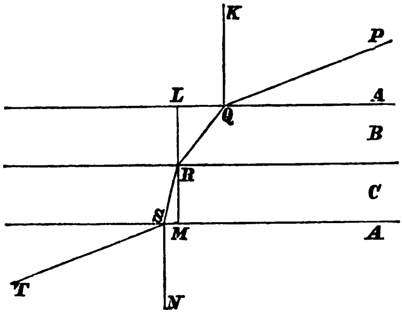


plain glass, putting water into it, and holding it a little above the eye, so as to look at the under side of the surface of the water. Let E denote the position of the eye, and $ABCD$ a section of the tumbler. Then rays entering the water through CD, proceed in such a direction as PQ, are totally reflected at Q, pass out through AB, and enter the eye. The under side of the surface of the water seems to shine with a kind of metallic lustre.

240. We have hitherto supposed that light passes from vacuum to a medium, such as water or glass; but we may now consider the case in which light passes from one medium to another medium, as from glass to water. The laws of refraction are found by experiment to be precisely the same as in Art. 228, and the only thing required is the value of the index of refraction. The Rule for finding this is best put in the following form, in order to be easily remembered; *the index of refraction from* A *to* C *is equal to the product of the index of refraction from* A *to* B *into the index of refraction from* B *to* C. Here A, B and C may denote any media, vacuum included. For example, suppose we require the index of refraction from water to glass; we will let A denote vacuum, B denote water, and C denote glass. The index of refraction from vacuum to glass is $\frac{3}{2}$; the

index of refraction from vacuum to water is $\frac{4}{3}$; hence according to the Rule, $\frac{3}{2}$ is equal to the product of $\frac{4}{3}$ into the required index, so that the required index is $\frac{3}{2}$ divided by $\frac{4}{3}$, that is $\frac{9}{8}$.

241. The preceding Rule is founded on an experimental fact which may be stated thus: *when a ray of light passes through any number of media separated by parallel plane surfaces, if any two of these media are identical the directions of the ray in them are parallel.* Let *PQRST* be the course of a ray which passes from a medium *A*, through the media *B* and *C*, into a medium *A*

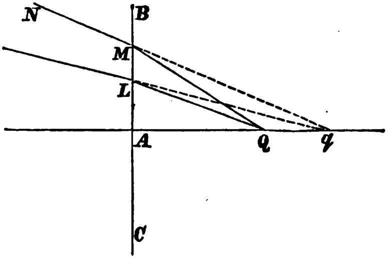


again; and let these media be separated by parallel plane surfaces. Draw *QK*, *LRM*, and *SN* respectively at right angles to the plane surfaces. Then these three straight lines are all parallel; and therefore the angle *LRQ* is equal to the angle between *RQ* and *KQ* produced, and the angle *SRM* is equal to the angle between *RS* and *NS* produced: see Art. 155. Thus if we let *A*, *B*, and *C* stand for vacuum, water, and glass respectively, as in Art. 240, we have, sine of $TSN = \frac{3}{2}$ sine of *SRM*, and sine of $PQK = \frac{4}{3}$ sine of *LRQ*. Now by the experimental fact stated above the angles *TSN* and *PQK* are equal; thus $\frac{3}{2}$ sine of $SRM = \frac{4}{3}$ sine of *LRQ*; and from this it follows that sine of $LRQ = \frac{9}{8}$ sine of *SRM*. This *demonstrates* the result obtained from the Rule in Art. 240.

242. The index of refraction from vacuum to air is found to be very nearly 1·0003; and thus strictly speaking the index of refraction from air into any medium is not the same as from vacuum into that medium. Suppose, for instance, that the index of refraction from vacuum into diamond is 2·439, then the index of refraction from air into diamond will be the quotient of 2·439 divided by 1·0003; but the correction thus introduced is scarcely sensible when we restrict ourselves to three places of decimals. Thus we may henceforth consider the refraction between air and a medium as practically identical with the refraction between vacuum and that medium.

243. The index of refraction just stated for air applies to air in the ordinary state; when the density of air diminishes so does the index of refraction. Small however as the index of refraction of air may seem to be, yet it is known from Astronomy that the results are striking and important. Thus by means of the refraction of the air the Sun and Moon become visible to a spectator before they have really risen, and continue visible after they have really set: the air renders them visible when they are actually below the horizon in the same manner as the water renders the coin visible in the experiment described in Art. 231.

244. *Rays diverge from a point and fall on a plane refracting surface: it is required to find their course after refraction.* Let BC denote the plane refracting surface, Q the origin of the light; draw QA from Q perpendicular



to the refracting surface. Let QM represent a ray incident at M on the refracting surface; let MN denote the direction of the ray after refraction at the surface, and produce NM to meet AQ produced at q. The angle AQM is equal to the angle of incidence of the ray QM at M, and the angle AqM is equal to the angle of refraction: see Art. 241. Hence the sine of the angle AQM bears to the sine of the angle AqM the proportion expressed by the index of refraction. This leads by a little mathematical investigation to the result that Mq bears to MQ the proportion expressed by the index of refraction; for example, if the medium is glass Mq is $\frac{3}{2}$ times MQ, and if the medium is water Mq is $\frac{4}{3}$ times MQ. If the angle MQA is very small MQ is nearly equal to AQ, and Mq is nearly equal to Aq; so that we obtain the following result as very nearly true: *Aq is equal to the product of AQ into the index of refraction.* Thus if rays from a point fall very nearly in a perpendicular direction on a plane surface of glass they appear after refraction to come very nearly from a virtual focus $\frac{3}{2}$ times as distant from the surface as the origin; if the plane surface is that of water the virtual focus is $\frac{4}{3}$ times as distant from the surface as the origin; and so on.

245. If the diagram of the preceding Article be carefully drawn on a large scale it will be found that, the further M is from A, the further is the point denoted by q, where the direction of the refracted ray crosses AQ produced. Suppose we draw two incident rays, as for instance QM and QL; then their directions after refraction will not strictly meet on the *axis*, as we may call AQ, but at some point beyond the axis. It is only approximately, and on the supposition that the angle MQA is small, that the directions of the refracted rays meet at a single point on the axis: this point is then the *virtual focus* of the refracted rays.

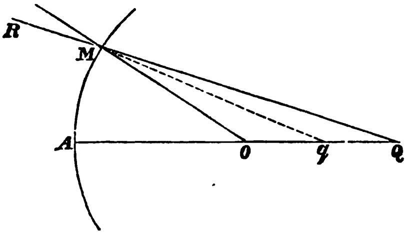
246. In the preceding Article we have supposed the rays to pass from air or vacuum into a medium; that is, we suppose air or vacuum on the right-hand side of BC, and the medium on the left-hand side. The diagram however will serve for the case in which the rays pass from a medium to air or vacuum. We have only to suppose q to be the origin of the light, and Q the point at which the directions of the rays meet after refraction: the medium must now be supposed to be on the right-hand side of BC, and air or vacuum on the left-hand side.

247. *A pencil of parallel rays consists of parallel rays after refraction.* This proposition is strictly accurate, and can be demonstrated by the aid of geometry; but the reader may be content to take it as established by experiments. Some impression of its truth may however be obtained by considering such a diagram as that of Art. 244. For suppose M and L to remain fixed, and also the *direction* MQ, but let Q retire continually from the surface; then the corresponding point q retires continually from the surface: thus the incident rays QM and QL approach continually to a state of parallelism, and so also do the directions of the refracted rays qM and qL.

XXIV. REFRACTION AT CURVED SURFACES.

248. Refraction may take place at curved surfaces as well as at plane surfaces. Any very small portion of a curved surface may be considered as coinciding with a corresponding portion of the plane which would touch the surface there. We shall confine ourselves to spherical surfaces; and, as we have stated in Art. 195, a plane surface touching a sphere at any point is at right angles to the radius of the sphere there. The *axis* and the *vertex* of a spherical refracting surface are defined in the same way as for a spherical reflecting surface: see Art. 195.

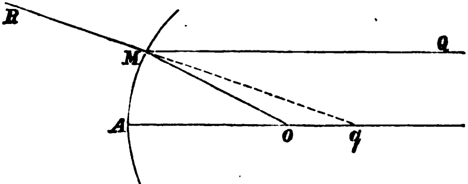
249. *Rays of light fall on a concave spherical refractor from a point on the axis: it is required to determine their course after refraction.* Let O represent the



centre of the sphere, OA the axis of the refractor, A the vertex. We will suppose the origin of light to be further from the vertex than O is; let it be at Q in AO produced. Let QM represent one of the rays, incident at M on the spherical refracting surface; and let MR be the direction of the ray after refraction. Produce RM to meet the axis at q. Join OM. Then QMO is the angle of incidence, and qMO is equal to the angle of refraction. If the diagram be carefully constructed it will be found that the rays after refraction do not in general proceed as if they came exactly from a single point; but if the angle MOA does not exceed 8^0 or 10^0 the rays will proceed very nearly as if they came from a single point. This point is the *virtual focus* of the refracted rays; we denote it by q. The following is the Rule which connects the distances of Q and q from A: *the difference of the reciprocals of the distances of O and Q is equal to the product of the index of refraction into the difference of the reciprocals of the distances of O and q.* For example, suppose AO to be 12 inches, AQ to be 18 inches, and the index of refraction to be $\frac{4}{3}$. Then $\frac{1}{12} - \frac{1}{18} = \frac{3}{36} - \frac{2}{36} = \frac{1}{36}$; divide by $\frac{4}{3}$; thus we obtain $\frac{1}{36} \times \frac{3}{4}$, that is $\frac{1}{48}$. This then is the difference of the reciprocals of the distances of O and q; hence the reciprocal of the distance of q is $\frac{1}{12} - \frac{1}{48}$, that is $\frac{4}{48} - \frac{1}{48}$, that is $\frac{3}{48}$, that is $\frac{1}{16}$. Therefore Aq is 16 inches. For another example suppose AO to be 12 inches, AQ to be 16 inches, and the index of refraction to be $\frac{3}{2}$. Then $\frac{1}{12} - \frac{1}{16} = \frac{4}{48} - \frac{3}{48} = \frac{1}{48}$; divide by $\frac{3}{2}$; thus we obtain $\frac{1}{48} \times \frac{2}{3}$, that is $\frac{1}{72}$. This then is the difference of the reciprocals of the distances of O and q. Hence the reciprocal of the distance of q is $\frac{1}{12} - \frac{1}{72}$, that is $\frac{6}{72} - \frac{1}{72}$, that is $\frac{5}{72}$. Therefore Aq is $\frac{72}{5}$ inches, that is $14\frac{2}{5}$ inches.

250. Suppose that instead of a pencil of rays proceeding from a point on the axis we have a pencil of rays all *parallel* to the axis; the rule of Art. 249 will enable us to assign the position of the virtual focus q from which the rays appear

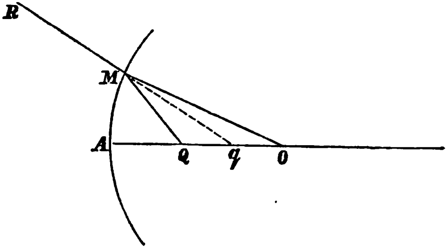
very nearly to proceed after refraction. For this case is practically the same as that of a pencil of rays proceeding from a point on the axis which is excessively remote, so that the reciprocal of the distance of Q is zero: see Art. 202. Let O represent the centre of the sphere, OA the axis of the refractor, A the vertex. Let QM represent one of the incident rays parallel to the axis, MR its direction after refraction; produce RM to meet AO produced at q. The position of the point q is then called the *principal focus* of the refracting surface, and its distance from the vertex is called the *focal length;* this position is thus determined according to the Rule of Art. 249: *the reciprocal of the radius is equal to the product of the index*



of refraction into the difference of the reciprocals of the radius and of the focal length. Thus, suppose the radius to be 12 inches, and the index of refraction to be $\frac{4}{3}$; then we have the following calculation: the reciprocal of the radius is $\frac{1}{12}$; divide this by the index of refraction, and we get $\frac{1}{12} \times \frac{3}{4}$, that is $\frac{1}{16}$; this therefore is the difference between the reciprocals of the radius and the focal length, so that the reciprocal of the focal length is $\frac{1}{12} - \frac{1}{16}$, that is $\frac{4}{48} - \frac{3}{48}$, that is $\frac{1}{48}$; the focal length therefore is 48 inches.

251. In various Rules of Optics we have to use the number which is obtained by *subtracting unity from the index of refraction*, and accordingly we shall give a name to this number, and call it the *index of deviation;* the propriety of the name will be more obvious when we treat of refraction through a prism. We may then give this simple Rule for finding the *focal length* of a spherical refracting surface: *multiply the radius by the index of refraction, and divide the product by the index of deviation.* Thus, taking the same elements as in the Example of Art. 250, we have the index of refraction $\frac{4}{3}$, and therefore the index of deviation is $\frac{1}{3}$; we must then multiply 12 by $\frac{4}{3}$ and divide the product by $\frac{1}{3}$: the result is 48.

252. In the next place suppose that the origin of light is between the centre and the vertex. The diagram is then drawn in a similar manner to that in Art. 249; and the Rule which connects the distances of Q and q from A



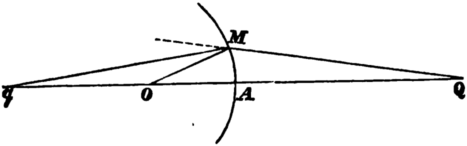
may be given in the same words as in that Article; but in applying the Rule we must remember, what the diagram shews, that AQ and Aq are now less than AO. For example, suppose as before that AO is 12 inches, and the index of refraction $\frac{4}{3}$; and let AQ be 6 inches. Then

$\frac{1}{6}-\frac{1}{12}=\frac{2}{12}-\frac{1}{12}=\frac{1}{12}$; divide by $\frac{4}{3}$; thus we obtain $\frac{1}{12}\times\frac{3}{4}$, that is $\frac{1}{16}$. This then is the difference between the reciprocals of the distances of O and q from A; hence the reciprocal of the distance of q is $\frac{1}{12}+\frac{1}{16}$, that is $\frac{4}{48}+\frac{3}{48}$, that is $\frac{7}{48}$. Therefore Aq is $\frac{48}{7}$ inches, that is $6\frac{6}{7}$ inches. For another example suppose AO to be 12 inches, AQ to be 8 inches, and the index of refraction to be $\frac{3}{2}$. Then $\frac{1}{8}-\frac{1}{12}=\frac{3}{24}-\frac{2}{24}=\frac{1}{24}$; divide by $\frac{3}{2}$; thus we obtain $\frac{1}{24}\times\frac{2}{3}$, that is $\frac{1}{36}$. This then is the difference between the reciprocals of the distances of O and q from A; hence the reciprocal of the distance of q is $\frac{1}{12}+\frac{1}{36}$, that is $\frac{3}{36}+\frac{1}{36}$, that is $\frac{4}{36}$, that is $\frac{1}{9}$. Therefore Aq is 9 inches.

253. The results of Arts. 249, 250, and 252 may be summed up thus. The points Q and q always move in the same direction. As Q moves from a very remote distance on the right-hand side up to the refracting surface, q moves from the principal focus up to the refracting surface; and the points coincide when Q is at O and when it is at A. It is obvious that when Q is at O the rays which fall on the refracting surface proceed in the medium as if they came *accurately* from a point, namely from O. It can be shewn by mathematical investigation that there is another position of Q such that the rays from it will after refraction proceed as if they came *accurately* from a point: this is when Q is at the distance from A which is equal to the product of the radius into the index of refraction increased by unity; and then q is at a distance from A which is equal to that of Q divided by the index of refraction. For instance, if the radius is 12 inches, and the

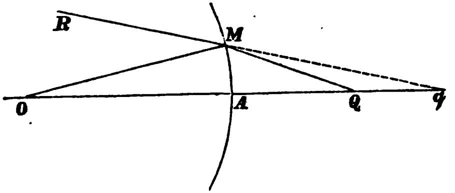
index of refraction is $\frac{4}{3}$, then to obtain the distance from A of this position of Q, estimated in inches, we add $\frac{4}{3}$ to 1 and multiply by 12; thus we get $\frac{7}{3} \times 12$, that is 28: the corresponding distance of q estimated in inches is 28 divided by $\frac{4}{3}$, that is $28 \times \frac{3}{4}$, that is 21.

254. *Rays of light fall on a convex spherical refractor from a point on the axis: it is required to determine their course after refraction.* We shall treat this rather briefly after the full investigation with respect to the concave refractor. Let O represent the centre of the sphere, OA the axis of the refractor, A the vertex.



There are two cases to be considered; we first suppose the origin of light at a great distance from the refractor. Let Q represent the origin of light, QM one of the rays incident at M on the spherical refracting surface, Mq its direction after refraction. When Q is at a very remote distance from A the rays after refraction converge very nearly to a point on the left-hand side of A, which is called the *principal focus*, and the distance of which from A is called the *focal length* of the refractor: this distance is given by the same Rule as in Art. 251. As Q moves towards the surface q moves away from it, the two continuing to move in the same direction. The Rule which connects the distances of Q and q from A is the following: *the sum of the reciprocals of the distances of O and Q is equal to the product of the index of refraction into the difference of the reciprocals of the distances of O and q.* When Q arrives at a certain point q is at a very remote distance on the left-hand side, or, in other words, the rays after refraction are *parallel* to the axis. The distance of Q from A when this takes place is equal to the *radius divided by the index of deviation:* thus for glass the radius must be divided by $\frac{3}{2} - 1$, that is by $\frac{1}{2}$; so that the distance is equal to twice the radius.

Next suppose the origin of light to be nearer to the surface than the point just determined. Let Q represent



the origin, QM one of the rays incident at M on the refracting surface, MR the direction of the refracted ray; then if RM be produced it will meet the axis at some point q to the right of A. The distances of Q and q from A are connected by the Rule of Art. 249, changing the word *difference* to *sum*, so that it is: the *sum of the reciprocals of the distances of O and Q is equal to the product of the index of refraction into the sum of the reciprocals of the distances of O and q.* As Q moves up to the refracting surface from the point for which q is very remote, so also does q, starting, as it were, from a point on the right-hand side very remote.

255. For an example suppose that the convex refracting surface is formed of glass, so that the index of refraction is $\frac{3}{2}$; and let the radius be 12 inches. The *index of deviation* is here $\frac{1}{2}$; thus the focal length is $12 \times \frac{3}{2} \div \frac{1}{2}$, that is $12 \times \frac{3}{2} \times 2$, that is 36: the *principal*

focus then is 36 inches to the left of A, and to this point the rays will converge when Q is at a very remote distance on the right-hand side. The position of Q which separates the two cases of Art. 254 is at a distance from A equal to twice the radius, that is at a distance of 24 inches. Suppose Q at a *greater* distance, say 36 inches from A. Then $\frac{1}{36}+\frac{1}{12}=\frac{1}{36}+\frac{3}{36}=\frac{4}{36}=\frac{1}{9}$; divide by the index of refraction; thus we get $\frac{1}{9}\times\frac{2}{3}$, that is $\frac{2}{27}$. This therefore is the difference between the reciprocals of the distances of O and q; hence the reciprocal of the distance of q is $\frac{1}{12}-\frac{2}{27}$, that is $\frac{9}{108}-\frac{8}{108}$, that is $\frac{1}{108}$: so that q is 108 inches to the left of A. Next suppose Q at a *less* distance, say 12 inches, from A. Then $\frac{1}{12}+\frac{1}{12}=\frac{2}{12}=\frac{1}{6}$; divide by the index of refraction; thus we get $\frac{1}{6}\times\frac{2}{3}$, that is $\frac{1}{9}$. This therefore is the sum of the reciprocals of the distances of O and q; hence the reciprocal of the distance of q is $\frac{1}{9}-\frac{1}{12}$, that is $\frac{4}{36}-\frac{3}{36}$, that is $\frac{1}{36}$: so that q is 36 inches to the right of A.

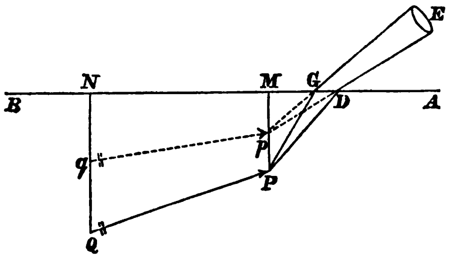
256. Other cases which may be proposed will be found to be really included in what we have given. Thus suppose rays to be *converging to* a point on the axis of a convex spherical refracting surface; the diagram of Art. 249 will be applicable when the point is further from the surface than the centre is, and the diagram of Art. 252 when the point is nearer to the surface: in both cases the light must be supposed to come from the left-hand side, and the medium to be on the right-hand side of the spherical surface. Similarly the diagrams of Art. 254 can be rendered applicable to the case in which rays are *converging to* a point on the axis of a concave spherical refractor. Also our diagrams suppose that light passes from a vacuum into a medium, or from one medium into another which has a greater refractive index; but they can be readily adapted

to the case where light passes from a medium into vacuum, or from one medium into another which has a less refractive index. Thus in Art. 249 if light is to pass from a medium into vacuum we have only to suppose that q is the origin of the light, and that QM produced through M is the course of the ray refracted at M.

257. Hitherto we have considered the origin of light to be *on the axis* of the refracting surface. If the origin is not on the axis we must join the origin with the centre and treat this straight line as if it were the axis: see Art. 207. The straight line passing through the origin and the centre is called a *secondary axis*, and the pencil of light from the origin is called a *secondary pencil*. When a pencil of rays is refracted by a spherical surface the directions of the rays never pass accurately through a single point, except in the special cases noticed in Art. 253. This fact is expressed, as in the case of spherical reflectors, by saying that there is *spherical aberration*.

XXV. IMAGES FORMED BY REFRACTION.

258. *An object is placed before a plane refracting surface: it is required to determine the image of it.* Let



PQ be an object placed before a plane refracting surface AB; we will suppose that on the other side of AB there is a vacuum, or a medium having a less refractive index than the medium in which PQ is situated. Consider any point of the object as P; draw PM perpendicular to the plane refracting surface. The rays which fall from P on the surface will after refraction proceed very nearly as if they came from a single point p in PM; the distance of p from M is obtained by dividing the distance MP by the index of refraction for the medium: see Art. 246. Thus p may be regarded as the image of P. Similarly take any other point Q of the object and determine its image q; and in like manner proceed with the whole object. In this way we obtain the image pq of the object PQ; it is virtual and erect. If PQ is a straight line then pq is also a straight line. If PQ is parallel to the refracting surface so also is pq, and pq is of the same length as PQ. If, as in the diagram, PQ is inclined to the refracting surface so also is pq, and pq is less than PQ; if pq and PQ be produced the straight lines will meet at some point of the refracting surface.

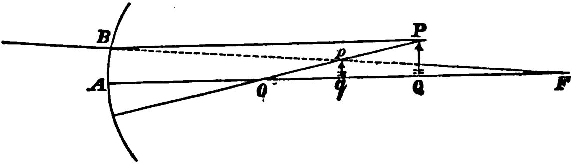
259. *To trace the course of a pencil by which an assigned point of the image is seen.* Let p denote the point of the image. The position of the eye must lie somewhere within the angle formed by straight lines drawn from p to the extreme points of the refracting surface, and produced. Let E be the position of the eye; join the extreme points of the eye, say the highest and lowest points, with p; and let the straight lines cut the refracting surface at G and D respectively: draw PG and PD. Then PG and PD are the extreme rays of the pencil which proceeds from the point P and after refraction reaches the eye in the assigned position.

260. Suppose, for example, that the medium in which AB is situated is water, so that the index of refraction is $\frac{4}{3}$. Then Mp is the quotient of MP divided by $\frac{4}{3}$; that is Mp is $\frac{3}{4}$ of MP. Similarly the image of any other point of the object is at three-fourths of the distance of that point from the refracting surface. This case of the formation of an image is very familiar. For if a straight stick be partly immersed in water, in an oblique position, the

image of the immersed part seems nearer to the eye than the corresponding part of the stick would be if there were no water; so that the stick appears as if it were bent at the point where it enters the water. We observe too that the apparent depth of a stream or pond of clear water is less than the real depth, namely about three-fourths of it.

261. If instead of a vacuum above AB there is any medium which has a less refractive index than the medium in which PQ is situated the diagram is of the same nature as in Art. 258; we must now use the refractive index from the upper medium into the lower instead of the refractive index from vacuum into the lower medium, in connecting the distance pM with the distance PM. If however the upper medium have the greater refractive index the diagram becomes altered: the image is then further from the refracting surface than the object.

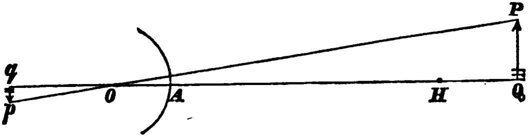
262. *An object is placed before a concave spherical refracting surface: it is required to determine the image of it.* Let PQ denote an object placed before a concave



spherical refracting surface; we will suppose it more distant from the surface than the centre is. Consider any point P of the object; the rays which fall from P on the refracting surface will after refraction proceed very nearly as if they came from a single point p on the straight line PO; so that p is the virtual image of P. Similarly to every other point of the object there is its corresponding image; and in this way we obtain pq the image of the whole of the object: this image is virtual, erect, and less than the object. The position of this image may be determined by calculation: see Arts. 249 and 257. It may also be determined by the aid of the *principal focus*, as in the case of images formed by spherical reflectors. Suppose for instance that the refracting surface is composed of glass; then the focal length is three times the radius: see Art. 251. Thus if F denote the principal focus AF is three times AO. Draw PB parallel to AO, then the direction of the ray PB after refraction will pass through F; hence p is at the intersection of PO and BF, so that its position is determined. As in Art. 218 it is found that the image is not *exactly* similar to the object; but it is nearly so if the angle POQ does not exceed a few degrees. In like manner, if the object is nearer to the surface than the centre is, the image can be determined; as before it will be found virtual, erect, and less than the object.

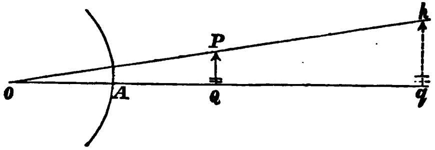
263. *An object is placed before a convex spherical refracting surface: it is required to determine the image of it.* This may be treated briefly after the investigation of the preceding Article: there are two cases as in Art. 254.

If the object is at a very remote distance on the right-hand side the image is real, inverted, and less than the object; it is on the left-hand side nearly at the *principal*



focus. As the object approaches the surface the image moves towards the left hand, and when the object is at a certain distance the image is as far from O as the object is, and just equal to it. In this position the distance of Q from the vertex is equal to the diameter of the sphere divided by the index of deviation. We will denote this position by H. Thus if the refracting surface is supposed to be of glass the distance of H from A will be four times the radius; if it be of water the distance will be six times the radius. As the object moves from H to a point midway between H and A the image continues to move towards the left hand, and is greater than the object; becoming at last very great and very remote.

As the object moves from the point just noticed up to the refracting surface the image is erect, virtual, and



larger than the object, and it moves from a very remote distance on the right-hand side up to the refracting surface.

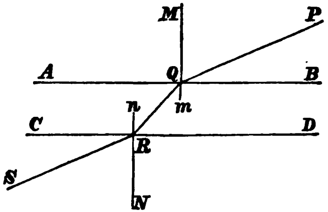
As in former cases we may draw the pencil by which any point of the image is seen by an eye in an assigned position: see Arts. 217, 220, 222, and 258.

XXVI. SUCCESSIVE REFRACTION AT PLANE SURFACES.

264. Rays of light after being refracted by a plane or spherical surface may fall on another surface and be refracted again. The most common and the most important case is that in which the rays proceed from air into a medium, pass through that medium, and come out into the

air again; this case occurs perpetually in the construction of optical instruments, such as telescopes and microscopes: the medium is then almost always glass. We will consider in the present Chapter successive refraction at *plane* surfaces; there are two cases to notice; for the second surface may be parallel to the first, or may be inclined to it at some angle.

265. A portion of any medium bounded by parallel planes is called for shortness a *plate;* a piece of common window glass is an example. *The direction of a ray of light after passing through a plate is parallel to its direction before entering the plate.* Let *PQRS* represent the course of a ray which passes through the plate bounded



by *AB* and *CD*. Draw *MQm* at right angles to *AB*, and *NRn* at right angles to *CD*. Then, since *MQm* and *NRn* are parallel straight lines, the angle *RQm* is equal to the angle *QRn*: see Art. 155. Since *PQR* is the course of a ray, it follows by the nature of light that a ray would also go along the course *RQP*. Then *RQ*, regarded as incident at *Q*, makes with the normal to the surface there an angle equal to that which *QR* makes at *R* with the normal to the surface there; and therefore the angles at emergence, *PQM* and *SRN*, must be equal. Hence by the nature of parallel straight lines *RS* is parallel to *PQ*. In the particular case in which the ray is incident perpendicularly on *AB* there is no deviation at either surface, so that the direction after passing through the plate is coincident with the direction before entering the plate.

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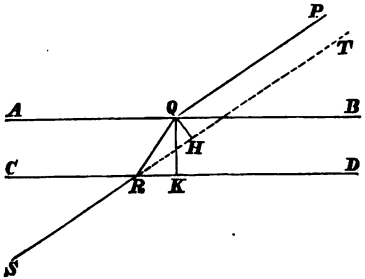
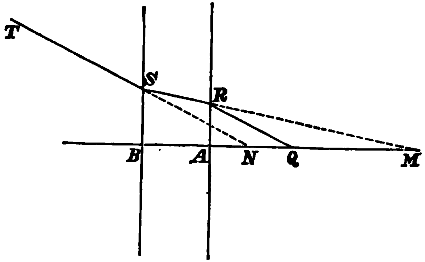
266. Let *PQRS* represent the course of a ray through the plate bounded by *AB* and *CD*; then as we have just



shewn *RS* is parallel to *PQ*. Produce *SR* backwards to *T*. From *Q* draw *QH* perpendicular to *ST*, and *QK* perpendicular to *CD*. Then by the action of the plate the course of the ray is changed from *PQ* to the parallel position *RS*; that is, it is shifted through the distance *QH*. We may calculate the length of *QH* if we know the thickness of the plate, the index of refraction, and the angle of incidence. Or we may draw the diagram very carefully to scale when we know these things; and thus *QH* will become known. The most important point to notice is that if the medium and the angle of incidence remain the same, *QH* will always be in proportion to the thickness of the plate, and will be very small when this thickness is very small.

267. Suppose now that a *pencil* of parallel rays falls on a plate. If the incidence is perpendicular the pencil passes through the plate without deviation on the part of any ray. If the incidence is oblique the course of each ray after leaving the plate is parallel to its course before entering the plate; so that the whole pencil after leaving the plate consists of rays parallel to the incident rays, and therefore to each other. The pencil is shifted from its original position through a space which is very slight if the plate be very thin.

268. *Rays of light fall from a point on a plate: it is required to determine their course after refraction through the plate.* Let Q denote the origin of light. From Q draw a perpendicular on the plate, meeting the first

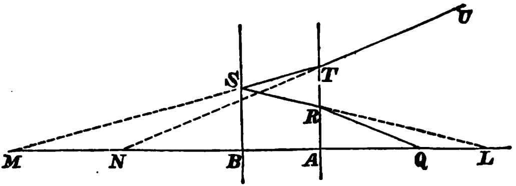


surface at A, and the second surface at B. Let $QRST$ be the course of a ray refracted by the first surface at R, and by the second surface at S. Produce SR to meet AQ produced at M; and produce TS to meet AQ at N. Then, by Art. 245, if the angle RQA is small for the extreme ray of the pencil, all the rays after refraction at the first surface will proceed very nearly as if they came from a single point M. Similarly the rays which appear to diverge from M will after refraction at the second surface proceed very nearly as if they came from a single point N. By Art. 266 we know that SN is parallel to RQ; and therefore it follows by Geometry that QN bears the same proportion to QM that RS bears to RM. When the angle RQA is very small RM differs very little from AM. Now suppose the plate to be made of glass; then AM is $\frac{3}{2}$ times AQ; see Art. 244. Therefore QM is $\frac{1}{2}$ of AQ, and consequently $\frac{1}{3}$ of AM. Thus when the angle AQR is very small QN is about $\frac{1}{3}$ of RS, that is $\frac{1}{3}$ of the thickness of the plate. The reader may go through the process on the supposition that the plate is of water instead of glass; the result will be that QN is about $\frac{1}{4}$ of AB. And in this way by examining various cases it will be found that QN is always a certain *fraction* of the thickness of the plate; the numerator of the fraction is the *index of deviation*, and the denominator is the *index of refraction*.

269. *An object is seen through a transparent plate: it is required to determine the image of the object.* By the aid of the result obtained in the preceding Article we can easily determine the image. It will be found that the image is erect, virtual, and equal to the object; it is slightly nearer to the spectator than the object is: the amount of approach is the fraction of the thickness of the plate given by the result of the preceding Article.

270. We are now able to trace the course of the rays by which an image is formed of an object placed before a common looking-glass. Some of the rays which fall on the first surface of the glass are reflected by it, and form an image in the manner explained in Art. 214. This image however is usually very faint compared with that which is formed by the silvered back of the looking-glass, and thus in general it is not noticed. But it may be readily observed if a bright light, as that of a kindled match, is held near the first surface of the glass. We are however now concerned with the rays which pass into the glass at the first surface, are reflected by the silvered back, and then come out again through the first surface: these rays form the image which is usually observed. In the diagram $QRSTU$ represents the course of a ray proceeding from Q, refracted by the first surface of the glass at R, falling on the silvered back at S and reflected along ST, then refracted by the first surface at T, and proceeding out in the direction TU. Draw a straight line from Q perpendicular to the plate, meeting the first surface at A, and the second surface at B. Produce SR to meet BA produced at L; produce TS to meet AB produced at M, and produce UT

to meet AB produced at N. Then the rays proceeding from Q and falling with small obliquity on the first surface of the glass will after refraction have a virtual focus at L; after reflection at the silvered back they will have a virtual focus at M; and after refraction again at the first

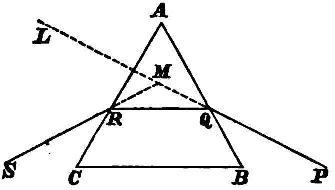


surface they will have a virtual focus at N. It will be found on investigation that AN slightly exceeds AQ; the excess is equal to twice the thickness of the plate divided by the index of refraction. For an example let us suppose that AQ is 18 inches, and that AB is $\frac{1}{4}$ of an inch, the plate being formed of glass. Then, by Art. 244, we have $AL = \frac{3}{2}$ times $AQ = \frac{3}{2} \times 18 = 27$. By Art. 214 we have $BM = BL = BA + AL = \frac{1}{4} + 27$. By Art. 244 we have $AN = \frac{2}{3}$ of AM; and $AM = AB + BM = \frac{1}{4} + \frac{1}{4} + 27 = \frac{1}{2} + 27$; therefore AN is $\frac{2}{3}$ of $27\frac{1}{2}$, that is $18\frac{1}{3}$. Thus AN exceeds AQ by $\frac{1}{3}$; and $\frac{1}{3}$ is equal to twice the thickness of the plate divided by the index of refraction.

271. After thus considering the passage of light through a *plate*, that is through a medium bounded by *parallel* plane surfaces, we proceed to the case in which light passes through a medium bounded by plane surfaces which are *inclined* to each other. Such surfaces are pre-

sented to us very conveniently by a body called in Optics a *prism*. The formal definition may be given thus: a *prism* is a body bounded by five plane figures; of these two are triangles, and are called the ends; the other three are quadrilaterals. The *prism* thus has the same form as the *wedge* in Mechanics: see Vol. I. Art. 252. The triangles are usually parallel and equal; and the quadrilaterals are usually rectangles: what is called the *edge* of the wedge is called the *refracting edge* of the prism. The light is supposed to pass in succession through the two faces which meet at the edge, and the angle between these faces is called the *refracting angle* of the prism. Unless the contrary is stated the entire course of a ray of light is supposed to be in one plane which is perpendicular to the refracting edge.

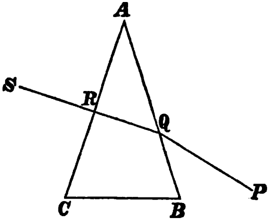
272. Let *PQRS* denote the course of a ray which is refracted at *Q* by the face *AB* of a prism, and at *R* by the



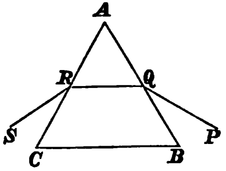
face *AC*. Produce *PQ* to *L*, and produce *SR* to meet *PL* at *M*. Then by the two refractions the direction of the ray has been changed from *ML* to *MS*, so that it has been turned through the angle *LMS* towards the *thick part* of the prism. In the present diagram the ray both at *Q* and at *R* undergoes deviation towards the thick part of the prism, and thus of course the whole deviation is of the same character. But even when the deviation at one of the surfaces is towards the *thin* part of the prism, and at the other surface towards the *thick* part of the prism, on the whole the deviation is towards the *thick* part of the prism. This statement might be verified by actual experiment, or by accurate diagrams shewing the courses of rays which are incident at various angles; but it may also be established by reasoning of an elementary kind, and to this we now proceed.

273. *When a ray passes through a prism of greater refractive index than the medium in which the prism is placed, the deviation of the ray is on the whole towards the thick part of the prism.* Let $PQRS$ be the course of a ray meeting the faces of the prism at Q and R respectively. Three different cases may occur, according as the triangle ARQ formed by the course of the ray within the prism and the sides of the prism is right-angled, or acute-angled, or obtuse-angled.

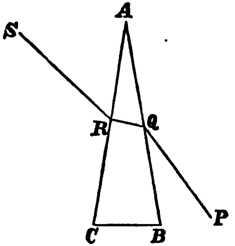
I. Let the triangle ARQ be *right-angled* at R. Then at Q the ray is turned from its original direction towards the thick part of the prism; and at R it undergoes no deviation. Thus on the whole the deviation is towards the thick part of the prism. In like manner if $SRQP$ be the course of the ray there is no deviation at R, and the deviation at Q is towards the thick part of the prism; so that, as before, the deviation on the whole is towards the thick part of the prism.



II. Let the triangle ARQ be *acute-angled*. Then the deviations both at Q and R are towards the thick part of the prism, and so of course the whole deviation is towards the thick part of the prism.



III. Let the triangle ARQ have an *obtuse angle* at R. Then at Q the ray is turned towards the thick part of the prism, and at R towards the thin part. Now it is known by Geometry that the angle QRC is greater than the angle RQA: see Art. 156. Hence QR makes with the normal to AC at R a *less* angle than it makes with the normal to AB at Q; therefore the amount of deviation at R is *less* than the amount of deviation at Q: see Art. 230. Hence on the whole the deviation is towards the thick part of the prism. The same result follows if we regard $SRQP$ as the course of the ray.



274. If the angle of the prism, that is the angle denoted by A, is too large, a ray of light will not pass through the prism in the way supposed in the preceding Articles. In I. the angle ARQ is a right angle; the least value of the angle AQR is the complement of the critical angle, by Art. 238: hence the angle A cannot be greater than the critical angle: see Art. 156. In II. the least value of each of the angles AQR and ARQ is the complement of the critical angle, and therefore the angle A cannot be greater than twice the critical angle. In III. the angle A is, by Art. 156, equal to the excess of the angle QRC above the angle AQR; the former angle is *at most* a right angle, and the latter angle is *at least* the complement of the critical angle: hence the angle at A cannot be greater than the critical angle. Thus we see that in Cases I. and III. the refracting angle cannot be greater than the critical angle, and in Case II. it cannot be greater than twice the critical angle; so that we have the following result: *no ray of light will pass by two refractions in a plane perpendicular to the edge through a prism in which the refracting angle is greater than twice the critical angle.* If the refracting angle of the prism falls slightly below this value, and a ray passes through the prism, the angles of

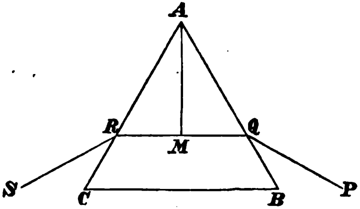
incidence at Q and of emergence at R must be very nearly right angles; so that PQB and SRC will both be very small angles: the ray then very nearly grazes the surface at entering the prism and at leaving it.

275. In the diagram which belongs to Case II. of Art. 273 suppose the angles AQR and ARQ to be equal; then it is found by mathematical investigation that the whole deviation which the ray undergoes is *less* than for any other case, so that it is the least possible. This result may be verified by experiment. Let a beam of light pass through a small hole into a darkened room, and be allowed to fall on a prism held so that its edge is perpendicular to the direction of the beam. The course of the beam before entering the prism, and after leaving it, can be perceived by means of the bright specks of dust in the air; thus the deviation can be estimated, and it is found that this is least when the angle which the beam on entering the prism makes with the face on which it falls is equal to the angle which the beam on leaving the prism makes with the other face. Two important facts are connected with the adjustment of the prism which makes the deviation least: these we proceed to notice in the next two Articles.

276. Suppose a slender pencil of rays to fall from a point on a prism in such a manner as to be almost entirely in a plane perpendicular to the edge of the prism. If the rays are incident at very nearly the angle which secures the *least* deviation, theory shews that after refraction through the prism they proceed very nearly as if they came from a single point. If the rays pass close to the edge of the prism, so that their path inside the glass is very short, the virtual focus on leaving the prism is at about the *same distance* from the prism as the origin of the light; the two points however do not coincide because the rays undergo deviation in passing through the prism.

277. The index of refraction of any transparent solid is best determined by constructing a prism out of the solid, and allowing a ray to pass through it so as to have the least deviation. The refracting angle and the whole deviation must be carefully measured; then the index of refraction is the quotient obtained by dividing the sine

of half the sum of these angles by the sine of half the former angle. This depends on simple geometrical facts. Suppose the angles AQR and ARQ to be equal; and draw AM perpendicular to QR: then AM divides the angle at



A into two equal parts. And the angle MAQ is equal to the angle of refraction at Q; for the angle between QR and the normal to AB at Q is equal to the angle between AM, which is perpendicular to MQ, and AQ, which is perpendicular to the normal. Thus *half the angle of the prism is equal to the angle of refraction* at Q. The deviation at Q is equal to that at R, and is therefore half the whole deviation; add this to the angle of refraction at Q and we obtain the angle of incidence at Q: namely the angle of incidence is half the sum of the deviation and the refracting angle. Then the index of refraction is equal to the quotient obtained when we divide the sine of the angle of incidence by the sine of the angle of refraction.

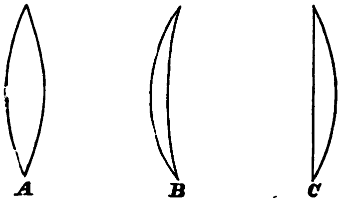
278. For glass prisms in which the refracting angles are respectively 20^0, 40^0, and 60^0, the corresponding least deviations of a ray are found to be about 11^0, 23^0, and 40^0. If the refracting angle of a prism is *very small* the least deviation is nearly equal to the product of the *index of deviation* into the refracting angle: thus for glass the index of deviation is $\frac{1}{2}$, so that the least deviation is about half the refracting angle; for water the index of deviation is $\frac{1}{3}$, so that the least deviation is about one-third of that angle. For example, if the refracting angle of a glass prism is 4^0 the least deviation is about $2^0\,7'$. Moreover if the

refracting angle is very small the deviation increases very slightly when the incident ray is inclined several degrees to the direction which corresponds to the least deviation: thus for a glass prism having a refracting angle of 4^0 the ray may be inclined as much as 5^0 to the direction which corresponds to the least deviation, and the deviation will not be increased by more than 1′. The facts stated in the present Article explain the ground for introducing the definition given in Art. 251.

XXVII. LENSES.

279. We have treated of successive refraction at two *plane* surfaces, and we proceed to consider successive refraction at two curved surfaces, or at two surfaces one of which is curved and the other plane; in practice the curved surfaces are always spherical. A portion of a medium bounded by surfaces of which one is spherical and the other plane or spherical is called a *lens;* in practice lenses are almost always formed of glass. There are six kinds of lenses known in Optics, which form two classes.

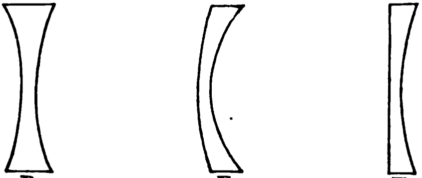
The first class is that of *convex* lenses. There are three varieties of convex lenses. In *A* both surfaces are spherical, and the convexities are turned in opposite ways; this



A *B* *C*

is called a *double-convex* lens. In *B* both surfaces are spherical, and the convexities are turned in the same way; this is called a *meniscus*. In *C* one surface is plane; this is called a *plano-convex* lens.

The second class is that of concave lenses. There are three varieties of concave lenses. In *D* both surfaces are spherical, and the convexities are turned in opposite ways;



D *E* *F*

this is called a double-concave lens. In *E* both surfaces are spherical, and the convexities are turned in the same way; this is called a *concavo-convex* lens. In *F* one surface is plane; and this is called a *plano-concave* lens.

280. The *axis* of a lens is a straight line which passes through the centres of the spherical surfaces; if one surface be plane the axis is the straight line which is perpendicular to the plane surface and passes through the centre of the spherical surface. The *thickness* of a lens is the breadth estimated at the axis, that is the portion of the axis between the surfaces; the thickness will be supposed small in all the lenses we consider. It will be seen from the diagrams that a convex lens is broadest at the axis, and that a concave lens is narrowest at the axis. The lenses of the convex class have one important property in common, namely, rays from a very distant point, after passing through the lens, are brought very nearly to a *real* focus on the *other* side of the lens; and the lenses of the concave class have a corresponding property in common, namely, rays from a very distant point, after passing through the lens, are made to proceed very nearly from a *virtual* focus on the *same* side of the lens. In general it will not be necessary for us to specify any particular kind of convex lens, or any particular kind of concave lens; when we speak of a convex lens it may be any one of the first three varieties of Art. 279; and when we speak of a concave lens it may be any one of the second three varieties.

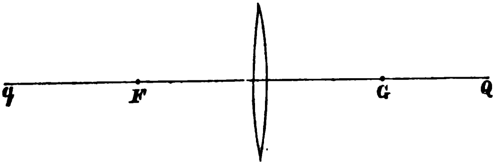
281. The forms B and E of Art. 279 have some degree of likeness; but there is a decisive difference between them. In B the left-hand surface is part of a *smaller* sphere than the right-hand surface; in E the left-hand surface is part of a *larger* sphere than the right-hand surface. Moreover in E, the two spherical surfaces do not meet, so that the lens is bounded, at the part most remote from the axis, by a portion of a cylinder; this remark applies also to the other concave lenses D and F.

282. We begin with the discussion of the properties of *convex* lenses. Now we might trace the course of a pencil of rays through such a lens by considering the refractions at the two surfaces of the lens, in the manner exemplified for a plate in Art. 268. But it will be simpler for the purposes of the present work to state facts and leave them to be verified by experiment. Suppose rays of light which are parallel to the axis to fall on a convex lens; or, what is practically the same thing, let the rays of light come from a very remote point on the axis: the rays after passing through the lens are brought very nearly to a real focus at a single point on the axis, on the other side of the lens. This point is called the *principal focus* of the lens; and its distance from the lens is called the *focal length* of the lens. The focal length of every lens is determined by the following Rule: the reciprocal of the focal length is equal to the product of the index of deviation into the *sum* or the *difference* of the reciprocals of the radii of the surfaces, according as the surfaces turn in *opposite* ways or in the *same* way. If a surface is plane the corresponding reciprocal must be taken to be nothing. The focal length is the same whichever of the two faces of the lens is exposed to the incident light. For example, let the radius of one surface be 12 inches, and the radius of the other surface 18 inches; and let the lens be of glass. Suppose it of the form A of Art. 279; then $\frac{1}{12}+\frac{1}{18}=\frac{3}{36}+\frac{2}{36}=\frac{5}{36}$; and $\frac{5}{36}\times\frac{1}{2}=\frac{5}{72}$: this is the reciprocal of the focal length, so that the focal length is $\frac{72}{5}$ inches, that is $14\frac{2}{5}$ inches. Suppose the lens to be

of the form B of Art. 279; then $\frac{1}{12} - \frac{1}{18} = \frac{3}{36} - \frac{2}{36} = \frac{1}{36}$; and $\frac{1}{36} \times \frac{1}{2} = \frac{1}{72}$: this is the reciprocal of the focal length, so that the focal length is 72 inches. To test such a result experimentally we proceed thus. Take a lens and put a bright point at a very great distance from it; hold a piece of white paper behind the lens and perpendicular to the direction of the axis of the lens. The rays refracted by the lens will form an illuminated round spot on the paper; move the paper to or from the lens until a position is obtained for it in which the round spot has its smallest size, so as to be scarcely more than a bright point: this then is the focus of the refracted rays, and the distance of it from the lens is the required focal length. Practically the sun may serve for the origin of the light, as we shall see when we treat of images formed by lenses; then the lens becomes in fact a *burning glass*, and the principal focus is the point at which the rays of the sun are brought most nearly to a point. A kind of burning glass may be formed by a globe of water, for example by such a vessel as is used for holding gold-fish; and it is said that window curtains have been sometimes set on fire when the vessel has been left exposed to sunshine.

283. Let rays coming from any point on the axis fall on a convex lens; after being refracted by the lens they will be brought very nearly to a focus, real or virtual. The origin of light and the real or virtual focus are together called *conjugate foci.* We shall proceed to indicate their relative positions in various cases. Let Q denote the origin of light, and q the focus after refraction by the lens. Let Q be at a very remote distance on the right-hand side of the lens; then q coincides with the *principal focus* which we denote by F. As Q moves towards the lens q also moves towards the left-hand side, and when Q is at a point G, which is as far from the lens as F is, then q is at a very remote distance: thus the focus of the refracted rays is *real* up to this position. As soon as Q passes G the focus of the refracted rays becomes *virtual*, and is on the right-hand side of the lens, being at

first at a very remote distance; then as Q moves up to the lens so also does q, and they meet at the lens. We may now suppose that Q is on the left-hand side of the lens, and that, instead of being an origin of light, rays are converging to it before they fall on the lens; then after being refracted by the lens they will come to a real focus between Q and the lens; and while Q moves from the

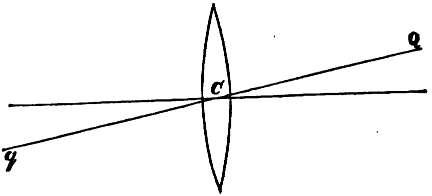


lens to a very remote distance on the left-hand side q moves from the lens up to F. The points Q and q are called *conjugate* foci for this reason: let rays passing from right to left have Q as their focus *before* refraction through the lens, and q as their focus *afterwards*, then rays passing from left to right, and having q as their focus *before* refraction through the lens, will have Q as their focus *afterwards*; here the focus in every case may be, according to circumstances, real or virtual.

284. The statement of the preceding Article may be verified by experiment in the manner of Art. 282. The distances of Q and q from the lens are connected by the following Rule: the reciprocal of the focal length of the lens is equal to the *difference* or the *sum* of the reciprocals of the distances of Q and q from the lens, according as Q and q are on the same side of the lens or on opposite sides. For an example, suppose Q on the right-hand side of the lens at a distance equal to twice the focal length, then by the process in Art. 282 it follows that q is on the left-hand side of the lens; therefore q will also be at a distance equal to twice the focal length: for this makes the sum of the reciprocals of the distances of Q and q equal to the reciprocal of the focal length.

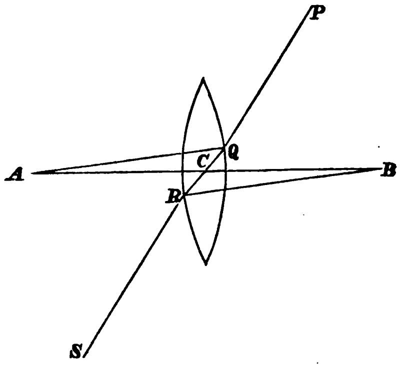
285. We must now consider the case in which rays falling on a lens proceed from a point which is not on the axis. If we fix our attention on the part of the lens which is very near the axis we see that it may be regarded roughly as a *plate;* the spherical surfaces at the points where the axis meets them are, as it were, parallel. Hence if a ray be incident in such a manner that its course within the lens crosses the axis at a small angle it may be considered to have passed through a plate; it will therefore have undergone no angular deviation but only a slight shifting of position: see Art. 266. As the part of the lens near the axis is not *exactly* a plate it may seem that our statement hitherto is rather vague; but it is found by mathematical investigation that there is a certain point on the axis of a lens, such that if the course of a ray while within the lens passes through this point the incident ray and the emergent ray are *exactly* parallel. This point is called the *centre* of the lens. The thickness of the lens is usually so small that we may practically neglect the slight *shifting of position* which occurs by the refraction of the ray; and thus we arrive at the following result: *when a pencil of rays falls on a lens from a point not on the axis, the straight line joining the point with the centre of the lens may be taken to represent a ray which goes through the lens without any change of direction.*

286. Thus if a pencil of rays falls on a lens from a point not on the axis the straight line which joins the point with the centre of the lens may be called a *secondary axis.* Let Q denote the origin of the light, C the centre



of the lens, q the focus of the refracted rays, real or virtual; and suppose QC makes only a small angle with the axis of the lens: then theory shews that Q, C, and q are on one straight line, and that the distances of Q and q from C are connected by the same Rule as the distances of the conjugate foci from the lens when the origin of the light is on the axis.

287. We will now shew how the position of the centre of a lens is exactly determined. Let A be the centre of



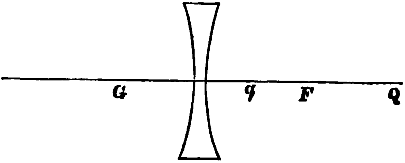
the first surface of a double convex lens, B the centre of the second surface. Draw *any* radius AQ of the first surface; draw the radius BR of the second surface parallel to AQ; join QR cutting AB at C: then C is the centre of the lens. It may be shewn by careful drawing that, whatever may be the position of the radius AQ with which we start, we always arrive by the process at the *same* point C. This is also clear from a little geometry; for ACQ and BCR are *similar* triangles, so that AC bears to BC the same proportion as AQ bears to BR: that is AC bears to BC a *fixed* proportion. Let PQ represent the course of a ray which enters the lens at Q and is refracted along QR; and let RS represent the course of the ray on leaving the lens: then RS is parallel to PQ. For since AQ and BR are parallel the angles AQR and BRQ are equal, by Art. 155; hence by the laws of refraction PQ must make an angle with AQ equal to that which RS makes with BR: and therefore PQ and RS are parallel.

288. In the case of a double convex lens the diagram shews that the centre falls inside the lens; if the two radii are equal the lens is called an *equi-convex* lens; the centre then is midway between the surfaces. Diagrams can be easily drawn for the other two varieties of convex lenses; it will be found that for a plano-convex lens the centre is at the point where the axis meets the convex surface, and that for a meniscus the centre is outside the lens on the side which is convex.

289. We proceed to discuss the properties of *concave* lenses. Suppose rays of light which are parallel to the axis to fall on a concave lens; or, what is practically the same thing, let the rays of light come from a very remote point on the axis: the rays after passing through the lens proceed very nearly from a virtual focus on the axis on the same side of the lens. This point is called the *principal focus* of the lens; and its distance from the lens is called the *focal length* of the lens. The focal length is determined by the same Rule as in Art. 282. To find the focal length by experiment we may proceed thus: take the lens and put a bright point at a very great distance from it; then the rays after passing through the lens will spread out and their course will be rendered visible, as usual, by the illuminated specks of dust in the air. If the courses of the rays be mentally produced towards the side from which they came they will form a cone having its vertex at the point assigned by theory as the principal focus.

290. Let rays coming from any point Q on the axis fall on a concave lens; after being refracted by the lens they will be brought very nearly to a focus, real or virtual. The origin of light and the real or virtual focus are together called *conjugate foci*. We shall proceed to indicate their relative positions in various cases. Let Q denote the

origin of light, and q the focus after refraction by the lens.



Let Q be at a very remote distance on the right-hand side of the lens, then q coincides with the principal focus which we denote by F. As Q moves towards the lens q also moves towards it, and they meet at the lens: thus the focus of the refracted rays is *virtual* up to this position. We may now suppose that Q is on the left-hand side of the lens, and that instead of being an origin of light, rays are converging to it before they fall on the lens. Take G as far to the left of the lens as F is to the right. As Q moves from the lens to G the focus of the refracted rays is *real*, and q moves from the lens to a very remote distance on the left-hand side, being always further from the lens than Q is. As soon as Q passes G the focus of the refracted rays becomes *virtual*, and is at a very remote distance from the lens on the right-hand side of it; then as Q moves from G to a very remote distance on the left-hand side of the lens q moves from a remote distance on the right-hand side up to F.

291. By comparing Articles 283 and 290 we may note some general results which can be easily remembered. For both convex and concave lenses Q and q always move in the *same* direction, that is we may say from right to left; but in both cases when Q is at G then q suddenly changes from a very remote distance on one side of the lens to a very remote distance on the other. The characteristic property of the *convex* lens is to bring *parallel* rays to a *real* focus; and it always brings rays to a real focus except when Q lies between the lens and G. The characteristic property of the *concave* lens is to bring *parallel* rays to a *virtual* focus; and it always brings rays

to a virtual focus except when Q lies between the lens and G.

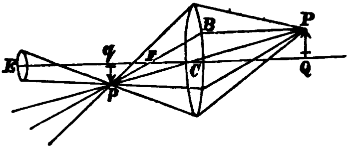
292. The distances of Q and q from the concave lens are connected by the same Rule as in Art. 284. As an example, if rays be converging to a point which is distant twice the focal length from a concave lens they will after refraction proceed from a virtual focus at the same distance on the other side of the lens.

293. The remarks of Art. 285 apply to a concave lens as well as to a convex lens. Also a concave lens has a *centre* determined in the same manner as for a convex lens. If the lens is double-concave the centre is within it; if the two radii are equal the lens is called *equi-concave*, and the centre is midway between the surfaces. If the lens is plano-concave the centre is at the point where the axis meets the concave surface; if the lens is concavo-convex the centre is outside the lens on the side which is concave.

294. Convex lenses are sometimes called *converging* lenses, and concave lenses *diverging* lenses, from the way in which they behave with respect to parallel rays. Convex lenses are also called *negative* lenses and concave lenses *positive* lenses; the reason being that when the origin of light is very distant a convex lens brings the rays to a focus on the *other* side of the lens, whereas for a concave lens the focus is on the *same* side of the lens. One particular case may be noticed in which there is apparently a lens, though without any of the properties of a lens; it is that which resembles the concavo-convex lens, but where the radii are *equal:* the reciprocal of the focal length is zero in this case, and the apparent lens may be considered to act on light somewhat in the manner of a *plate.*

XXVIII. VISION THROUGH A LENS. MAGNIFICATION.

295. If an object be placed before a lens an image of it may be formed and viewed by an eye in a suitable position : the subject requires careful attention, on account of its application to optical instruments.

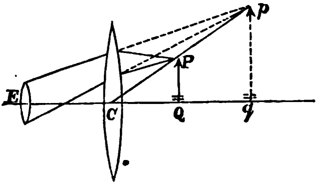


296. Suppose for instance that an object is placed before a convex lens at a distance *greater* than the focal length; then a real inverted image is found on the other side of the lens. Let PQ denote the object placed before a lens whose centre is C; then the rays proceeding from any point P of the object will after refraction through the lens converge very nearly to a point p on PC produced; thus p is the image of P. Similarly for every other point of the object there is its corresponding image; and thus we obtain the real inverted image pq of the whole object: if the angle PCQ is small the image will be a tolerably accurate copy of the object. This image will be visible to an eye suitably placed; for instance if the eye is to see the point p of the image it must be placed within the angle formed by the extreme rays which pass through p. If E be the position of the eye the point p will be seen by means of rays falling from P on the lower part of the lens; in this position the eye would receive rays also from every other point of the image, so that the whole image would be visible.

297. The precise place which the image pq will occupy can be determined if we know the distance of PQ from the lens, and also the focal length of the lens; for then we

can use the Rule of Art. 284. Or we may adopt the same method as in former cases: let F denote the principal focus of the lens, and draw PB parallel to the axis of the lens; then a ray represented by PB would after refraction by the lens pass through F, and so the intersection of BF and PC produced will determine the position of the point p. As several examples of this method of finding the place of an image have now been given we may assume that the reader is familiar with it, and so we need not in future repeat it.

298. For another example suppose an object PQ placed before a convex lens whose centre is C, at a distance



less than the focal length; then the image is erect, virtual, and behind the lens. The rays falling on the lens from any point P of the object will after refraction proceed as if they came from a point p in CP produced, and will be received by an eye on the other side of the lens if placed within the angle formed by straight lines drawn from p to the extreme points of the lens and produced. If the eye is on the axis, as at E, the whole image will be visible to it.

299. It is easy to perceive that the image in the preceding Article is larger than the object, and thus we obtain some comprehension of the familiar fact that lenses are used to magnify our view of objects; but some further consideration will be necessary in order to understand the matter fully. It is obvious that in general we see an object more distinctly the nearer we are to it. We may discern something in motion a mile off, and make out that it is a man; when we approach within the distance

of a few yards we can recognize every peculiarity of dress and feature, and may discover that the man is a well-known friend. So also in order to distinguish minute objects, as the letters when we read small print, we bring them very near to our eyes. But experience teaches us that we cannot see distinctly any thing which is *too near* our eyes; the reason for this will become apparent when we explain the constitution of the eye: the limit within which vision ceases to be distinct is different for different persons, but may be taken at about 10 inches on an average.

300. Now in the diagram of Art. 298 we perceive that the image is really larger than the object, but on the other hand it is more remote from the eye than the object is; thus we may be for a moment in doubt whether the object appears larger when seen through the lens than when seen by the naked eye. Suppose straight lines drawn from the points P and p to E where the eye is placed; then it is clear that the angle pEq is greater than the angle PEQ: and consequently the image seems larger to the eye than the object would if it were left in its place and the lens were removed.

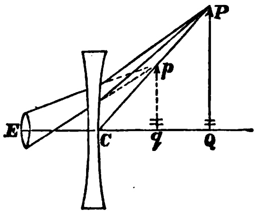
301. It is usual to give numerical expressions for the *magnifying power* of lenses; thus one lens is said to magnify 6 times, and another 8 times. Methods somewhat different have been proposed for estimating the magnifying power of lenses; we will explain that which seems to be the best. In this method the observer is supposed, whether he uses the lens or not, to put the object in what may be called the *most advantageous position.* Thus when the lens is not used the object is supposed to be placed at the least distance from the eye consistent with distinct vision; this we may take for example to be 10 inches. When the lens is used we arrange it so that the image shall be at the least distance from the eye consistent with distinct vision; that is at the same distance of 10 inches. We also put the eye close to the lens, because it is found that then as much light as possible from each point of the image reaches the eye. Thus, turning to the diagram of Art. 298, we must suppose Cq to be 10 inches, and

the eye to be placed close to C. Then the angle PEQ becomes equal to pEq; and thus it might for the moment appear as if there were no magnification. But it must be remembered that in this position of the object the eye could not see it distinctly without the aid of the lens, for it would be within the limit of distinct vision; and so there is practically a magnification. The amount of this may be estimated by the proportion of the angle under which the image appears to the angle under which the object would appear if placed where the image is, that is at 10 inches from the eye. It is found by investigation that this proportion is the same as that of Cq to CQ, or is the same as that of the *limit of distinct vision to the distance of the object when the image is at that limit.*

302. Suppose, for example, that the focal length of the lens is 4 inches; then the difference of the reciprocals of the distances of Q and q from the lens must be $\frac{1}{4}$; and, as the reciprocal of the distance of q is $\frac{1}{10}$, the reciprocal of the distance of Q is $\frac{1}{4}+\frac{1}{10}$, that is $\frac{5}{20}+\frac{2}{20}$, that is $\frac{7}{20}$: thus CQ is $\frac{20}{7}$ inches. The magnifying power is obtained by dividing 10 by $\frac{20}{7}$, so that it is $10\times\frac{7}{20}$, that is $\frac{70}{20}$, that is $\frac{7}{2}$, that is $3\frac{1}{2}$. Again, suppose that the focal length of the lens is 1 inch; then it will be found that CQ is $\frac{10}{11}$ of an inch. The magnifying power is obtained by dividing 10 by $\frac{10}{11}$; so that it is 11. In this way we can shew that if the focal length is small the magnifying power is nearly equal to the quotient obtained by dividing the limit of distinct vision by the focal length of the lens.

303. Images may also be formed by concave lenses though these are not so frequently employed as convex

lenses in optical instruments. Let PQ denote an object placed before a concave lens whose centre is C. The rays



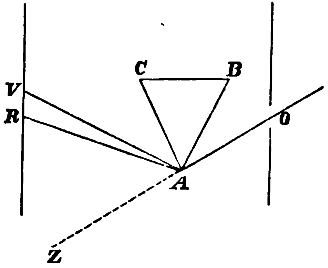
falling on the lens from any point P of the object will after refraction proceed as if they came from a point p on CP, and will be received by an eye if placed on the other side of the lens within the angle formed by straight lines drawn from p to the extreme point of the lens. If the eye is on the axis the whole image will be visible to it. The image is erect, virtual, and less than the object.

304. We have supposed throughout this Chapter that the lenses are so thin that we may use the Rules given in Chapter XXVII. But if the lens is very thick these Rules must not be used. It will be possible then by carefully drawing diagrams on a large scale to determine the place of the image formed by an object: as a simple example a diagram might be drawn shewing the course of a beam of parallel rays incident on a sphere of glass, and passing through it.

XXIX. COLOUR.

305. We have hitherto considered light as varying only in *intensity*, so we have implicitly supposed that all visible objects present to us only different combinations of brightness and shade; but this sombre and monotonous aspect is really, as we know, relieved by the beauty and variety of colour. We proceed to describe the way in which Newton unravelled the texture of the solar beams.

306. Let O denote a small hole through which the light of the sun is admitted into a room otherwise dark.



If the light is not in any way deviated a round white spot will be seen on the floor at a point Z, such that ZO produced would pass through the sun. But suppose that a prism of glass is held near the hole with its edge horizontal and downwards; then the light is refracted and forms a bright patch RV on the opposite wall, instead of the spot at Z. The patch instead of being round, like the spot at Z, may be about five times as long as it is broad; and instead of being white will exhibit various colours in bands: these colours were thus named by Newton, beginning from the upper end, violet, indigo, blue, green, yellow, orange, and red. Thus the general result is that a ray of the sun's light is not of one simple kind, but consists of various coloured rays, each having its own index of refraction, the index being greatest for the violet rays, least for the red rays, and of intermediate values for the other colours. The coloured patch is called the *prismatic spectrum*, or the *solar spectrum*, or briefly the *spectrum*. Having given this general account of the spectrum we must now advert to some matters of detail.

307. We have spoken of the length of the spectrum as about *five* times the breadth; but the exact length will in reality depend on various circumstances which we must

notice. In the first place the length changes with the angle which the rays from O make with the face of the prism on which they fall. It is usual to adopt as a standard position for the prism that in which the rays on passing through the prism undergo the *least* deviation: see Art. 275. The bright patch then takes the *lowest* possible position on the wall; and so by gradually turning the prism round its edge this position can be easily found. Again, the length of the spectrum depends on the refracting angle of the prism, increasing when this increases, and nearly in the same proportion, provided the angle be not very large. Finally the length of the spectrum depends on the nature of the material of which the prism is composed, as may be conclusively shewn in the following manner. Remove the prism hitherto used and substitute one of another material, and of such a refracting angle that the violet end of the spectrum occupies exactly the same position on the wall as before: then the red end of the spectrum will in general be higher or lower than before, instead of being at the same place. This fact escaped the careful scrutiny of Newton; or at least, as we shall see, an obvious result from it was unknown to him.

308. It will be convenient to put the experimental fact just noticed into technical language. The angle between OZ and the direction of any ray after passing through the prism is called the *deviation* of that ray. Consider a ray which occupies about the middle position of those which form the spectrum, a ray for instance on the confines of the blue and the green: the deviation of this ray may be called the *mean deviation*. The excess of the deviation of the violet ray over the deviation of the red ray may be called the *dispersion* of the spectrum. Then experiment shews that the *mean deviation* may be the same for two different prisms while the *dispersion* is not the same; and in like manner the dispersion may be the same for two different prisms while the mean deviation is not the same. Suppose, for example, that prisms are made of water, diamond, and flint glass respectively, so as to give the same mean deviation; then it is found that the dispersions are in the proportion of 35, 38, and 48.

309. The division of the spectrum into *seven* coloured spaces is somewhat arbitrary. The colour at one end is bright red, and gradually changes to violet at the other; but observers coming fresh to the subject might very easily divide the spectrum in different ways, depending on the greater or less sensibility of their eyes, and on the richness or poverty of their vocabulary for expressing tints of colour. Newton availed himself of the services of an assistant whose eyes for distinguishing colours were more critical than his own, and divided the whole spectrum into parts the proportions of which are expressed in the following Table; for the sake of comparison the result given, according to Brewster, by Frauenhofer for a flint glass prism is added.

	Newton.	Frauenhofer.
Red	45	56
Orange	27	27
Yellow	48	27
Green	60	46
Blue	60	48
Indigo	40	47
Violet	80	109
	360	360

310. The rays into which a ray of white light is thus separated do not admit of dispersion again by a prism. Let a small hole be made in the wall at the point denoted by V in Art. 306, so that a violet ray alone passes through the hole, and let it be refracted by a second prism in the same way as the original white ray was refracted by the first prism; then it is found that there is no second dispersion, that is no second change of a round spot into a long patch. The same remark applies to the red ray or to any intermediate ray between the violet and the red.

311. Newton's experiment then *resolves*, or *analyses*, or *decomposes* white light into light of various colours. On the other hand we may *compound* various colours so as to produce white by their mixture. The following is a rough example: mix together seven powders of the

colours of the spectrum, taking the quantity of each proportional to the length of the spectrum which that colour occupies; then the result of the whole is found to be a sort of greyish white. It is however not necessary to mix *all* the colours of the spectrum in order to obtain the sensation of white; a *pair* of colours suitably chosen will suffice, and the two colours in such cases are called *complementary*. Thus red and greenish blue are complementary; so also are yellow and dark blue. In general for every colour in that part of the spectrum which extends from the red end to the beginning of the green there is a complementary colour in that part of the spectrum which extends from the beginning of the blue to the violet end: the green colour of the spectrum alone has no simple complementary colour. But it must be observed that the mixing of two prismatic colours is not the same thing as the mixing of two artificial colours which seem to correspond with them; thus prismatic blue and yellow make white, while blue and yellow pigments produce green; the reason is said to be that the pigments are not *pure* colours, the blue and the yellow pigments each containing green. Newton says that he could never by mixing only two primary colours produce a perfect white.

312. The question may be proposed whether the seven prismatic colours are not reducible to a smaller number. Brewster held that they were reducible to three, namely red, yellow, and blue, which he called the primary colours. Others hold that red, green, and violet are the primary colours.

313. We must now point out the modifications which will have to be made in some of the statements of preceding Chapters in consequence of the compound nature of the sun's light. The main fact is that we can no longer with strict accuracy speak of the *index of refraction* for an assigned medium, as glass, inasmuch as the index of refraction is different for all the different rays which make up a ray of white light. Indeed the index of refraction is not the same for all the rays which may be said to be of the *same colour:* thus the index of refraction is really less for the red ray which is at the end of the spectrum than for the red ray which borders on the orange. Never-

theless we may for purposes of general explanation speak of the index of refraction of the red rays, and of the index of refraction of the violet rays, and so on; and if extreme precision is required we may understand this to mean the index for the *mean* red ray, for the *mean* violet ray, and so on. To furnish an idea of the accuracy with which this matter has been investigated we will give the index of refraction for the *mean* rays of all the colours with respect to a piece of crown glass, as determined by Frauenhofer, beginning with the mean red ray:

1·525832, 1·526849, 1·529587, 1·533005, 1·536052, 1·541657, 1·546566.

314. All that has been said with respect to the *reflection* of light remains unaffected by the discovery of the compound nature of the sun's light; in other words the laws of reflection are absolutely true, and the consequences deduced from them also. Let us pass on to the case of a pencil of rays falling from a point on a *refracting* surface, plane or spherical. We have stated that the rays after refraction have, at least nearly, a focus real or virtual. This is however on the assumption that light is *homogeneous*, so that there is one fixed index of refraction for every medium. If we consider real light as made up of rays of various colours, we may say that the rays of any assigned colour will have, at least nearly, a focus; but the focus for rays of one colour will not coincide with the focus for rays of another colour: the result is that images formed by refracting surfaces are far less sharp and distinct than they would be if light were really homogeneous.

315. Consider, for example, the case of a convex lens, and suppose rays all parallel to the axis to fall upon it; these rays will not be brought to a single focus by refraction through the lens: each system of rays of which the incident white light is really composed will be brought to its own focus, at least nearly. The index of refraction is greatest for the violet rays, and so the focus for these rays will be the nearest to the lens; the index of refraction is least for the red rays, and so the focus for these rays will be the furthest from the lens; the focus for any intermediate

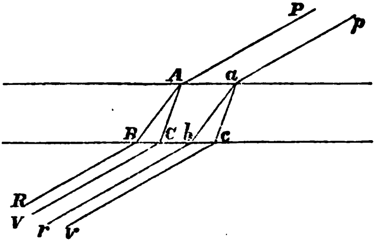
colour will lie between these two: see Art. 282. This fact, that the foci of the different coloured rays do not all coincide, is expressed by saying that there is *chromatic aberration.* The same remarks may be made if the rays instead of being parallel come from a point on the axis at a greater distance than the focal length of the lens. In consequence of this separation of the refracted rays of various colours the image of an object formed by a convex lens is not sharp and distinct: instead of a single image we have in fact a series of images which do not coincide, and so cause confusion and loss of brightness. Now, as we shall see hereafter, the construction of a telescope, one of the most important of optical instruments, assumes that a bright and distinct image of an object can be formed by a convex lens. Even if light were homogeneous this end could not be *exactly* attained by the use of a single lens, as the rays coming from a single point would not be brought precisely to a single point as focus; but calculation shews that the deviation from accuracy owing to the compound nature of the sun's light is vastly greater than would otherwise exist. In technical language the *chromatic* aberration is far greater than the *spherical* aberration.

316. The remarks made in the preceding Article suggested themselves to Newton as soon as he had unfolded the nature of light, and made a strong impression on him. He calculated the deviation from accuracy in some cases of the formation of images by convex lenses: in one case he found that the error arising from the different refrangibility of the rays of light was 1200 times as great as would otherwise exist; in another case it was more than 5000 times as great. In consequence of this Newton pronounced the improvement of refracting telescopes of given lengths to be *desperate:* nevertheless the difficulty has been overcome, and the principle on which success depends will be explained in the next Chapter.

XXX. ACHROMATISM.

317. It might be supposed from the preceding Chapter that when we view an object through any transparent medium it will necessarily appear coloured and indistinct; but we shall now shew that images may be formed which are practically without colour, and which are therefore said to be *achromatic*. We shall first explain how this is brought about when objects are viewed through a *plate;* we know as a fact that this does take place from our experience in looking through a common window pane.

318. Let us first consider the case in which a single *ray* passes through a plate. Suppose PA to represent a

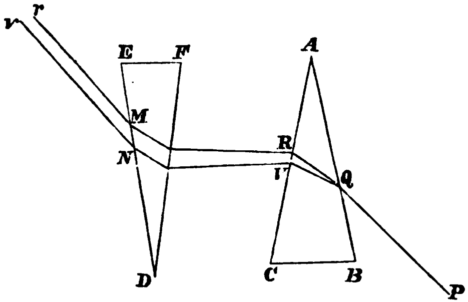


ray incident on a plate at A; by refraction this ray is decomposed into various coloured rays of which AB may represent the red ray, and AC the violet ray; the other coloured rays will occupy intermediate positions. At the second surface of the plate the rays are again refracted, the red ray along BR, and the violet ray along CV, where BR and CV *are each of them parallel* to PA: the other coloured rays will occupy intermediate positions. Thus a single ray of white light will be spread out into a band of coloured rays, and the band will be of the same width at all distances from the plate. But now suppose that we have incident on the plate, not a single ray, but a pencil of parallel rays, of which PA and pa denote the extreme rays. The ray *pa* will give rise to the red ray *pabr* and to the violet ray *pacv* in the same manner as before, and to rays of intermediate colours. Thus after refraction through the plate all the red rays will constitute a pencil parallel to the incident pencil, of which *BR* and *br* will represent the extreme rays; and all the violet rays will constitute a pencil parallel to the incident pencil, of which *CV* and *cv* will represent the extreme rays: the refracted rays of other colours will occupy intermediate positions. The result is this: the space between *CV* and *br* contains rays of all colours in the same way as for the incident pencil, and the mixture constitutes *white light;* the space between *CV* and *BR* does not contain all kinds of rays, and so there is a tinge of colour becoming more decided towards the end *BR*, where it is red; and in like manner there is a tinge of colour in the space between *br* and *cv*, becoming more decided towards the end *cv*, where it is violet. In practice the tinges of colour are obliterated by the superior brilliance of the white light over the larger space; for the angles *BAC* and *bac* are much exaggerated in the diagram in order to ensure clearness. If the rays instead of being parallel at incidence come from a point not very near, the final result will be of a similar character; and thus if an object is viewed through a plate there is scarcely any trace of colour sensible to the eye.

319. In the preceding Article the rays on leaving the plate are parallel to their directions before entering the plate; they experience no angular deviation. The image formed is very nearly at the same place as the object itself; and therefore, as we shall find when we explain the construction of telescopes, vision through a plate is of no use for the purpose of such an instrument: thus we have not yet obtained an achromatism of practical utility. The difficulty which embarrassed Newton is overcome by using *two* lenses instead of one. Let two lenses be constructed, one of the kind of glass called flint glass, and the other of the kind of glass called crown glass, and let their focal lengths be in a certain proportion assigned by theory; the lenses are placed close together and so as to have a common axis: then when we use the two lenses instead of

the single lens of Art. 315, the violet rays, the red rays, and the rays of intermediate colours, are brought very nearly to a common focus. A pair of lenses constructed in this manner may be called a *compound achromatic lens*, or simply an *achromatic lens.*

320. The possibility of constructing an achromatic lens depends on the fact stated in Art. 308 that dispersion and deviation are not necessarily in the same proportion for different substances. It is easy by a little mathematical investigation to trace the connection between this fact and the practical construction; but as such a process is beyond the range of our book we must be content with an illustrative case. Consider the passage of a ray of light through two *prisms* in succession. Let PQ represent



a ray which is incident at Q on the surface AB of a prism BAC; this will be separated into various rays, of which QR may represent the red ray and QV the violet ray. Let these rays then fall on a second prism DEF, which has its refracting angle turned in the contrary direction to that of BAC; after passing through this prism Mr may represent the red ray and Nv the violet ray, the other coloured rays occupying intermediate positions. If the original ray PQ were homogeneous, Mr and Nv would coincide; but according to the actual constitution of light they will not coincide; they will not even be parallel in general, but will be inclined at an appreciable angle. Now in order to form an *achromatic combination of two prisms* we endeavour to make Mr and Nv *parallel.* It might at first consideration seem that this does not afford much advantage, inasmuch as the rays in the space bounded by Mr and Nv will still be all really distinct. But, as we have explained in Art. 318, under such circumstances if a *pencil* of light composed of parallel rays be incident on the first prism parallel to PQ then the emergent rays will consist of a pencil parallel to Mr which will be sensibly achromatic. We cannot make Mr and Nv coincide, and so we must be content with making them parallel, and by this adjustment we practically secure the advantage we require.

321. In the preceding Article we have stated the end which we wish to gain by the use of two prisms: let us now consider how this is accomplished. Suppose the prisms to be of the *same* material, and to have equal refracting angles, and let DF be parallel to AC; then the end is gained *in a certain sense*, for Mr and Nv will be parallel: in fact, the combination of the two prisms is now equivalent to a plate, and the case becomes that of Art. 318. But, as in that Article, the result is of no use for the construction of optical instruments, inasmuch as the emergent rays are now parallel to the incident ray; and so we cannot obtain any useful image. But by properly choosing two prisms which differ in material and in refracting angle, it is found that we can make the emergent rays Mr and Nv parallel to each other, but not parallel to the incident ray PQ: the refracting angles must be turned in *opposite* directions, as in the diagram. In this way we obtain a practical and useful achromatic combination of two prisms.

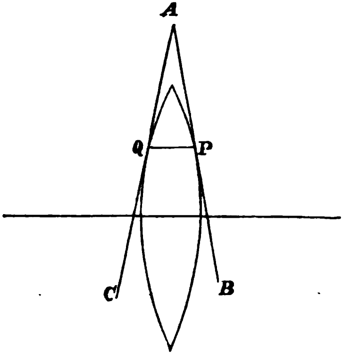
322. In the application of the principles to optical instruments there are two favourable circumstances, which we have hitherto left unnoticed in order to secure clearness in the diagrams: the refracting angles of the prisms are usually *small*, and the rays at incidence and emergence make *small* angles with the corresponding normals to the

surfaces. Let us now take a numerical example. Suppose one prism to be made of *crown* glass and to have a refracting angle of 10^0. In this kind of glass the index of refraction is about 1·55 for the violet ray, and 1·53 for the red ray. The mean deviation of a ray incident at a small angle after passing through such a prism is about 5^0; and the difference of the deviations of the extreme rays is about $\frac{1}{5}$ of a degree: see Art. 278. Now take another prism; let it be of *flint* glass, and have a refracting angle of 5^0. In this kind of glass the index of refraction is about 1·67 for the violet ray, and 1·63 for the red ray. Thus the difference of the index of refraction for the extreme rays is about *double* of what it was before; namely ·04 instead of ·02. The mean deviation of a ray incident at a small angle after passing through this prism is about 3^0; and the difference of the deviations of the extreme rays is about $\frac{1}{5}$ of a degree. The two prisms will thus form an achromatic combination; this arises from the fact that the difference of the deviations of the extreme rays is the *same* for the two prisms, namely about $\frac{1}{5}$ of a degree. The mean deviation on the whole will be about $5^0 - 3^0$, that is 2^0; and it will be towards the thick part of the prism of crown glass.

323. The proportion between the refracting angles of two prisms when these angles are small and the prisms are to form an achromatic combination may be inferred from the preceding example: the product obtained by multiplying the difference of the index of refraction for the extreme rays into the refracting angle must be the same for the two prisms.

324. The investigation of the course of a ray of light through a prism is important because it admits of application to the case of a lens. Thus suppose PQ to represent the course of a ray within a lens, and draw straight lines AB and AC, touching at P and Q respectively the arcs of circles which represent the surfaces of the lens. Then a prism denoted by BAC will act with respect to the light passing along PQ, or very near PQ, in the same way as the lens does. By drawing large and accurate diagrams

practical men can trace the course of a ray of light through two lenses, and can obtain rules for assigning the nature of



the lenses so as to bring to a common focus the rays of different colours which proceed from a point.

325. We will state the most important fact with respect to an achromatic combination of two lenses placed together; we shall require for this purpose a new definition. Let the difference of the index of refraction for the two extreme rays of the spectrum be divided by the index of deviation for the mean ray: the quotient is called the *dispersive power* or the *index of dispersion.* For example, in a piece of crown glass the index of refraction for the extreme violet ray was 1·5466, for the extreme red ray 1·5258; the difference is ·0208: also the index of refraction for the mean ray was 1·5330, and so the index of deviation was ·5330. Therefore the *index of dispersion* is $\frac{\cdot 0208}{\cdot 5330}$, that is about ·039. Suppose now that two lenses placed together are to form an achromatic image of an object before them; *then one lens must be concave and the other convex, and the product of the index of dispersion into the reciprocal of the focal length must be the same for each lens.*

326. When *two* lenses are placed in contact it is found that in some respects, such as the magnification they produce, they act like a *single* lens of a certain focal length: hence this single lens is said to be *equivalent* to the two. If the two lenses are both convex the equivalent lens is convex, and the reciprocal of its focal length is equal to the sum of the reciprocals of the focal lengths of the two. If the two lenses are both concave the equivalent lens is concave, and the reciprocal of its focal length is equal to the sum of the reciprocals of the focal lengths of the two. If one lens is concave and the other convex the equivalent lens is of the same character as that which has the less focal length, and the reciprocal of its focal length is equal to the difference of the reciprocals of the focal lengths of the two.

327. In the case of two lenses which are to form a *real* achromatic image of a distant object the lens which has the shorter focal length must be convex; and this must therefore be made of the substance of *least* dispersive power. Usually the convex lens is made of crown glass and the concave lens of flint glass. If we take the rough numbers given in Art. 322 we find that for crown glass the index of dispersion is $\frac{\cdot 02}{\cdot 54}$, that is $\frac{1}{27}$; and for flint glass the index of dispersion is $\frac{\cdot 04}{\cdot 65}$, that is about $\frac{1}{16}$. Hence the focal length of the crown glass lens must be to the focal length of the flint glass lens in the proportion of 16 to 27.

328. Thus we have endeavoured to explain how an achromatic combination of two prisms, or of two lenses, may be obtained. The need of such a combination arises from the circumstance that, owing to the compound nature of the sun's light, rays proceeding from a point, after refraction by a prism or a lens, will not have one and the same point as focus: in consequence of this the image formed of an object will not be bright and well defined, but comparatively dull and confused. The image at the same time will be *coloured*, that is instead of being white

it will be red, or yellow, or blue, or some other colour, which will depend upon the position where it is supposed to be formed. It must be observed that the *colour* would not in itself be a very serious objection; the real disadvantage is the confusion and loss of brightness which occur simultaneously with the colour, and arise from the same cause.

XXXI. THE SPECTRUM.

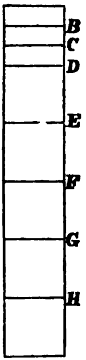
329. We have described the spectrum as it appeared to Newton who first studied it; we proceed to mention some additional facts which have been discovered by philosophers since. The spectrum has been examined as formed by many transparent substances, solid and fluid; a solid substance can always be cut into the shape of a prism; a fluid substance can be put into this shape by confining it between plates of glass which are inclined at any angle that may be required.

330. When the spectrum, as formed by various substances, is examined it is found that the colours succeed each other always in the same fixed order; but the lengths of the various coloured spaces are *not in a fixed proportion.* We have already given an example of this in Art. 309; we see for instance that in a spectrum examined by Newton the blue occupied one-sixth of the whole coloured space, while in a spectrum examined by Frauenhofer it occupied two-fifteenths. For another example we may compare the spectrum formed by a prism of flint glass with the spectrum formed by a prism of water having the same refracting angle. The glass spectrum is much more deviated than the water spectrum, and is nearly

three times as long; but the various coloured spaces are not equally enlarged; for the red and yellow spaces are not three times as large, while the violet space is four times as large. Let the refracting angle of the glass prism be diminished until the two spectra have the same length, then the water spectrum is much more deviated than the glass spectrum; and the various coloured spaces have not corresponding lengths: in the water spectrum the red, orange, yellow, and green spaces are larger than in the glass spectrum; the blue is of about the same length in both; the violet is much shorter in the water spectrum than in the glass spectrum. For another example we will compare the spectrum formed by oil of cassia with that formed by sulphuric acid: in the former the red colour occupies about one-fourth of the whole, and in the latter one-fifth; on the other hand in the former the violet colour occupies about one-sixth of the whole, and in the latter about one-fifth. The middle ray for the former falls within the blue space, and for the latter within the green space. Thus we may say that the dispersive action of oil of cassia is the stronger for the less refrangible rays, and that the dispersive action of the sulphuric acid is the stronger for the more refrangible rays. The general fact illustrated by these three examples is called the *irrationality of the spectrum*, the coloured spaces being not always in the same *ratio* or proportion. When this fact is traced to its consequences by mathematical investigation it is found that achromatic combinations of prisms or lenses are less satisfactory than they would otherwise be. For instance, in the case considered in Art. 320, if we adjust the prisms so that the red and the violet rays corresponding to a single incident ray are parallel when they emerge from the second prism, then in general any other ray, as the yellow ray, will not be strictly parallel to these.

331. When the solar spectrum is formed by very transparent prisms and examined with great care it is found to be traversed at right angles to its length by a large number of *dark lines.* Dr Wollaston announced their existence in 1802; but the matter excited little attention until it was taken up by Frauenhofer in 1814. Seven of

these dark lines have been specially distinguished on account of their distinctness and the comparative ease with which they can be found; they are called *Frauenhofer's lines*, or the *fixed lines*, and are denoted by the letters *B*, *C*, *D*, *E*, *F*, *G*, *H*: their positions are thus described by Sir D. Brewster. "Of these *B* lies in the *red* space, near its outer end; *C*, which is broad and black, is beyond the middle of the *red*; *D* is in the *orange*, and is a strong double line, easily seen, the two lines being nearly of the same size, and separated by a bright one; *E* is in the *green*, and consists of several, the middle one being the strongest; *F* is in the *blue*, and is a very strong line; *G* is in the *indigo*, and *H* in the *violet*." These lines are seen in the same order with respect to the coloured spaces, whatever be the nature of the prism, provided the light is directly or indirectly that of the sun. Frauenhofer observed nearly 600 of these lines; and the number has been carried by other investigators, as Brewster and Kirchhoff, up to 2000. Great precautions are necessary in order to observe these lines; the light is admitted through a very narrow slit, and after refraction by the prism is viewed with the aid of a telescope.



332. The dark lines thus described are formed in the sun's light; but the system of lines is very different for other kinds of light: this subject we shall consider in the next Chapter.

333. We will now advert briefly to the *heating* power of the sun's rays. It appears that the quantity of heat in various parts of the spectrum is not in the same proportion as the quantity of light, so that the hottest part is not necessarily the brightest part. Sir W. Herschel found in some experiments that the heating power gradually increased from the violet end to the red end of the spectrum; it did not however stop there but increased to some further point. Hence he drew the inference that there are rays in the sun's beams which convey heat though they are invisible; and that these rays are less refrangible than the red rays. Seebeck has

shewn by experiment that the situation of the point of greatest heat depends on the material of which the prism is composed; thus for water it is in the yellow part, for crown glass in the middle of the red part, and for flint glass beyond the red end.

334. Light produces effects of a certain kind which may be described in general by the term *chemical*. Thus the growth and health of plants and animals are much influenced by the action of light, as is shewn in works on physiology. But besides such general results there are special phenomena due to the operation of special kinds of light. For instance, let a mixture of equal parts of chlorine and hydrogen be put into a thin glass ball, and exposed to day-light: then the two gases gradually combine to form a gas the aqueous solution of which is called muriatic acid. But instead of sun-light let the light of a magnesium lamp fall on the glass ball; then it instantly bursts, with a loud explosion, into a large number of fragments. It is found too that the various coloured rays of the solar spectrum possess in very different degrees the power of producing chemical effects. Thus, for instance, muriate of silver is blackened by exposure to the sun's light; if the phenomenon is examined in detail to ascertain the situation of the rays which chiefly produce it, we find that the less refrangible rays towards the red end of the spectrum exercise scarcely any influence, but that the more refrangible rays near the violet end are very powerful. Experiments on this and similar points are very interesting on account of their application to photography; but they belong rather to Chemistry than to our subject. Sir J. Herschel remarks in his *Familiar Lectures on Scientific Subjects*, "the immense variety and extent of the chemical agencies of light as displayed in its action both on organic and on inorganic matter, revealed to us by the late discoveries in photography, assign to it a rank among natural agents of the highest and most universal character; and have even rendered it exceedingly probable, if they have not actually demonstrated, that vision itself is nothing but the mental perception of a chemical change wrought by its action on the material tissue of the retina of the eye."

XXXII. SPECTRUM ANALYSIS.

335. We have described the *solar* spectrum, that is the spectrum formed by ordinary sun light; we must now observe that every kind of light when passed through a prism exhibits a characteristic spectrum: these spectra may be arranged in three classes, considered in the next three Articles.

336. *The Solar Spectrum.* This consists of a bright ground interrupted by dark lines. The spectra produced by the light of the fixed stars resemble the solar spectrum, though they are not identical with it. Thus according to Sir D. Brewster; "In the spectrum from the light of *Sirius*, no fixed lines could be perceived in the *orange* and *yellow* spaces; but in the *green* there was a very strong streak, and two other very strong ones in the *blue*."

337. *Continuous Spectra.* In the spectrum formed by the light of a lamp, and in general of all white flames, the brightness is continuous without any interruptions of dark lines. The most remarkable examples are those of the most powerful artificial lights, namely the oxyhydrogen or Drummond light, the magnesium light, and the electrical light.

338. *Discontinuous Spectra.* These consist of one or more bright lines standing out on a dark ground. Thus the flame produced by burning the metal sodium is a brilliant yellow, and the spectrum reduces to a *single yellow line:* this shews that the yellow sodium light is homogeneous, that is consists of rays having all the same index of refraction. Sodium is one of the components of common salt, and the yellow flame may be obtained by salting the wick of an ordinary lamp. The flame produced by burning the metal lithium is red, and the spectrum reduces to two bright lines, one red and one orange. The light of incandescent hydrogen produces a spectrum of three bright lines, one red, one greenish-blue, and one violet.

339. Now by careful observation it is found that the spectrum produced by the light of any assigned burning substance is always the same; and hence when we see a certain spectrum we infer that the light must have proceeded from a certain definite substance. By this test two lights may be distinguished which appear identical to the unassisted eye. If a particle of sodium not larger than a speck of dust be put in a flame it is sufficient to produce in the spectrum the yellow line which is characteristic of sodium. New metals have been discovered by examinations of spectra; for example in this way Mr Crookes discovered a lead-like metal to which the name *thallium* has been given: the spectrum formed by burning *thallium* reduces to a single green line. The name *spectrum analysis* is now applied to the collection of facts and inferences relative to the spectra of different lights.

340. An exception should be noticed to the general law that every glowing substance has its permanent characteristic spectrum. Gases are enclosed in tubes and rendered incandescent by sending electrical discharges through the tubes; then it has been found that by varying the pressure of the gas, and the kind of the electrical discharge, two different spectra may be obtained for the same gas: some obscurity remains on this part of the subject.

341. Suppose we take one of the lights of the class mentioned in Art. 337; this would of itself give a continuous spectrum. Let this light when used with a prism to form a spectrum pass from the prism through the vapour arising from burning sodium; the result is that we have in the spectrum a *dark* line just at the point where a *bright* line occurs in the spectrum formed by the light of burning sodium. In other words the vapour of sodium is almost opaque for the rays of its characteristic colour, while it freely transmits all the other rays. The fact thus established for sodium is found to hold for other substances, and the general law may be thus expressed: *every vapour absorbs precisely those kinds of rays which it emits when it is in a glowing condition.*

342. The comparison of the solar spectrum with the spectrum formed by glowing iron establishes a remarkable

connection between the two. Many *dark* lines in the solar spectrum correspond in position precisely with *bright* lines in the iron spectrum; not less than 490 of such coincidences have been counted. Hence it has been inferred that the dark lines in the solar spectrum indicate the absence of rays which have been absorbed by passing through the vapour of iron near the sun's surface. In the spectra of other glowing bodies there are bright lines which correspond in position to certain dark lines in the solar spectrum; thus the bright sodium line is precisely in the place of the dark line *D*; so also the red hydrogen line is in the place of the dark line *C*, and the greenish-blue hydrogen line is in the place of the dark line *F*. In this manner we infer the presence near the sun of iron, sodium, hydrogen, copper, zinc, and various other elementary substances with which chemists are familiar. It is supposed that these substances exist in the form of vapour near the sun, just beyond the surface from which the light streams; so that they absorb the rays which are missing where the dark lines of the solar spectrum occur. Some of the dark lines however correspond to rays which have been absorbed by our atmosphere; this is established by the fact that their appearance changes with the position of the sun, being darkest at sun-rise and sun-set when the rays pass through the greatest breadth of the atmosphere: the dark line *B* belongs to this class.

343. It has been known for some time that during a total eclipse of the sun rose-coloured flames appear to stand out from various parts of the boundary of the sun's disc. These flames are not visible in ordinary day-light, being lost in the blaze of the sun's light. During the total eclipse of the sun on August 18th, 1868, the rose-coloured flames were examined with the aid of a prism, and a series of bright lines was observed in the spectrum; among these the three characteristic of hydrogen were noticed. Hence the inference is that the rose-coloured flames consisted chiefly of incandescent hydrogen. Methods have since been devised for observing the rose-coloured flames even when the sun is not eclipsed.

344. In Art. 74 we have noticed the fact that a sound exerted by a musical instrument which is moving *to-*

wards us appears a little *sharper*, and by a musical instrument which is moving *from* us a little *flatter*, than it would if excited by the same instrument at rest: now something of a similar kind occurs with respect to light. On examining the spectrum formed by the light of the star Sirius a dark line is found so near the line *F* of the solar spectrum that we may reasonably assume them to be the same; but in the Sirius spectrum this line is a little nearer to the red end of the spectrum than it is in the solar spectrum. Hence it may be conjectured that this is owing to a motion of Sirius away from the solar system; and a calculation founded on the amount of this displacement leads to the result that Sirius is moving away from the solar system at the rate of about 20 miles in a second. Similar observations have been made on the spectra of other stars, and similar inferences drawn; thus, for example, the star Arcturus is held to be approaching the solar system at the rate of 50 miles in a second.

345. The subject of *absorption* of light thus introduced in connection with spectrum analysis, leads us to give a few remarks on two other classes of facts which also depend on absorption, and are known by the names of *fluorescence* and *phosphorescence*.

346. *Fluorescence.* Let a solution be prepared by putting a small quantity of the bark of the horse-chestnut tree in water. Let light be sent through a convex lens and brought to a focus just at the surface of the solution, so that it diverges from this point and passes on; then the course of the rays through the solution will be indicated by a light blue shimmer: the particles of the substance held in solution appear as if they were self-luminous, and send out this delicate blue light in all directions. The light is brightest where it enters the solution, and gradually diminishes in intensity as it advances further into the solution. There are numerous bodies, solid and fluid, which exhibit a similar appearance; and in particular fluor spar, from which the name *fluorescence* is derived.

347. Suppose light is passed through a glass vessel containing a solution of horse-chestnut bark, refracted by a convex lens, and brought to a focus at the surface of such

a solution in another vessel: then the sky-blue shimmer is *not* perceived. Thus the power of exciting the fluorescence is lost by being exercised; or in other words fluorescence in a body is due to rays which are *absorbed*.

348. We have hitherto supposed that *white* light was employed to produce fluorescence; but by employing in succession the various coloured rays of the solar spectrum we can determine which of them are capable of producing the effect. Now it is found that, starting at the red end, the red rays and the others down to the indigo, do not produce fluorescence; the sky-blue shimmer begins with the light in the neighbourhood of the dark line G, and reaches not only to the end of the spectrum, as usually visible, but also far beyond. Thus we learn the existence of a new portion of the solar spectrum, extending in length about as much as the ordinary visible portion; it is called the *ultra violet* portion. There are *fixed lines* in this ultra violet portion as in the ordinary visible portion.

349. The fluorescent light may be examined by means of the prism, and a very remarkable result is thus obtained, namely that the *refrangibility of rays can be changed:* for incident rays of a definite refrangibility give rise to fluorescent rays of *various refrangibilities*. It is found that the refrangibility of any ray is *never exceeded* by that of the fluorescent ray to which it gives rise.

350. *Phosphorescence.* Some bodies when they have been exposed to the action of a brilliant light will continue to shine for a time in the dark; the duration of this shining is different for different bodies, but in general is very brief for all. This property belongs to various compounds of sulphur with calcium, barium, and strontium; and also to the diamond and to fluor spar. It is called *phosphorescence*, and is, like fluorescence, an effect of absorbed light. Phosphorescent light when examined by the aid of the prism exhibits the same peculiarity as fluorescent light; namely the rays are in general less refrangible than the rays which excited them. To prevent mistakes, which might arise from the similarity of name, it should be stated that the light of *phosphorus* itself does not come into the class now before us; phosphorus is self-luminous

by virtue of a chemical action, and in somewhat like manner the *glow-worm* is self-luminous by virtue of a vital process.

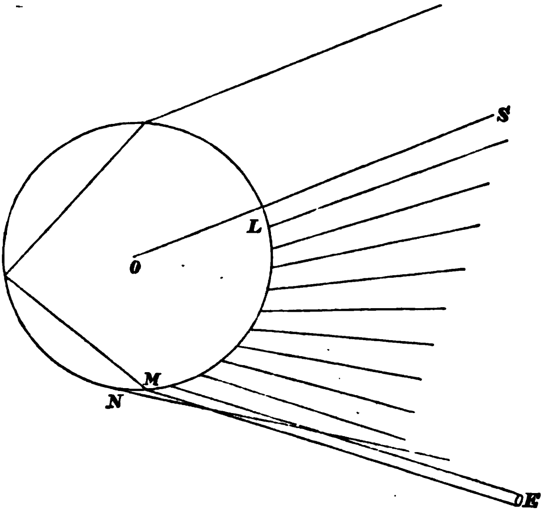
XXXIII. THE RAINBOW.

351. Newton applied his knowledge of the compound nature of light to explain the formation of the rainbow, concerning which only very imperfect notions had been previously entertained. The phenomena may be taken as well known from observation. When a shower is falling and the sun is shining at the same time, a person with his back to the sun may see on the opposite sky a bright band of colours in the form of a circular arch; the colours are arranged in the order of the solar spectrum, red being at the outer boundary, and violet at the inner boundary, and the other colours in intermediate positions: this is called the *primary bow*. Outside of this may frequently be seen a second band of colours, fainter than the former, having red at the inner boundary, and violet at the outer boundary: this is called the *secondary bow*.

352. We take the *primary bow* first. The explanation is made most clear by considering separately the different coloured rays of which sun-light is composed; and we begin by supposing this light to consist of homogeneous *red* rays.

353. Let the circle represent a rain drop on which the sun's light falls. On account of the enormous distance of the sun, rays falling from any point of its surface on the drop may be considered parallel. Some of these rays will be refracted into the rain drop and so proceed to the back of the drop; some of those that thus reach the back of the drop will then be reflected towards the front of the drop: there some of them will be refracted, and thus pass out of the drop and reach an eye suitably placed. Now the course of the rays which are thus twice refracted, with an intermediate reflection, can be correctly determined, either by mathematical calculation or by a diagram carefully drawn on a large scale. Let *O* denote the centre of the drop, *OS* the straight line drawn from *O* towards the sun; let the circle denote the boundary of the drop. Let

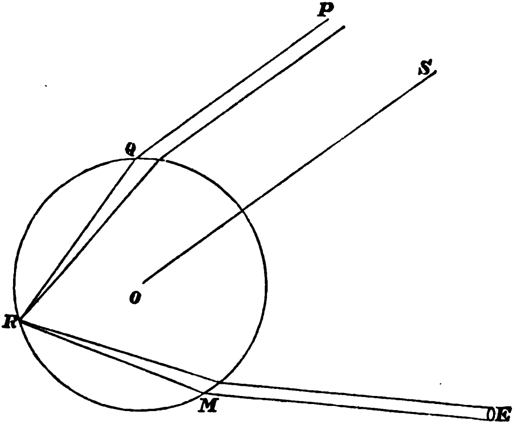
the straight lines drawn from the circle denote rays which issue from the drop; these rays are supposed to have fallen parallel to *SO* on the part of the drop which is above *SO* and towards *S*, and to have been first refracted, then reflected at the back of the drop, and finally refracted by the part of the drop which is below *SO*: the course of one of the incident rays through the drop is traced out as an example, the others are omitted to avoid complication in the diagram. Now by calculation or by careful drawing,



it is found that the rays which issue from a certain part *LM* spread out from each other, but the divergence of the rays gradually diminishes until at a certain point *M* two adjacent rays emerge almost parallel, and at this point the angle between *SO* and the direction of the ray which leaves the drop has its greatest value. Rays issuing from the part *MN* of the drop spread out from each other, and also make a smaller angle with *SO* than the ray at *M*.

Thus an eye placed so as to receive the rays from *M* gets more light than it would if placed in any other position, since it receives parallel rays instead of diverging rays. The angle between two straight lines drawn from the drop, one to the eye of the spectator and one to the sun, has then its greatest value ; this angle is found to be about 42° for the red rays.

354. The diagram shews the course of the rays which leave the drop at *M* and enter the eye at *E*. Two parallel



rays are drawn falling on the drop, of which *PQ* is one ; these rays are refracted so as just to meet at *R* on the back of the drop ; then they are reflected to *M*, are refracted there, and enter the eye as parallel rays. If *EM* and *SO* be produced to meet they will include an angle of about 42°.

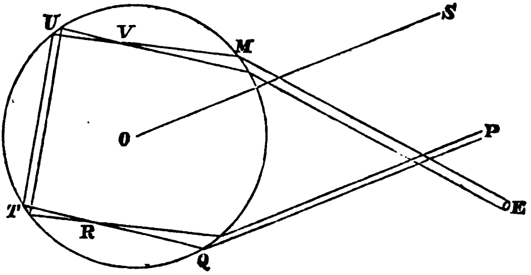
355. We must now consider the drops which are situated in various directions from the eye of the spectator ; from all of these rays will reach the eye provided the drop be not at too great angular distance from the

eye. The limit is thus determined. Suppose a straight line drawn from the eye E downwards, parallel to SO; this may be called the *axis:* then the straight line from the eye to the drop must not make with the axis a greater angle than 42°. But although light from all the drops within this limit reaches the eye, yet the light is faint except that which comes from the drops just at the extreme angular distance; because it is only from these that the rays are parallel when they reach the eye. Thus the eye would see a large patch of the sky of a red colour, bounded by an arc of a circle; but the colour would be comparatively faint except at the boundary: beyond the boundary the sky would appear dark.

356. Having thus determined the appearance corresponding to homogeneous red light, it is easy to infer the results for homogeneous light of other colours, and then finally for real light which is composed of these various colours. For if we consider the violet light we find in like manner that there must be a large patch of the sky of a violet colour, bounded by an arc of a circle, but comparatively faint except at the boundary. This boundary corresponds to an angle of about $40\frac{1}{4}°$ with the *axis*, instead of the 42° which occurs for red light. Similarly for the intermediate colours we should have the relative coloured patches, larger than that for the violet and less than that for the red. Now suppose all the coloured rays to exist together. Then we obtain white sky over the large space where all the coloured patches exist together; the outer boundary will be red, because at this boundary the red alone occurs, and moreover is at its brightest state; the inner boundary will be violet, because there the violet is at its brightest state and so prevails over the other colours which are present there but are comparatively faint; at intermediate points the tint will be mainly that of the colour which is there at its brightest state. Beyond the red boundary the sky will appear dark.

357. The *secondary bow* is now to be considered. This is formed by light which has been twice refracted with two intermediate reflections. It is found that if homogeneous parallel rays fall on a rain drop, pass into the

drop, and after two reflections emerge, they will in general spread out from each other; but that two adjacent rays emerging in a certain direction will be almost parallel. The course of the rays which thus emerge is shewn in the diagram. Two parallel rays are drawn falling on the drop, of which *PQ* is one; they are refracted by the drop and



proceed to the back at *T*, crossing at the point *R* such that *QR* is three-fourths of *QT*; at *T* the rays are reflected and proceed parallel to each other to *U*, where they are again reflected; they proceed to *M*, crossing at the point *V* which is such that *MV* is three-fourths of *MU*; at *M* they are refracted out of the drop, parallel to each other, and so may reach an eye *E* suitably placed. The line *PQ* is parallel to *OS* passing from the centre of the drop to the sun. The angle at the drop between straight lines drawn to the sun and to the eye has its least value when the eye is so placed as to receive parallel emergent rays: for red rays this angle is about 51^0 and for violet rays about 54^0.

358. Thus if we consider homogeneous red light we find that drops at a greater angular distance from the *axis* than 51^0 will send light to the eye; but the light will be faint except in the case of drops which are just at the limit of the angular distance. Thus the sky *beyond* a certain arc of a circle appears red, but the colour is faint except at the boundary. Similarly for

the violet rays there will be a violet colour on the sky beyond a certain arc of a circle corresponding to the angular distance of 54^0 from the *axis;* but the colour will be faint except at the boundary. For intermediate colours the boundaries will be determined by intermediate angles. Hence when we suppose all the coloured rays to exist together we obtain a band of colours, red at the lower edge, and violet at the upper edge; above the upper edge there will be white light, and below the lower edge a dark part of the sky. The secondary bow is not so vivid as the primary bow, because more of the light which enters the drop is lost, as there are two reflections instead of one.

359. We have hitherto treated the sun as if it were a *point.* To be accurate we must consider each point of the visible surface of the sun as giving rise to its special rainbow; these will be blended together, and as they will not exactly coincide the colours will be in some degree confused in consequence; but the general appearance will remain as we have described it.

360. Theoretically a *third* bow might be formed by light which has undergone *three* reflections within the drop; but the light would in general not be strong enough to make an impression on a spectator.

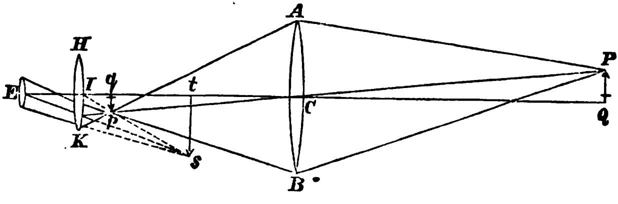
361. It will be observed that when various persons see a rainbow simultaneously, it is, strictly speaking, not the same object which they contemplate: each eye really forms its own rainbow, with the aid of the sun and the drops of water. Again, it may happen that a spectator looking at a smooth reflecting surface, like that of a quiet lake, sees a rainbow there: this however is not, strictly speaking, the *image* of that rainbow which he can see by direct vision at the instant. In the diagram of Art. 189 let the straight line BD represent the level of the lake, and Q the position of the eye; then the bow which the eye sees in the lake is the same as would be visible to an eye placed at R, if the obstruction caused by the ground were removed: and this is not the same as the bow seen in the sky by the eye at Q.

362. We may place here a remark which will be understood hereafter: the truth of the theory of the rainbow is confirmed by the fact that the light from any part of the bow is found to be *polarised* in the plane through the *axis* and that part.

XXXIV. REFRACTING TELESCOPE.

363. We have already discussed the formation of images by reflecting surfaces, and by lenses; and also the magnification of visual objects by means of lenses. We shall find that by a combination of these two principles, namely, the formation of an image and the magnification of it, telescopes and microscopes are formed. Telescopes are of use for viewing objects which are too distant, and microscopes for viewing objects which are too small, for unassisted vision. We consider in the present Chapter telescopes where the image is formed by a lens, and which are therefore called *refracting* telescopes.

364. *The common Astronomical Telescope.* AB and HK are two convex lenses which are so placed as to have



a common axis; C and I denote their respective centres. AB is the larger, and is called the *object* glass; HK is called the *eye* glass. Let PQ be a distant object which is situated nearly on the common axis of the two lenses. Rays falling from any point P on the object glass will after refraction through it converge very nearly to a focus p on PC produced; and as PC is supposed large the distance of this focus from C will be very nearly equal to the focal length of the object glass. Similarly the rays from any other point of the object will after refraction be brought very nearly to a focus; and thus a real inverted image pq of the object will be formed. The eye glass HK is so placed that this image is at a little less distance from it than its focal length; hence the rays which diverge from p will after refraction through the eye glass appear as if they came from a virtual focus s on Ip produced. Similarly the rays which fall from any other point of the image pq on the eye glass will after refraction appear as if they came from a virtual focus; thus a virtual image st is formed, which is erect with respect to pq, and is therefore inverted with respect to PQ: this can be seen by an eye placed, as at E, on the common axis of the lenses.

365. *Magnification.* Since PQ is supposed to be a very distant object, the angle which it subtends at the eye is practically equal to that which it subtends at C; that is, it is equal to the angle PCQ, which is equal to the angle pCq. The angle which the image subtends at the eye is sEt; and supposing, as is usually the case, that the distance EI is very small compared with the distance It, this angle is practically the same as the angle pIq: see Art. 301. Thus the magnifying power of the telescope may be taken as equal to the proportion of the angle pIq to the angle pCq. Now it may be shewn by measurement, or by calculation, that this proportion is very nearly the same as that of Cq to Iq. The former length is not sensibly different from the focal length of the object glass; the latter distance is somewhat less than the focal length of the eye glass, but may be taken as roughly equal to it. Thus finally the magnifying power of the astronomical telescope is equal to the proportion of the focal length of the object glass to the focal length of the eye glass. For example, if the focal length of the object glass is 30 inches, and the focal length of the eye glass is $\frac{1}{2}$ an inch, the magnifying power is $30 \div \frac{1}{2}$, that is 60.

366. It need scarcely be said that the lenses are usually placed in a tube of wood or of light metal, so that they may be kept steadily on a common axis; the length of the tube is a little greater than the sum of the focal lengths of the lenses. The inside of the tube is blackened in order to absorb any stray light, which might otherwise be reflected from its sides and tend to confuse the impression made on the eye by the image. The object glass is fixed firmly in the tube while the eye glass is capable of being pushed out or in a little; thus any observer can adjust the position of the eye glass so as to make the distance of the image *st* from his eye just that which best suits his habit of sight.

367. It is usual with writers on Optics to suppose the telescope to be adjusted so that the distance between the lenses is equal to the *sum* of their focal lengths. This seems to be taken as a standard arrangement which is satisfactory to many eyes; the image *st* must then be supposed to have moved to an extremely remote distance on the right-hand side of the eye glass, and to have become excessively large. If then an observer who sees best when objects are at a distance of 10 inches comes to a telescope which is in the standard arrangement, he pushes the eye glass in a little; if he sees best when objects are still nearer than 10 inches, he pushes the eye glass in somewhat further.

368. The eye glass is small, and it is found convenient to put it inside a little cylindrical box, which has holes at its ends to allow a passage for the light; this box is so constructed that the eye when placed close to one end is at the proper distance from the eye glass: this distance is the focal length of the lens. For consider the ray *PCp*; the angle between this and the axis of the lenses is so small that we may treat the ray as if it were parallel to the axis, and so after refraction by the eye glass it will proceed very nearly to the principal focus of the lens. Thus we find that the *secondary* axes of the pencils from all points of the object, after passing through the eye glass, meet at a common point, namely, the principal focus of the glass; therefore if the eye is placed here it can receive pencils from all points of the object so as to see the whole of it at once.

369. The diagrams which are used to explain the construction and use of telescopes are necessarily, from the want of adequate space, considerably distorted. Thus *PQ* ought to be much further from the object glass, and the angle *PCQ* much less, than in our diagram. In an astronomical telescope of any value the object glass *AB* would be at least two inches in diameter; the opening of the eye by which light enters into it is not more than a quarter of an inch in diameter. Still in spite of the imperfection of the diagrams, the student ought not to fail to notice the two principles which are employed in the construction of the telescope: first, an image is formed by the object glass in the manner of Art. 296; and next this image is magnified by the eye glass as explained in Art. 301.

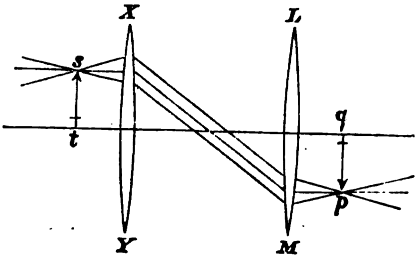
370. The magnifying power of the telescope, as we have seen, is equal to the proportion of the focal length of the object glass to the focal length of the eye glass; hence we may theoretically make this as great as we please, but practically there are serious difficulties. It can be shewn by geometry that the magnifying power is also equal to the proportion of the diameter of the object glass to the diameter of the pencil of light which enters the eye from any point of the image. Now unless the eye receives ample light from each point of the image, that image appears *faint*, and consequently not adapted for accurate vision. Thus it is important that the pencil of light from any point of the image should fill the opening of the eye, at least as nearly as possible: it would be of no use for the pencil to be larger, as the excess of light could not enter the eye, and so would be wasted. Hence if we want high magnifying power we must have very large object glasses; and much trouble and expense are incurred in producing clear pieces of glass of sufficient size, and in grinding them into lenses.

371. The main purpose of the object glass is to form an image of the object which may be examined and magnified; and incidentally the object glass is of great importance by collecting vastly more light from every point of the object than the unassisted eye could obtain. When the image is formed by the object glass we can, if we please, view it by the eye alone, but the eye glass enables

us to magnify the image; incidentally too the eye glass secures this advantage, that by means of it more of the image can be seen at once. In the diagram of Art. 364 if we suppose the eye placed at the position there indicated the whole of the image as drawn can be seen without shifting the eye, and also an equal portion on the other side of the axis; but if we withdraw the eye glass, it will be necessary to move the eye up and down in order that it may in succession receive pencils broad enough to fill it from various points of the image pq. The extent of image visible to the eye in one position is called the *field of view;* it depends on the size of the eye glass. If the ray PA, after refraction at A, passes just in the direction AK, then the whole pencil from this point falls on the eye glass and is transmitted to the eye. If the ray PB, after refraction at B, passes just below the direction BK, then none of the pencil from this point falls on the eye glass, and so none of it reaches the eye. If the ray PC produced just passes through K, then of the pencil from P which falls on the object glass, the lower half falls on the eye glass and is transmitted to the eye, and the upper half is lost: P is then said to be visible by a *half pencil.*

372. After explaining the general principle of the refracting telescope we must notice some matters of detail. The telescope, as we have described it, exhibits objects *inverted;* this is of no consequence to astronomers, but is inconvenient in the case of other observers. Accordingly it is usual to interpose two convex lenses between the object glass and the eye glass by the aid of which the fixed image is rendered erect. Let LM and XY denote two convex lenses on the same axis as the object glass and the eye glass of the telescope. Let pq denote the image of an object formed by the object glass. The lens LM is so placed that its distance from pq is about equal to the focal length of the lens. Hence the rays diverging from the point p, after refraction by the lens LM, will proceed nearly as a pencil parallel to the straight line which joins p with the centre of LM. These rays falling on XY will be brought to a focus at a point s, such that the straight line which joins it to the centre of XY

is parallel to the rays incident on XY: moreover the distance of this point from XY is very nearly the focal length of the lens. In this way a real image *st* is formed,



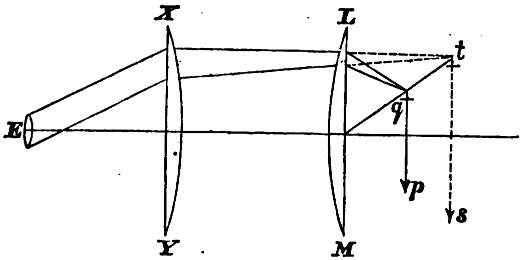
which is *inverted* with respect to *pq*, and therefore *erect* with respect to the original object; and this image may be viewed through an eye glass, and so magnified, in the usual way.

373. The image formed by a single object glass will be confused, mainly because of the compound nature of light; and to remedy this instead of a single lens *two* lenses are used. These consist of a convex lens of crown glass, and a concave lens of flint glass, placed in contact; the former is the nearer to the object. The focal lengths of the two lenses must be in the same proportion as the dispersive powers of the two kinds of glass: see Art. 325. Now in the two lenses there are *four* spherical surfaces, so that the *four* radii of the spheres are quantities at our disposal, with only *one* condition to be regarded, namely, that just stated with respect to the focal lengths. Hence by taking the four radii properly it is possible to attend to other conditions besides that of *achromatism;* and accordingly opticians are guided by theory and by trial to give such values to the radii as to alleviate other defects to which we have already made some allusion. Thus a single lens will not bring accurately to a focus even homogeneous rays which fall upon it; this fact is called *spherical aberration:* also the image which a lens

forms of an object is not exactly similar to the object; this defect is called *distortion*. These defects are alleviated by a suitable construction of the two lenses which constitute the compound achromatic object glass.

374. Again, vision through a single eye glass is defective for reasons like those which apply to the image formed by the single object lens; theory and practice shew that a great improvement is effected by the use of an *eye piece* instead of a single eye lens. The eye piece consists of two lenses placed on a common axis at a suitable distance apart; the lens nearest the object is called the *field lens*, and the other the *eye lens*. There are two eye pieces which have been much used, Ramsden's eye piece, and Huygens's eye piece: these we will now describe.

375. *Ramsden's eye piece.* This eye piece is named after a celebrated maker of optical instruments. It con-



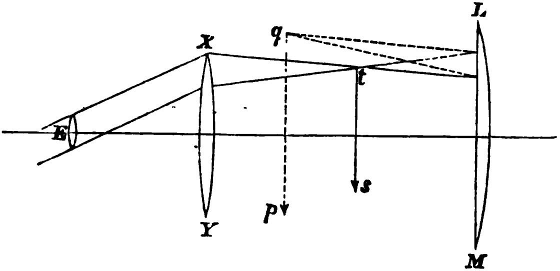
sists of two plano-convex lenses of the same material, and the same focal length, placed at a distance from each other equal to two-thirds of the focal length of either. Let LM and XY denote the lenses, and let pq be the inverted image of a distant object which has been formed by the object glass of a telescope. The eye piece is so placed that pq is in front of LM, at a distance from LM equal to one-fourth of the focal length. Then a virtual image of pq will be formed by the rays after refraction through LM; this image will also be in front of

LM and at a distance from *LM* equal to one-third of the focal length: let *st* denote this image. Then as the distance between *LM* and *XY* is two-thirds of the focal length, the distance between *st* and *XY* will be equal to the focal length. Thus rays which diverge virtually from *t*, and fall on *XY*, will after refraction by *XY* form a pencil parallel to the straight line which joins *t* with the centre of *XY*. Hence to an eye at *E*, which is adapted for the reception of parallel rays, such rays will form an image of the point *t*; and in this way the eye will perceive an image of the whole object.

376. Ramsden's eye piece is much used for large telescopes in cases where the visible field is required to be marked out into equal spaces; a system of very fine wires is placed at *pq*, and so the relative position and magnitude of various parts of the image can be estimated. The eye piece is not quite achromatic: it might, according to theory, be rendered nearly achromatic by making the distance between the two lenses equal to the focal length of either: but then *pq* and *st* fall on *LM*, and it is found that dust or moisture collected on *LM* is seen magnified by *XY*, so that vision is not distinct. Ramsden's eye piece may be used as a magnifier apart from the object glass of a telescope to which we have supposed it attached; we have only to place the thing which we wish to examine in the position *pq*. In estimating the magnifying power of a telescope furnished with Ramsden's eye piece, it is found by theory that the result is the same as if instead of the eye piece a single lens had been used with a focal length equal to three-fourths of the focal length of either of the lenses of the eye piece. This is expressed by saying that the focal length of a lens *equivalent* to the eye piece is three-fourths of the focal length of either of the lenses of the eye piece. Ramsden's eye piece is sometimes called the *positive* eye piece, apparently because it can be used to magnify a real object.

377. *Huygens's eye piece.* This consists of two convex lenses placed on a common axis; let *LM* denote the field lens and *XY* the eye lens. The focal length of *LM* is three times that of *XY*; and the distance between the lenses is half the sum of their focal lengths. The eye

piece is so placed that *LM* is in front of the inverted image *pq* of a distant object formed by the object glass of a telescope, and at a distance from it equal to half the focal length of *LM*. The rays which would form this image if the eye piece were absent, being refracted by the field glass, form an image *st* which is just midway between the



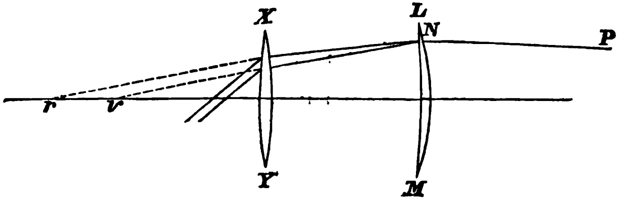
field lens and the eye lens: this image is therefore at a distance from *XY* equal to the focal length of *XY*. Thus rays which diverge from *t* and fall on *XY*, will after refraction by *XY* form a pencil parallel to the straight line joining *t* with the centre of *XY*. Thus to an eye at *E* which is adapted for the reception of parallel rays such rays will form an image of the point *t*; and in this way the eye will perceive an image of the whole object.

378. Huygens's eye piece cannot be used independently as a magnifier, and apparently on this account is called the *negative* eye piece. When used in conjunction with the object glass of a telescope it is found that with respect to magnifying power Huygens's eye piece is *equivalent* to a lens of half the focal length of the field glass.

379. Huygens's eye piece was devised with the view of diminishing the distortion which exists when an image is formed by a single lens. It happens that the confusion arising from the compound nature of light is at the same time so much diminished that the eye piece may be said to be achromatic: this can be shewn by theoretical investigations upon which we do not enter here. It must how-

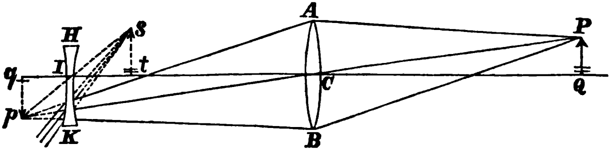
ever be remarked that there is considerable difference between the two problems of making an achromatic object glass, and of making an achromatic eye piece; this arises from the difference in the character of the pencils of rays which pass through them from a distant object: the pencil which falls from any point of the object on the object glass fills the whole of it, whereas that which after refraction through the object glass falls on the field glass of the eye piece fills only a very small part of it. The next Article will give some notion of the problem of constructing an achromatic eye piece.

380. We will now illustrate the way in which by Huygens's eye piece the confusion which would arise from the compound nature of light is alleviated. Let a ray of white light fall in the direction PN on the field glass of Huygens's eye piece. The ray is separated by refraction into its components, and we may denote the red ray by Nr and the violet ray by Nv. The rays now fall on the eye lens



XY, and the red ray meets it at a point *more distant from the axis* than the violet ray. The effect is the same as if the red ray passed through a prism of greater refracting angle than the violet ray: see Art. 320. On *this account* the red ray undergoes *more* angular deviation than the violet ray; while by reason of its smaller index of refraction it would undergo *less* angular deviation from the eye lens as well as from the field lens. Hence if the distance between the two lenses is properly adjusted the two rays may emerge from XY *parallel*, as drawn in the diagram; and thus we shall obtain a practical achromatism of the kind considered in Art. 324. Theory shews that if the incident ray PN is very nearly parallel to the common axis of LM and XY, then the distance between the lenses should be equal to half the sum of their focal lengths.

381. *Galileo's Telescope.* This differs from the common Astronomical telescope in having a *concave* eye glass instead of a *convex* eye glass. AB is a convex lens called the object glass, C its centre; HK is a concave lens called



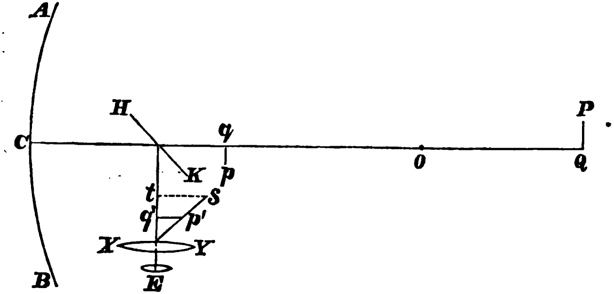
the eye glass, I its centre; let PQ denote a distant object, which is situated nearly on the common axis of the lenses. Rays falling from any point P of the object on the object glass will after refraction through it converge very nearly to a focus p on PC produced. Similarly the rays from any other point of the object will after refraction be brought very nearly to a focus; and thus a real inverted image pq of the object would be formed. But the concave lens HK is placed in front of the position of pq, and at a distance from it somewhat greater than its focal length. Hence the rays which were converging to p, appear after refraction through the lens as if they came from a point s on pI produced. In this way a virtual image st is formed, which is inverted with respect to pq, and therefore erect with respect to the original object PQ. The eye of the observer is usually placed close behind the eye glass, and consequently the diameter of this eye glass need not be greater than that of the opening of the eye; thus the field of view is not very large, which is a disadvantage in this instrument. On the other hand, it avoids the loss of light which is caused by the use of two additional lenses to obtain an erect image, as explained in Art. 372. The object glass ought to be compound in order to secure achromatism;

but this is not found to be so necessary for the eye glass. It is usual to suppose that the telescope is so adjusted that the distance between the lenses is equal to the difference of their focal lengths: see Art. 367. The magnifying power is found, as for the common Astronomical telescope, to be equal to the proportion of the focal length of the object glass to the focal length of the eye glass. This construction is in common use for opera glasses, which, as is well known, consist of two small telescopes, placed side by side, one for each eye. Galileo, from whom this telescope received its name, though not the original inventor of an instrument for assisting the view of distant objects, set the example of employing it for important purposes, such as for astronomical observations. His first telescope magnified three times, his second eight or nine times, and finally he constructed one that magnified above thirty times.

XXXV. REFLECTING TELESCOPES.

382. In refracting telescopes, as we have seen, an image of the distant object is formed, and the necessary amount of light is collected by means of a *lens;* in reflecting telescopes these ends are attained by means of a concave *reflector*. Reflecting telescopes differ from one another only in the contrivance used to transfer the image to a position which is convenient for observation; and the image is then viewed, as in the case of refracting telescopes, by an eye glass or an eye piece. Newton was the first who actually constructed a reflecting telescope; as we have seen in Art. 316, he despaired of obtaining a good refracting telescope, and this led him to recommend the use of reflectors.

383. *Newton's telescope.* ACB is a concave spherical reflector; O the centre of the surface; OC the axis of the reflector. Let PQ denote a distant object placed nearly on the axis of the reflector. Rays from any point P of the object would after reflection from ACB be brought very nearly to a focus p which is in the straight line PO produced; and as PC is supposed large the distance of p

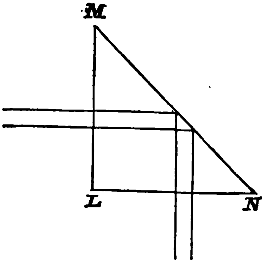


from the reflector is about half the radius of the reflector. Similarly the rays from any other point of the object would after reflection be brought very nearly to a focus, and thus a real inverted image pq of the object would be formed. HK is a small plane reflector inclined at an angle of 45^0 to the axis of the spherical reflector, and placed between this reflector and the position where the image pq would be formed. The rays falling from the spherical reflector on HK are reflected, and thus an image $p'q'$ is formed, which is situated, with respect to the front of HK, at the same distance and in the same position as pq with respect to the back. This image $p'q'$ is then viewed by an eye at E with the aid of a convex lens XY, which produces the virtual image st of $p'q'$.

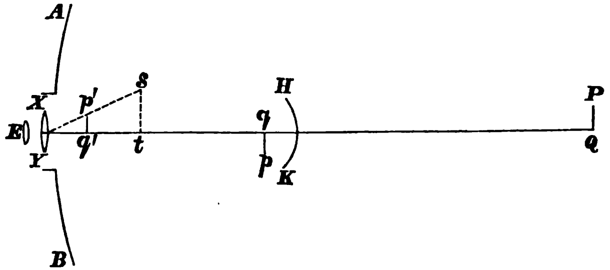
384. The spherical reflector ACB is placed at the end of a tube the axis of which coincides with that of the spherical reflector; the small plane reflector HK is supported by an arm which is attached to this tube. The lens XY is placed in a second tube which can move out-

wards or inwards through a small hole in the first tube, in a direction at right angles to the axis of it. The plane reflector intercepts some of the light which falls on the spherical reflector *ACB* from every point of the object; but its size is in reality far less than would appear from the diagram. The boundary of it is of an oval form so that it may just catch all the rays reflected from *ACB*, without intercepting more of those which fall on *ACB* than is necessary. When the telescope is used the lens *XY* is on the upper side of the large tube, so that the observer looks downwards through the small tube and sees an inverted image of the object. It is usual to suppose, as a standard arrangement, that the lens *XY* is placed so that $p'q'$ may be at the distance of the focal length from it; and then the image *st* becomes very large and at a very remote distance: see Art. 367. The magnifying power is found to be equal to the focal length of the concave reflector divided by the focal length of the convex lens.

385. In the two reflections which occur in this telescope it is found that about half the light incident on the concave reflector is lost. In order to diminish this loss Newton proposed to use the internal reflection of a glass prism instead of the reflection from *HK*. The prism has one angle a right angle, and each of the other angles half a right angle; it is placed so that its back *MN* takes the position of *HK*. The rays of light reflected from *ACB* enter the face *ML* nearly at right angles, so that they undergo scarcely any deviation; then they fall on *MN* and are totally reflected; next they come out through the face *LN* nearly at right angles to it, and proceed to form the image $p'q'$ as in Art. 383. It appears from experiment that the loss of light, by the two refractions and the total reflection in the prism, is less than the loss by the single reflection at the surface *HK* of Art. 383.



386. *Gregory's telescope.* AB is a concave spherical reflector, and HK another, much smaller. The two are

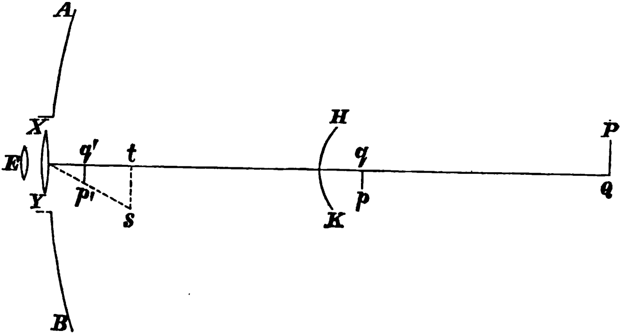


placed on the same axis and have their concavities turned towards each other. Let PQ denote a distant object, nearly on the axis of the reflectors. A real inverted image pq of PQ will be formed by AB nearly at its principal focus; the position of HK is so adjusted as to bring pq a little further from HK than its principal focus. Thus an image $p'q'$ of pq will be formed at a much greater distance from HK, on the same side of it: this image will be inverted with respect to pq, and therefore erect with respect to PQ. This image falls very near AB, and a small hole is made round the vertex of AB, so as to allow the rays which form the image to pass through; thus the image $p'q'$ may be viewed by an eye at E, with the aid of a convex lens XY, which produces the virtual image st of $p'q'$.

387. The reflector AB is placed at the end of a tube the axis of which coincides with that of the reflectors; and the reflector HK is supported by an arm which is attached to the side of this tube: the arm can be moved in such a manner as to allow the distance between the two concave reflectors to be a little increased or decreased at pleasure. The eye glass XY is placed in a small tube which has the same axis as the large tube and works through the small hole in AB. The small reflector HK intercepts some of the light which would otherwise fall on the large reflector; the hole in the large reflector does not cause much *additional* loss, for the rays which would fall on

it are mostly such as are intercepted by HK. It is usual to suppose, as a standard arrangement, that the lens XY is placed so that $p'q'$ may be at the distance of the focal length from it: see Art. 367. The magnifying power of the telescope is found to be given with sufficient accuracy by the following rule: take the square of the focal length of the large reflector and divide it by the product of the focal length of the small reflector into the focal length of the eye glass.

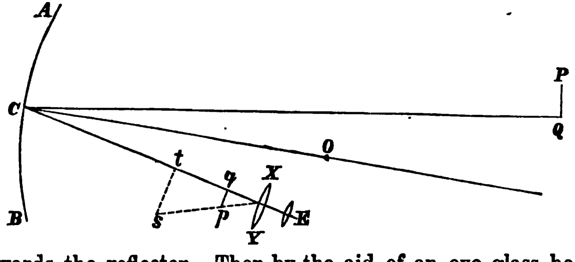
388. *Cassegrain's telescope.* This differs from Gregory's telescope in having the small reflector *convex* instead of concave. AB is a concave spherical reflector, and



HK a small convex reflector. The two are placed on the same axis, and the convexity of HK is turned towards the concavity of AB. Let PQ denote a distant object nearly on the axis of the reflectors. A real inverted image pq of PQ would be formed by AB nearly at its principal focus; but HK is placed between pq and AB, and so as to have pq a little nearer to HK than its principal focus. Thus the rays which would have formed the image pq are reflected, and form the real image $p'q'$ at a considerable distance from HK; this image is erect with respect to pq, and therefore inverted with respect to PQ. This image falls very near to AB, and a small hole is made round the vertex of AB, so as to allow the rays which form the image to pass through; thus the image $p'q'$ may be viewed by an eye at *E*, with the aid of a convex lens *X Y*, which produces the virtual image *st* of $p'q'$.

389. The arrangement of tubes is the same for Cassegrain's telescope as for Gregory's; and the same rule holds for the magnifying power. Cassegrain's telescope is considered to have a little advantage over Gregory's from the fact that the large tube is a trifle shorter, namely, to the extent of about twice the focal length of the small reflector. Moreover theory shews that the rays which form any point of the image $p'q'$ are brought rather more accurately to a single point by the convex and concave reflectors of Cassegrain's telescope than by the two concave reflectors of Gregory's telescope. But, on the other hand, Cassegrain's telescope exhibits the objects inverted, while Gregory's exhibits them erect.

390. *Herschel's telescope.* Sir W. Herschel constructed a great number of fine Newtonian telescopes, but his name is specially given to a very large instrument which he began in 1785, and finished in 1789. The reflector was about 4 feet in diameter, and had a focal length of 40 feet. In this telescope the observer sat at the mouth of the tube with his face turned to-



wards the reflector. Then by the aid of an eye glass he examined the image formed by the concave reflector, so that he did not have to shift the position of this image, as in the other reflecting telescopes, and thus he avoided the loss of light caused by this shifting. In order to prevent the head and shoulders of the observer from intercepting too much of the light falling on the concave reflector the image was formed out of the axis of the reflector. Thus let *ACB* represent the reflector, *O* its centre, *CO* its axis, and *PQ* a distant object. Then *PQ* is not on the axis of the reflector, and a real inverted image of it *pq* is found on the other side of the axis, so that the angle *OCq* is equal to the angle *OCQ*. Then this image could be viewed by an eye *E*, with the aid of the convex lens *XY* which produces the virtual image *st* of *pq*. But the rays proceeding from a point *P* which is at a considerable angular distance from the axis would not be brought very accurately to a single point as focus after reflection; and it is certain that the image formed by Herschel's large telescope was not distinct. It is usual to suppose, as a standard adjustment, that the lens *XY* is placed so that *pq* may be at the distance of the focal length from it: see Art. 367. The magnifying power of Herschel's telescope is found by the same rule as for Newton's.

391. For the sake of simplicity we have described the reflecting telescopes as furnished with an eye glass; but an eye piece may be used instead of an eye glass, as with the refracting telescopes. In all cases the eye glass or eye piece can be moved forwards or backwards a little so as to enable an observer to put the final image at that distance from his eye which best suits his habit of vision.

392. Numerous large reflecting telescopes have been made by zealous astronomers, and have rewarded the labour and thought expended on them by striking discoveries. It will be sufficient to notice two made by the late Earl of Rosse, and used by him in important observations, since continued by his son. The smaller of these telescopes had a reflector of 3 feet diameter, and of 27 feet focal length; the larger had a reflector of 6 feet diameter and of 50 feet focal length: both were of Newton's form. In such instruments as these, and that of Sir W. Herschel, much ingenuity is displayed in the erection of suitable mechanism for supporting the instrument and for directing it to various parts of the heavens; but these arrangements belong to Engineering rather than to Optics.

393. The main advantage of reflecting telescopes is that the image formed by the large reflector is free from

the confusion which exists in an image formed by a lens, owing to the compound nature of light. Hence before the manufacture of glass had reached its present state of excellence, which allows good achromatic lenses to be formed, reflecting telescopes were made as articles of trade and extensively sold. Thus Short was famous for the merit of his Gregorians; and afterwards Herschel produced very good Newtonians. But the reflector would never long retain its polish unimpaired, and thus the performance of these telescopes must have rapidly deteriorated, unless in the hands of a person who had the ability and the patience to renew the brightness of the surface from time to time. Sir J. Herschel when he went to the Cape of Good Hope, to spend some years in examining the southern stars, polished his reflectors before he set out, with the expectation that they would serve for his whole stay; but afterwards he became so fastidious and so skilful that he usually repeated the operation before any long series of observations. It is difficult to admit that all the fine reflecting telescopes made by Short and by Sir W. Herschel have completely perished; probably some of them may exist which could be rendered serviceable again with a little careful restoration. However the comparative ease and cheapness with which refracting telescopes can now be made, combined with their durability, seem to have almost banished reflecting telescopes from the shops of opticians. It has been said with respect to the few excessively large reflecting telescopes, that none of them ever really satisfied any observer except the maker himself; this is doubtless an exaggeration, but it is easy to see that a person who has himself superintended all the laborious processes involved in the casting of the metal and the grinding and polishing of the surface will feel a special interest in the result. With respect to the powers of refracting and reflecting telescopes see the *Monthly Notices* of the Royal Astronomical Society, XXXVI., 305 and XXXVII., 89.

394. The metal of which the reflector of a large instrument is made must be very carefully compounded, so as to ensure hardness and susceptibility of polish, without extreme brittleness; a mixture of copper and tin is found to answer best, the weight of the former being rather

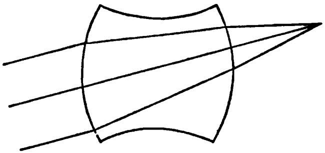
more than double that of the latter. A new method has been recently introduced by Foucault, by which the reflector of an astronomical telescope is constructed so as to resemble a common looking-glass. Suppose a thin convex glass, like a watch glass, to be silvered on the convex side; then when the other side is turned towards the light we have a concave reflector. The silvering is effected by an electrotype process, and the reflecting surface thus obtained is said to be brilliant and durable.

395. The manufacture of glass for scientific purposes has been much improved of late years, first on the continent, and afterwards in England. Thus telescopes with achromatic object glasses of three inches in diameter can be procured for a few pounds; and numerous fine instruments are to be found in public and private observatories with achromatic object glasses of a foot and upwards in diameter.

XXXVI. MICROSCOPES.

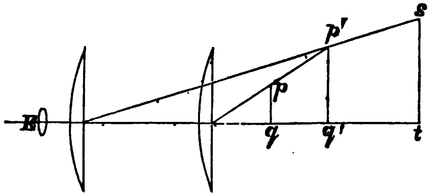
396. Microscopes are divided into two classes, *simple* and *compound*; in the former the object itself is viewed, and in the latter a *real* image of it is formed and viewed.

397. We have already explained the process of vision through a simple microscope: see Arts. 298...302. One of the most convenient forms of a simple microscope is that which is usually called the *Coddington lens*, from an eminent Cambridge writer on Optics; it seems however that the lens is attributed to him by mistake, and that it was really invented by Brewster. This consists of a sphere of transparent material in which the equatorial parts have been ground away. Thus the pencil of rays by which any point of a small object is seen will have its axis coincident with a diameter of the sphere; and the rays after passing through the sphere will appear to come very



nearly from a single point, or to be very nearly parallel. Thus all parts of a tolerably large field are seen equally well defined in all directions.

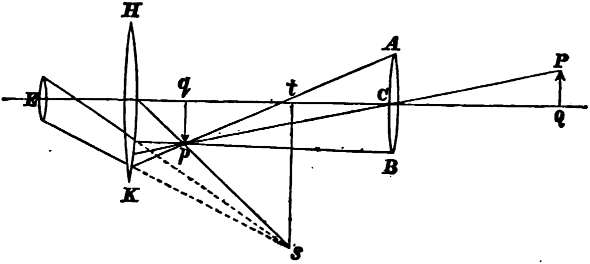
398. Instead of a single lens a combination of two lenses called a *doublet*, or of three lenses called a *triplet*, is often used; these rank with simple microscopes so long as there is no real image formed by part of the combination and magnified by the rest. One such combination is known as *Wollaston's doublet*. This consists of two



plano-convex lenses on a common axis, with their plane surfaces towards the object; the focal length of the lens nearest the eye is three times the focal length of the other: the distance between the lenses is about an inch and a half. When a small object *pq* is placed before the first lens, at a less distance than the focal length, an erect virtual image *p'q'* is formed at somewhat greater distance, but still at a distance from the second lens rather less than the focal length of that lens. Hence the rays after passing through this second lens appear to come from an erect virtual image *st*; and this therefore can be seen by an eye *E* on the common axis of the lenses. The magnifying power, as in Art. 301, is equal to the proportion of the angle under which the image *st* appears to the angle under which the object would appear if placed where the image is.

399. We proceed to *compound* microscopes; here a real image is formed by part of the instrument and magnified by the rest: in the simplest case the image is formed by one lens and magnified by another. *AB* and *HK* are convex lenses having a common axis; *AB*, which is nearest

to the object, is called the object glass, and *HK* is called the eye glass. *AB* is of short focal length. *PQ* is a small



object nearly on the common axis of the lenses, a little further from *AB* than the focal length of this lens; thus a real inverted image *pq* will be formed on the other side of *AB*. The eye glass *HK* is so placed that *pq* is at a little less distance from it than its focal length; hence the rays coming from *pq* will after refraction appear to come from a virtual image *st* which is erect with respect to *pq*, and therefore inverted with respect to *PQ*: this will be visible to an eye *E* on the axis of the lenses.

400. The magnification here is two-fold. In the first place the image *pq* is a magnified representation of *PQ*; and in the next place the lens *HK* magnifies *pq* in the manner explained in Arts. 298...302. The proportion of *pq* to *PQ* is the same as that of *Cq* to *CQ*; for example if *Cq* is six times *CQ* then *pq* is six times *PQ*. Again suppose that the focal length of *HK* is one inch, and that the distance of distinct vision is ten inches, then the eye glass magnifies *pq* about ten times. Hence the whole magnifying power is the product of 6 and 10, that is 60.

401. As in the case of telescopes it is supposed that for a standard arrangement *pq* is at a distance from the eye glass equal to its focal length, so that *st* is excessively large and very remote: see Art. 367.

402. It is found that a *single* object glass is objectionable for microscopes, as well as for telescopes, on account of the confusion occasioned by the compound nature of light; and accordingly a remedy has been devised for this. Great attention has been paid to the construction of microscopes in late years, on account of the demand for them made by physiologists engaged in minute investigations into the structure of animals and plants; and the result is that microscopes have been brought to a high degree of perfection. Instead of a single object glass the most powerful microscopes have three compound lenses; each of these consists of a concave and a convex lens, forming what may be roughly called an achromatic combination. We say *roughly* because the design is not that the confusion of the image should be removed by the action of one of the compound lenses, but rather by the action of all three in conjunction with that of an eye piece.

403. It is found necessary in some cases, in order to preserve minute objects for microscopic investigations, to cover them with thin plates of glass. Such plates, although they may not be more than one-hundredth of an inch in thickness will yet cause a difference in the vision of objects over which they are placed; for rays of light issuing from a point, after passing through a plate do not proceed *exactly* as if they came from a point, but only *nearly*. Now the modern microscopes of high power are so delicate that they clearly manifest the diminution in the excellence of the vision when the object is covered with the thin plate; and the defect is remedied by a slight adjustment of the distance of the front compound lens from the other two.

404. In microscopic investigations it is necessary to get a copious supply of light from each point of the object; accordingly provision is made by means of lenses or reflectors to throw a strong illumination on the object, so that sufficient light may proceed from it. Thus, for example, in conjunction with *Wollaston's doublet* a plane mirror and a lens are used for bringing light to the object. All the large compound microscopes have special arrange-

ments for the supply of light; and much ingenuity is shewn in making these arrangements, and in constructing stages on which the objects for investigation may be conveniently supported.

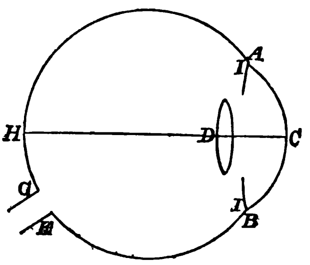
405. *Stops.* A *stop* or a *diaphragm* is a screen placed at any point of a telescope or microscope, pierced with a hole just large enough to allow the light which is wanted to go through; so that it prevents the passage of any other light. As an example we may take the Astronomical telescope; a stop is usually inserted at the place where the image *pq* is formed by the object glass: see Art. 364. Suppose the hole of the stop just large enough to allow the ray denoted by *AK* to get through it on the lower side; then the point *p* is the extreme point of the image below the axis which is seen by the eye; and the *full* pencil of rays from *P* which falls on the object glass reaches the eye. Hence in this case the stop limits the field of view to that part which is seen with tolerably uniform brightness, cutting off that part which would be seen with brightness gradually diminishing as the distance from the axis increased.

XXXVII. THE EYE.

406. Hitherto we have supposed the reader to possess that moderate acquaintance with the construction and action of the eye which the ordinary exercise of the power of vision cannot fail to impart; but we will now examine the nature of the eye more closely. We shall confine ourselves to the optical part of the subject, referring for what concerns physiology to works on that science.

407. The eye ball is nearly a sphere, and in an adult is about nine-tenths of an inch in diameter. The dia-

gram represents a horizontal section of the left eye made through its middle. About four-fifths of the circumference of the external coat is called the *sclerotica*, or in popular language the *white* of the eye; this is a tough, pearly, opaque membrane. The remainder of the external coat, forming the front part of the eye, is transparent, and is called the *cornea;* this is somewhat more curved than the sclerotica, so as to project forwards a little. In the diagram ACB denotes the cornea, while the other part AHB denotes the sclerotica. Behind the cornea is a transparent double convex lens D, called the *crystalline;* its axis coincides with a diameter of the eye ball, which also passes through the centre of the cornea, and is called the *optic axis:* it is denoted by CH. The space between the crystalline and the cornea is occupied by a transparent fluid called the *aqueous humour;* and the space on the other side of the crystalline, which constitutes by far the largest part of the cavity of the eye ball, is occupied by a transparent fluid called the *vitreous humour*. The sclerotica is lined by a dark-coloured membrane called the *choroid*. The optic nerve, proceeding from the brain, enters the eye ball through a small opening EG in the sclerotica and choroid; this opening is about one-tenth of an inch from the optic axis on the side towards the nose: inside the eye the optic nerve expands into a delicate transparent membrane called the *retina*, which is spread over the choroid. A little in front of the crystalline is a flat circular membrane, I, which is coloured blue, black, brown, or grey, and hence is called the *iris:* this is the part which, seen by a spectator through the transparent cornea, gives the characteristic tint of the individual eye. The iris has a circular hole, concentric with the optic axis, which is called the *pupil;* it is capable of becoming enlarged or contracted under the stimulus of light, so that



its diameter varies between one-eighth and one-fourth of an inch. At the extremity H of the optic axis there is a small transparent spot with a yellow margin, called the *foramen centrale;* it is free from the delicate membrane which constitutes the retina. As the eye is nearly spherical the diagram drawn as a horizontal section will serve for a *vertical* section if we omit the optic nerve.

408. The index of refraction for all the substances in the eye ball is nearly the same as that for water. The index for the aqueous humour is 1·337; for the vitreous humour 1·339; for the crystalline it is very nearly 1·377 at the surface, and increases to nearly 1·4 at the centre.

409. The crystalline is a more solid substance than either the aqueous or the vitreous humour. It is somewhat flatter before than behind, the radius of the front surface being three-tenths of an inch, and that of the back surface rather more than two-tenths of an inch. The thickness of the crystalline is about one-fifth of an inch, and the distance of its front surface from the cornea about one-tenth of an inch.

410. It is calculated that the whole refracting effect of the substances in the eye ball is equivalent to that produced by a single convex spherical surface having air before it and vitreous humour behind it; the radius of this surface would be about one-fifth of an inch, and the centre of the surface would be situated about one-fiftieth of an inch before the back of the crystalline.

411. Suppose rays to fall from any point of an object on the cornea of an eye; they pass through it, and through the various other substances within the eye ball, and come to a focus on the retina. This can be illustrated "by taking the eye of an ox, and paring away with a sharp instrument the sclerotic coat till it becomes thin enough to see the image through it." The statement may also be confirmed, to some extent, by drawing a careful diagram of the eye, and tracing the course of the rays. An image of any visible object is thus formed on the retina, and an impression is conveyed thence to the brain; but in what manner the last part of the process of vision is effected we do not know.

412. The image formed on the retina is believed to be *distinct*, when we disregard the compound nature of light; that is the rays from any point of an object seem to come accurately to a single point as focus. This is attributed by Brewster to the variation in density of the crystalline lens, "which, having a greater refractive power near the centre of its mass, refracts the central rays to the same point as the rays which pass through it near its circumference." The distinctness of the image is also promoted by the *pupil* of the eye, which acts like the stop in a telescope or a microscope, and cuts off rays that would come too obliquely. It has been held by some writers that the confusion arising from the compound nature of sun light is alleviated or removed by means of the different dispersive powers of the various substances within the eye ball; and we know that objects are in general seen without the coloured fringes which accompany the images formed by lenses that are not achromatic. But it is now generally admitted that the eye is not strictly achromatic; Brewster says: "No provision, however, is made in the human eye for the correction of colour, because the deviation of the differently coloured rays is too small to produce indistinctness of vision. If we shut up all the pupil excepting a portion of its edge, or look past the finger held near the eye, till the finger almost hides a narrow line of white light, we shall see a distinct prismatic spectrum of this line containing all the different colours; an effect which could not take place if the eye were achromatic."

413. In looking at an object we turn the eye towards it, so as to bring it on the direction of the optic axis; thus if the object is a mere point we receive a direct pencil from it, and the image of the point is on the optic axis at the back of the eye. If the object is very small, though not a point, the image of it is formed very near the optic axis. But the eye can see at once an object which is large enough to have the extreme parts of its image sensibly distant from the optic axis; thus from parts of the object the eye receives *secondary* pencils. Now when a secondary pencil passes through a lens there is one ray which suffers no angular deviation, and which may be

called the *axis* of the pencil; and the axes of all oblique pencils pass through a common point which is called the *centre* of the lens: see Art. 287. Similarly it is held by writers on Optics that there is a point which may be called the *optical centre* of the eye, situated at the geometric centre of the eye ball. The direction in which any point is seen is the straight line which joins that point with the optical centre of the eye, and the rays from the point come to a focus at the intersection of this direction produced with the retina.

414. The image of an object formed on the retina is *inverted* with respect to the object; and this has been considered a difficulty by some writers on Optics, who have puzzled themselves with the apparent inconsistency that we see objects erect while the images of them are inverted. It is sufficient to say that the retina is not the mind; seeing is performed by the aid of the eye, but it involves much more than a mere bodily operation, and of all beyond the image on the retina we are completely ignorant. There is no absurdity whatever in the simple fact that when rays from an external point are brought to a focus on the retina the mind is conscious that they come from an external point on the direction through the spot on the retina and the optical centre; so that an image on the *lower* part of the retina corresponds to an external point *above* the centre of the eye, and an image on the upper part of the retina to an external point *below* the centre of the eye.

415. We speak of images as formed on the *retina* in conformity with almost universal opinion; but it has been doubted by some writers whether this is the true seat of vision. Mariotte believed the choroid membrane to perform the functions usually attributed to the retina. The *foramen centrale* is the part of the eye where vision is most distinct; and the base of the optic nerve, that is the part of the back of the eye where this nerve enters the eye ball, is insensible to the action of light, as may be shewn by the following experiment. Place on a wall three wafers at the height of the eye, and two feet distant from each other; shut one eye, stand opposite the middle wafer at about ten

feet from the wall, and direct the other eye to the outside wafer on the same side as the shut eye: the middle wafer will then be invisible, and the other wafers visible. In this position of the eye the image of the middle wafer falls on the part *EG* of Art. 407, while the images of the other wafers are one on each side of it.

416. It is natural to suppose that for perfect vision it is necessary that the focus of the rays proceeding from any point of the visible object should fall precisely *on* the retina. Now it is a fact that most persons see clearly objects the distance of which from the eye ranges between a few inches and a large number of feet; hence the eye must possess some power of adjustment by which it can adapt itself to see objects at various distances. It appears by calculation that if parallel rays fall on the cornea they are brought to a focus at a point nine-tenths of an inch behind it: whereas if the incident rays instead of being parallel come from a point four inches in front of the cornea, the focus is one inch behind it. Thus the extreme power of adjustment required in the eye is equivalent to that of moving the retina through a space of about a tenth of an inch. Considerable diversity of opinion has existed with respect to the means by which the eye effects the necessary adjustment. Some have supposed that the form of the eye ball is changed so that the retina is removed from the crystalline lens or is brought nearer to it. Others have maintained that the crystalline lens is moved towards or from the retina. Others have suggested that the expansion and contraction of the pupil of the eye gave sufficient power of distinct vision at various distances. But finally the researches of Helmholtz and others have established, as had been previously conjectured, that the form of the crystalline lens is susceptible of change, and that by this means the eye can see and accommodate itself to the various distances of the objects it views.

417. But the power of adjustment for objects at different distances is not possessed to an adequate extent by all eyes; hence arise two common defects of vision. Some persons have eyes which are too strongly convergent, and thus the rays from a visible point are brought to a

focus at a spot in *front* of the retina; other persons have eyes which are too feebly convergent, and thus the rays from a visible point are brought to a focus at a spot *behind* the retina. The former class of persons are called short-sighted, or near-sighted; and the latter long-sighted or far-sighted: by some caprice of language the words *short-sighted* and *far-sighted* are used metaphorically with respect to the mind, as epithets of blame and praise respectively, and thus we have not an unobjectionable pair of correlative terms to denote the simple physical facts. With respect to the diversity between eyes as to the power of adjustment Coddington observes: "This depends almost entirely on habit. A North American Savage has a most perfect vision of very distant objects, but can hardly distinguish one held within arm's length."

418. The defect of short sight is remedied by the use of concave spectacles; the property of a concave lens is to render a pencil of divergent rays still *more* divergent, so that in spite of the too strong convergent power of the eye such rays may still come to a focus *on* the retina. The defect of long sight is remedied by the use of convex spectacles; the property of a convex lens is to render a pencil of divergent rays coming from a point more distant than the focal length *less* divergent, so that in spite of the too feeble convergent power of the eye such rays may still come to a focus *on* the retina. The defect of long sight happens to most persons as they grow old; for between the ages of 30 and 50 years a change takes place in the state of the crystalline lens by which its density and refractive power, as well as its form, are altered: the result is a difficulty in reading books printed with small type, and in distinguishing minute objects, especially by candle light. Besides the two common defects of vision which have been mentioned there are special defects of rarer occurrence. In some cases there is a difference between the refraction in a vertical and in a horizontal direction; so that rays in a vertical plane are brought by the eye to a focus at one point, and rays in a horizontal plane to a focus at another point, before or behind the other. This defect is remedied by the use of a lens which has one surface spherical and the other surface cylindrical.

419. In order that an object may be distinctly seen the image of it on the retina must occupy sufficient space. Suppose a circular disc, a foot in diameter, to be placed at the distance of about 57 feet from the eye; then it will subtend at the eye an angle of about one degree: that is if we suppose two straight lines drawn from the centre of the eye to the opposite ends of a diameter of the disc, the angle between these straight lines will be one degree. Now the distance of the optical centre of the eye from the retina is about half an inch, and consequently the diameter of the image on the retina will be in the same proportion to the diameter of the object as half an inch is to 57 feet, that is as half an inch is to 684 inches, that is as 1 is to 1368. Thus the diameter of the image is $\frac{1}{1368}$ of a foot, that is $\frac{1}{114}$ of an inch; and yet, small as this is, the object will be distinctly visible. The disc of the full moon subtends an angle of about half a degree at the eye of a spectator, and thus the diameter of the image on the retina will be half of that just determined, that is it will be $\frac{1}{228}$ of an inch. But it is well known that we can see distinctly, not only the form of the moon, but many varieties of light and shade constituting marks on her surface. Now the disc of a foot in diameter, which we mentioned before, might be moved to so great a distance that it would be invisible; this limit would vary with different eyes, and would depend partly on the power of the disc to send out light. It is said that a white disc one foot in diameter, illuminated by sun-light, will become invisible when at such a distance as to subtend at the eye an angle of about one three-hundredth part of a degree; its distance then would be about 300×57 feet, that is 17100 feet.

420. It is obvious that visibility depends partly on the quantity of light which an object sends out. If two objects of the same form and size, at the same distance, appear unequally bright, we naturally infer that the brighter sends out more light than the other. It might at first be supposed, that if we change our distance from a

luminous body, like the moon, the form of which we can distinctly discern, then the *brightness* will change: but this is not the case. Suppose we could place ourselves at half our present distance from the moon; then, by the law of Art. 172, four times as much light as at present would enter the eye from the moon: but this would be spread over an image on the retina four times as large as at present, and so would not give us the notion of a *brighter* moon, but of a *larger* moon. The diameter of the moon would seem to be twice as large as at present, and so the area of the visible circle would seem to be four times as large as at present. The fact illustrated by this example is stated briefly thus: *luminous objects of definite form appear equally bright at all distances.* This does not apply to cases where the origin of light is so distant or so minute that it appears as a mere *point*, without any definite form: thus a star would appear brighter as we approached it, so long as it was so distant as to look only like a point.

421. Common observation shews that light requires some appreciable time in order to produce an impression of visibility; though the time is excessively small. To this may be attributed the impossibility of following by the eye the course of a bullet through the air; the bullet does not remain long enough in one position to make a definite impression on the retina. In like manner the flight of a bat is so rapid that it is almost impossible to discern it in the dusk.

422. On the other hand the impression produced by light remains for a brief interval after the light has been removed; this is shewn by a familiar observation. Let a bright body, as for example a burning stick, be moved rapidly round in a circle; then the eye receives the notion of a circular rim of light: the impression produced by the burning stick in any position remains on the eye until the stick returns again to that position and so renews the impression. The interval during which an impression produced by light remains on the eye has been estimated at different amounts by different observers, ranging between half a second and one-seventh of a second. "Various interesting phenomena in the natural world have their

origin in this property of the retina. Jets of gas and of water appear as streams of light and of fluid, though they are detached portions only. A stone rapidly projected appears a continuous line. A descending meteor is seen as a long train of light, and in forked lightning, the eye combines the successive positions of an electrical discharge." Brewster's *Treatise on Optics.*

423. The impression of light from any point on the retina appears to extend for a minute distance round the spot which is the focus of the rays. In virtue of this bright objects appear somewhat larger than they really are. Thus the bright part of the new moon seems to extend beyond the faintly illuminated portion, as if clasping it. On the other hand dark objects on a bright ground appear smaller than they really are; the image of the bright ground encroaching as it were beyond its proper boundary. These effects are collected under the term *Irradiation.*

424. When the eye has been strongly impressed by a particular colour, and then is turned suddenly to a white wall or a white sheet of paper, it receives for a brief interval the impression, not of whiteness but of the colour which is *complementary* to that at first observed: see Art. 311. Thus if the first colour were that of a bright *red* wafer, the eye, on turning to the white ground, would see for a brief interval a *bluish green* light of the same size as the wafer. The colour thus produced by the eye is called an *accidental colour*, or an *ocular spectrum.* If the first colour is yellow the accidental colour is indigo: if the first colour is blue the accidental colour is orange red; and so on. The explanation of the fact is usually given in the following form. When the eye has gazed for a long time on the red wafer the retina becomes, as it were, fatigued and less sensible to the influence of red rays than to the influence of the other coloured rays; accordingly when the eye is turned to white light it receives with readiness the impression of the other coloured rays to the exclusion of red: thus it appears to see the colour which is complementary to red.

425. A singular defect has been detected in a few persons whose sight was otherwise sound, namely, an in-

ability to distinguish colours: this defect has been found to occur in the case of some very eminent men. Thus Dugald Stewart in daylight could not distinguish between the colours of the scarlet fruit of the Siberian crab, and its green leaves; Dalton was unable to distinguish blue from pink by daylight, and in the solar spectrum the red was scarcely visible to him, while the rest appeared to consist of two colours; Troughton fully appreciated only two colours, the more refrangible, which he called blue, and the less refrangible, which he called yellow. Since attention has been drawn to the subject the defect has been found to prevail more extensively than had been suspected. In an example which came under the notice of the writer the defect occurred in the case of a gentleman, of two of his daughters, and of some of his grandsons, but in none of his granddaughters. One of his grandsons when a youth had to pay attention to water-colour drawing for professional purposes; he carefully labelled his own box of paints so as to be able to use for any object the colours which his memory told him would be necessary, but on one occasion his box was mislaid by accident or by the contrivance of his companions; then he produced a picture with a bright red sea and green rocks. In the present day when railway travelling requires from the drivers attention to coloured signals it would be prudent to ascertain that they do not exhibit this defect, either permanently or occasionally. It appears that in all the cases of *colour blindness* which have been carefully examined the defect consisted mainly in a want of the sensation of *red* light. Inability to distinguish colour must not be confounded with an inadequate knowledge of the *vocabulary* by which tints are designated; men who have little occasion to use this vocabulary often neglect to master it fully, though they may be perfectly able to appreciate the most delicate shades of colour.

426. There are various interesting questions with respect to sight which are rather metaphysical than optical; such for example is that as to the way in which we learn to recognize by the aid of sight the solidity, the relative position, and the distance of the objects around us. It is usually held, with Berkeley, that the eye is directly cognizant only of light and colour; and that it is by experience

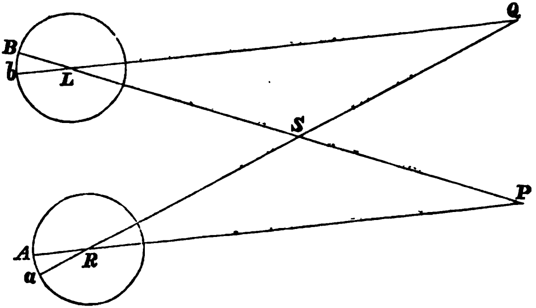
we associate combinations of tint, and varieties of brightness and shade, with the forms of bodies and their respective distances as ascertained by the sense of touch. The theory amounts to supposing that to an infant all visible objects may at first seem to be mere outlines, at the same distance; and that by touch and motion he corrects these errors. When there is no help by such means the eye continues unable to give us correct information. Thus the sun *appears* a flat disc, instead of a solid globe as we know it to be; and the eye gives no immediate notion of the vast distance at which this flat disc is situated. Berkeley's theory however is not without difficulty, especially from the case of some animals, which seem to have a correct appreciation of distance almost from their birth.

XXXVIII. THE STEREOSCOPE.

427. The question has been sometimes proposed, why with *two* eyes we have only single vision of objects. An obvious remark is that the question might be proposed with respect to many other cases in which the mind gains knowledge of external things by the aid of the bodily organs. Thus since we have two ears we might ask why our hearing is single; or if we feel an object with one hand and view it with two eyes, we might ask why we have not an impression of the existence of three objects instead of one. The fact is that the mind, aided by experience, puts the true interpretation on the perceptions of the senses in their standard condition.

428. We find on trial however that there is a certain condition with respect to the way in which we use our two eyes that must be regarded in order that we may have distinct single vision: the images formed in the two eyes must have *similar positions with respect to the optic axes of the eyes*. Suppose we view an object at the distance of several feet; let P denote a point of the object on which the attention is principally fixed. Then the optic axes of the eyes are so directed to P that the image of P falls for each eye on the *foramen centrale*, which we will denote by A for the right eye, and by B for the left eye. Now let Q

be a point near to P; then the image of Q will be formed for the right eye at a point a, and for the left eye at a point b. If Q be not far from P, and also be at about the same distance from the eyes as P is, then a and b will occupy



corresponding positions in the two eyes; that is, a will be as far to the right of A as b is to the right of B. Thus if the object consist of various points situated in the way we have supposed P and Q to be situated with respect to a spectator, the images in the two eyes will be alike and similarly placed; and experience shews that the vision in such a case will be single and distinct.

429. The reader who has a little knowledge of Geometry will be able to investigate under what circumstances there will be the *exact* correspondence of a with b. Let R denote the optical centre of the right eye, and L that of the left; and suppose Q to be so situated that the angle at Q is equal to the angle at P; then the angle BLb will be exactly equal to the angle ARa. For let S denote the point of intersection of QR and PB. The angle PSQ is equal to the sum of the angles SLQ and SQL; and the same angle PSQ is also equal to the sum of the angles SRP and SPR: see Art. 156. Hence the sum of the former two angles is equal to the sum of the latter two; and, as the angle at Q is equal to the angle at P, it follows that the angle SLQ is equal to the angle SRP. Therefore the angle BLb is equal to the angle ARa: see Art. 154.

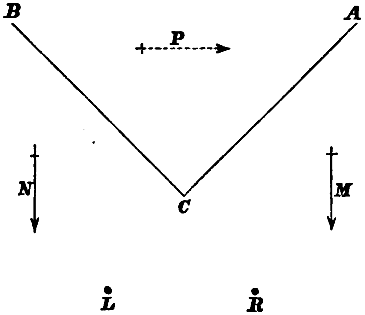
430. But when the eyes are so adjusted as to have single vision of P and also of Q, there will not at the same time be single vision of points which are much nearer to the eye, or much further from it, than P and Q. It is sufficient, for example, to advert to the point S; the image of this in the left eye will fall at B, and in the right eye at a; these are not corresponding points: we may therefore expect that S will appear double, and this is verified by experiment. Let the two forefingers be held up vertically before the eyes in front of the face, one near the face and the other more distant. Then if we look steadily at either finger, so as to make the directions of the optic axes meet on it, the other finger will seem double. Thus to see an object singly and distinctly we turn the eyes so as to cause the directions of the optic axes to meet at or near the object; this adjustment of the eyes is made unconsciously at the same time as that which is necessary in order to accommodate them for varying distance. In looking at a large object, as a portion of an extensive landscape, the eyes turn from point to point so rapidly that we obtain an impression of seeing instantaneously the parts which are merely glanced at in swift succession.

431. If a solid body be placed at a small distance from the face the images formed on the two eyes will generally be sensibly different, and the difference gives us a notion of the form of the object. For instance, suppose we put a thick book erect on a table, so that the back is towards the right eye; it will be easy to place the book so that the back alone is visible to the right eye, and therefore if the left eye be closed nothing more than the back will be seen. But if the left eye be open one side of the book may be completely visible to it; so that the image on the retina of the left eye is quite different from that on the retina of the right eye. When both images are produced at once the mind easily obtains an impression of the form of the solid object, that is, of the book.

432. Now imagine two pictures drawn of the book, one exactly as it appears to the left eye, and the other exactly as it appears to the right eye; then if these two pictures

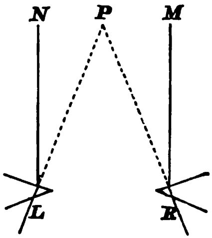
can be presented simultaneously, each to the appropriate eye, and each in its appropriate place, the idea of a solid figure is strongly impressed on the mind. There is no difficulty in drawing the two pictures, but some contrivance is necessary for bringing them into their appropriate places, and the name of *stereoscope* is given to the instruments by which this is effected.

433. The original stereoscope constructed by Professor Wheatstone was a *reflecting* instrument. *AC* and *BC* are plane reflectors which are inclined to each other at right angles. The right eye is placed at *R*, looking into the re-



flector *AC*, and the left eye at *L*, looking into the reflector *BC*. The picture *M*, suitable for the right eye, is placed before *AC*, and the picture *N*, suitable for the left eye, is placed before *BC*. If *M* and *N* are properly situated the image formed by *M* in *AC* and that formed by *N* in *BC* coincide in situation as at *P*; and thus the impression produced is that of a solid body placed at *P*. The whole apparatus is put into a convenient box, in a side of which holes are made at the places where the eyes should be applied.

434. The *refracting* stereoscope was invented by Brewster. In this the two pictures are placed in front of the spectator and viewed through two prisms, which have their edges turned towards each other. Let M be a point in the picture before the right eye, and N the corresponding point in the picture before the left eye. The rays from M in passing through the prism before the right eye, R, will be deviated towards the thick part of the prism; and in like manner the rays from N in passing through the prism before the left eye, L, will be deviated towards the thick part of the prism. Hence both sets of rays will seem to come from some common point P; and in this manner the combination of the two pictures is effected which gives the impression of a solid object. The whole apparatus is put into a convenient box or frame. It is found in practice that instead of two prisms we may advantageously use two halves of a convex lens, or even two quarters; in both cases the edges of the pieces of glass are turned towards each other, and the thick parts turned away from each other, as in the case with the prisms in the above diagrams. Brewster called his instrument a *lenticular* stereoscope.



435. There are various methods by which we may obtain the two pictures required for the stereoscope. If the object is very simple, as a cone or a pyramid, the two pictures may be drawn strictly by the rules of perspective. But for more complex objects they may be obtained by the aid of a *camera*, an instrument which will be described hereafter. The most common method however is to procure two photographs, placing the photographic apparatus first in one position and then in the other.

436. The wonderful effects which are produced by stereoscopic views of buildings involve a certain exaggeration. Suppose we are looking at a cathedral from a position distant about 200 or 300 feet. The two eyes are

about 2½ inches apart, and thus although there must be theoretically some difference between the images formed on the two eyes, yet the difference cannot be practically sensible. Our appreciation of the solidity of the object must then be derived from the varieties of light and shade as we contemplate successively different parts of the building, and not from the difference of the images formed simultaneously in the two eyes. The two photographic pictures which would be used in such a case for the stereoscope would be taken from points not at the distance of 2½ inches but at the distance of several feet. The result is that the person who sees the pictures in the stereoscope has strictly, not such a view as he would have by looking at the building itself from the distance of 200 or 300 feet, but rather such as he would obtain from an accurate *model* of it on a small scale placed at the distance of a few feet. Similar remarks hold with respect to the two portraits of a person which are combined in a sterooscope so as to give the effect of a solid bust.

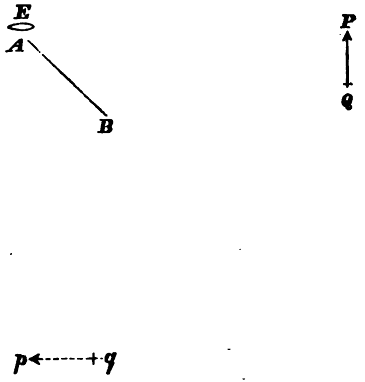
XXXIX. CAMERA OBSCURA AND CAMERA LUCIDA.

437. *Camera Obscura.* These words signify a *dark chamber;* and they are applied to an instrument invented by Baptista Porta. Originally it consisted of a dark chamber with a single hole in a window shutter, in which was placed a convex lens. If a piece of white paper is held vertically in the chamber before the lens, at about the distance of its focal length, an accurate picture of the external objects which could be seen from the window will be formed on the paper. Men, and trees, and clouds, will appear in their natural colours, and every change of position which takes place among these objects will be immediately reproduced in the image. The image is inverted, but if the observer looks over the top of the paper it will appear erect. If the paper is fixed firmly an outline of the image can be traced on it by a person who takes a front view; if instead of an opaque paper we use transparent material, as a piece of ground glass, an outline

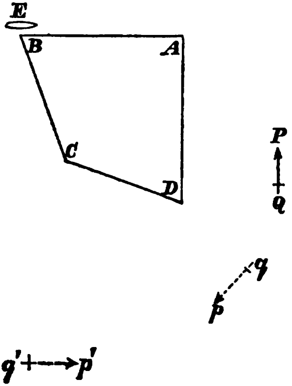
of the image can be traced on it by a person who takes a back view. Such is the *camera obscura* in its simplest form; but there are various modifications of it intended to render the process of tracing the image more convenient. For instance, a plane reflector is placed inclined at an angle of 45° to the rays which have passed through the lens, so as to serve the same purpose as the plane reflector in Newton's telescope: thus the image can be thrown on a horizontal table, and so can be traced by a person sitting near the table. In the *portable* camera obscura the chamber is reduced to the dimensions of a box, and the observer is outside the box. The lens is fixed in one of the vertical sides of the box, and the image is formed on the opposite side; supposing this side to be of ground glass the tracing can be performed by the observer on the outside; he may find it necessary to put a dark cloth over the box and over his head, so as to prevent the picture from being overpowered by the brightness of the light coming from external sources. Instead of receiving the image on the vertical face opposite the lens it is more convenient to insert a plane reflector inclined at an angle of 45° to the horizon, so that the rays may be reflected upwards, and the image formed on a part of the top of the box; and if this is constructed of ground glass the image can be traced as before. The *photographic* camera is like the portable camera, only instead of the ground glass we have paper or glass rendered sensitive by the application of some chemical solution, and instead of the tardy fingers of the draughtsman the light itself with almost immediate action stamps the image which it forms. In another variety of the camera the lens is put in a horizontal position outside a chamber, and above it is placed a plane reflector inclined at an angle of 45° to the horizon. Objects in front of the plane reflector send rays to it, which are reflected by it to the lens, and then refracted by the lens; so that an image is formed on a horizontal table suitably placed in the chamber. The plane reflector can be turned successively to every part of the horizon, and thus the whole surrounding landscape may be gradually brought to view.

438. *Camera Lucida.* This instrument, invented by Dr Wollaston, receives its name in contrast to the camera

obscura: it is mainly used for the same purpose, namely, for obtaining with ease an accurate drawing of any object. AB is a plane reflector inclined at an angle of 45^0 to the



horizon; PQ is any object situated before the reflector; pq is the virtual image of this formed behind the reflector. The image will be horizontal if the object be vertical, and will be visible to an eye E placed over the reflector. If this eye be just at the end of the reflector, the observer will be able to receive through part of his pupil the rays reflected from AB which form the image pq, and also to receive through the rest of his pupil light which will serve to guide his hand in tracing an outline of pq on paper. But it requires patience and dexterity to manage the instrument, and some persons entirely fail in their attempts to use it. The image formed is inverted to the eye of an observer placed in the manner we have supposed; the inversion may be removed by using two reflectors, and this is effected in the following manner. Let a rod of glass be taken such that $ABCD$ represents a section of it. The angle A is a right angle, the angle C is an angle of 135^0, and each of the angles B and D is an angle of $67\frac{1}{2}^0$. The side AD is vertical, and therefore AB is horizontal. Rays from an object PQ enter AD nearly at right angles,

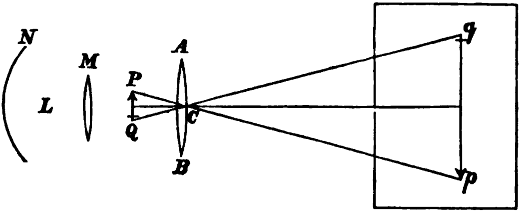


and so undergo no deviation; they fall on CD at an angle greater than the *critical angle*, and consequently are totally reflected: thus a virtual image pq is formed. This may be considered as an object in front of BC; the rays virtually proceeding from it fall on BC at an angle greater than the critical angle, and consequently are totally reflected: thus a virtual image $p'q'$ is formed. The rays virtually proceeding from it pass through AB nearly at right angles. If an eye E be placed close to B the observer can at the same time receive rays passing through AB, and also light which will serve to guide his hand in tracing an outline of the image $p'q'$ on paper: the image is in this case erect.

XL. MAGIC LANTERN. SOLAR AND OTHER MICROSCOPES.

439. In the camera obscura we have an image formed by the aid of a convex lens; the object is usually of large dimensions, as a landscape, and the image is a miniature of it. There is also a class of instruments in which the object is usually small, and the image is a magnified copy of it; in using these instruments it is necessary that the object should be rendered more luminous than it naturally would be, in order that the image may be reasonably distinct.

440. The *Magic Lantern.* This instrument was in-



vented by Kircher. AB is a convex lens of which C is the centre; a small object PQ is placed at a distance from the lens a little greater than its focal length. Rays proceeding from any point P of this object after passing through the lens are brought to a focus at some point p on PC produced, at a considerable distance from C. Similarly the rays from any other point Q of the object are brought to a focus at the point q; and in this way a real inverted magnified image pq is formed, and may be received on a screen placed just at this position. This screen is white, and may be either opaque or transparent; in the former case the image must be viewed by a spectator on the same side of the screen as the object, in the latter case the spectator may be on either side of the screen. In order to obtain sufficient light from every point a good lamp is placed at L near the object, and a convex lens M is interposed between L and the object, so as to concentrate the light on the object. Also a concave reflector N is placed behind L so as to throw forward these rays from L which would otherwise be lost. The object is usually a coloured drawing on glass, or a minute object fixed and preserved on glass; the object can be put into its place by sliding it through a groove made in the case which contains the reflector, the lamp, and the lenses; hence the objects receive the name of *slides*. To allow for the inversion produced by the lens AB the slides are put into the groove upside down.

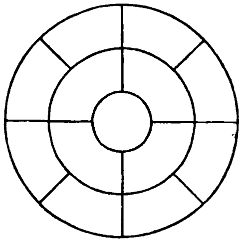
441. It will be sufficient to notice very briefly some modes of using the magic lantern for popular amusement. Suppose the screen to be transparent, so that the operator and the magic lantern can be concealed from the spectator by being placed on the other side of the screen; then, as the magnification is greater the further the screen is from the lens, the operator can vary this by approaching to or moving away from the screen: hence the spectator receives the impression of a gradual change in the distance of the object. Let the slide exhibit a landscape; then when the lens is very near the screen the image is very small, so that the scene may appear to the spectator lost in the distance; the exhibitor now may rapidly change the slide and remove the lens away from the screen, giving the impression of a new scene commencing at a great distance and gradually approaching. Effects of this kind have been called *phantasmagoria*. Sometimes two magic lanterns are used which form images on the same screen. Then a variety of effects may be produced by having two different pictures on the two slides; for by gradually covering or uncovering one or the other of the two convex lenses which form the images, we can throw one image alone on the screen, or render one image predominant over the other. Thus, for example, we may have a gradual change from a landscape viewed by day to the same viewed by night.

442. Hitherto we have spoken of the magic lantern chiefly as an instrument used for the purpose of amusement; but it has been employed more definitely as a microscope.

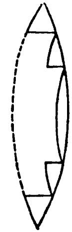
For example, the object may be a drop of impure water, and the magnified image may shew the presence of numerous living creatures of strange forms and habits. When the observations are to be made with accuracy greater precautions are necessary than those which will suffice for the exhibition of mere amusing spectacles; the single convex lens must be replaced by a compound achromatic lens, and special attention must be paid to the illumination of the object. The microscope receives different names according to the different modes of obtaining the illumination. Thus in the *solar microscope* the rays of the sun are received on a large convex lens, and the object is placed at the point where they are brought to a focus. It is necessary to intercept those rays which bring *heat* from the sun, for otherwise the object might be injured or destroyed; it is found that by allowing the rays to pass through water most of the *heat* is stopped, while little of the *light* is lost: the water is rendered still more efficacious by putting into it as much alum as it will dissolve. Instead of sun-light we may use artificial light; two such of great power have been thus applied. One is produced by a ball or cylinder of lime, rendered white hot by the flame of a blow-pipe from which proceed oxygen and hydrogen mixed in the same proportion as they are in water: the instrument with this illumination is called the *oxy-hydrogen microscope.* The other light, more recently introduced, is obtained by bringing two pieces of charcoal nearly into contact, after putting one piece in connection with one pole of a galvanic battery, and the other piece in connection with the other pole: the instrument with this illumination is called the *photo-electric microscope.*

443. A convex lens will produce intense heat by bringing the sun's rays to a focus. Brewster mentions an example in which a lens was constructed 3 feet in diameter, and the rays after passing through it were refracted by a second lens 13 inches in diameter: an image of the sun was formed half an inch in diameter at the distance of 63 inches from the larger lens. By means of this compound burning glass such substances as metals and stones were melted in a few seconds.

444. A more valuable application of convex lenses is for the purpose of lighthouses. A powerful lamp being placed at the principal focus of such a lens, the rays from the lamp after refraction through the lens proceed as a parallel pencil. If the lamp and the lens are fixed the direction of the pencil will be fixed. By making the lens move through an arc of a circle round the lamp, having its faces vertical and one always turned towards the lamp, the direction of the emergent pencil can be made to sweep round any assigned portion of the horizon. It would be very difficult and consequently very expensive, to make a large single lens of pure glass free from veins, and accordingly Brewster suggested that they should be built up of distinct zones, each zone consisting of distinct segments: lenses have been constructed in this way as much as three feet in diameter. Such a lens would in front present the appearance of the diagram; the lines indicate the junction of distinct pieces of glass. Here there is one central piece, then a zone of four segments, and beyond that a zone of eight segments. There are some advantages obtained incidentally by thus constructing the lens in pieces. The pieces need not form *one* spherical surface when joined; the radius of the spherical surface may vary from zone to zone in such a manner as to diminish the *spherical aberration:* see Art. 208.



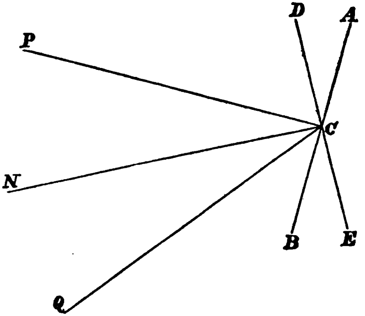
Moreover the thickness need not be so great throughout as that which a single lens would have; thus a section of the lens, made by a plane through its axis, would be of the form indicated by the dark lines: the dotted line shews what would be the boundary if the lens were an ordinary double convex lens, and so gives an idea of the quantity of glass saved, and of the consequent diminution of the loss of light by absorption.



XLI. THE SEXTANT. THE KALEIDOSCOPE.

445. The reflection of light in succession from two plane surfaces is applied in the *Sextant*, one of the most valuable instruments, and in the *Kaleidoscope*, one of the most attractive toys: we proceed to explain these two contrivances.

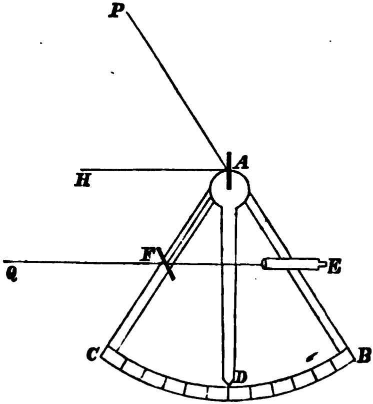
446. Let ACB represent a plane reflector which can



turn round an axis in its plane; the axis is supposed to be perpendicular to the plane of the paper, and to pass through the point C. Let DCE represent a second position of the reflector, so that it has been moved through the angle DCA. Suppose PC, in the plane of the paper, to be at right angles to ACB, and let it represent a ray of light. Then if PC falls on the reflector when it is in the first position, the ray will be reflected back along CP. Let CN be at right angles to DCE; and let the angle QCN be equal to the angle PCN. Then if the ray PC falls on the reflector when it is in the second position the ray will be reflected along CQ. Hence PCQ is the angle between the ray reflected in the first position of the reflector, and the ray reflected in the second. This angle is twice the angle *PCN*; but the angle *PCN* is equal to the angle *ACD*; therefore the angle *PCQ* is twice the angle *ACD*. Thus, for example, if *ACD* is an angle of 30° then *PCQ* is an angle of 60°; if the reflector is turned through a further angle of 5°, so that *ACD* becomes 35°, then *PCQ* will become 70°: so that a movement of the reflector through 5° increases the deviation of the reflected ray by 10°. In this way we arrive at the following result: when a plane reflector turns round an axis in its plane the deviation of a given ray incident in a plane perpendicular to the axis is increased by *double the angle through which the reflector has been turned.*

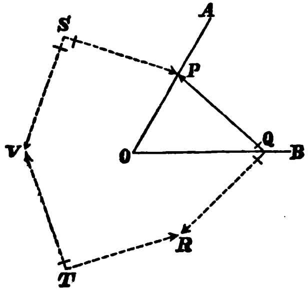
447. *The Sextant.* *ACB* is a framework consisting of rods *AC* and *AB*, which meet at *A*, the centre of the circular arc *BC*; all these parts are rigidly connected together, and the angle *BAC* is about 60°. A bar *AD* can turn round the centre *A*, and reaches to the arc *BC*. A plane reflector *F*, perpendicular to the plane of the instrument, is fixed to *AC*. At *A* another plane reflector, perpendicular to the plane of the instrument, is fixed to *AD*; this therefore turns with *AD*, and it is so arranged as to be parallel to the reflector *F* when the moveable bar *AD* coincides with *AB*. Only the lower part of *F* is silvered, so that the rays of light can pass through the upper unsilvered part. *E* is a small telescope fixed to *AB*, having the axis of its lenses parallel to the plane of the instrument and just directed through the line which separates the silvered from the unsilvered part of *F*. Let *P* and *Q* denote two distant objects; then the design of the instrument is to find the angle between two straight lines drawn from the observer, one to *P* and the other to *Q*. The observer holds the instrument so that its plane passes through *P* and *Q*, and that *Q* can be seen when he looks through *E*. Then he turns *AD* until *P* is visible, apparently in close contact with *Q*, by means of rays reflected first at *A* and afterwards at *F*; *so that AF and FE are equally inclined to the reflector F*. The required angle then is double the angle *BAD*. For draw *AH* parallel to *FQ*. Then since *P* and *Q* are supposed to be *distant* objects, the angle

HAP is practically equal to the required angle. Since a ray coming in the direction *PA* is reflected along *AF*, a ray going along *FA* would be reflected along *AP*. And if *AD* were moved to the position *AB*, as the reflector *A* would then be parallel to the reflector *F*, a ray *FA* would



be reflected in the direction *AH*: this will be clear if a diagram be carefully drawn, or it may be demonstrated by the aid of Arts. 153 and 155. Therefore by Art. 446 the angle *HAP* is double of the angle *BAD*. Hence if the arc *BC* be graduated from *B* we have only to observe the reading at *D*, and to double it; or if every half degree on *BC* be marked and counted as a degree, we have only to read off the number against *D* in order to obtain the required angle. A result obtained in this investigation deserves to be separately recorded; the angle *HAP* is double of the angle *BAD*, and we may therefore make the following statement: the deviation of a ray after reflection at two plane mirrors perpendicular to the plane containing the path of the ray, is double of the angle between the mirrors.

448. *The Kaleidoscope.* This very elegant toy was



invented by Brewster. Let OA and OB denote two plane reflectors inclined at an angle of 60°. Let PQ denote an object placed in any manner between the reflectors. The reflector OA will form an image, which we may denote by PS; and the reflector OB will form an image, which we may denote by QR. Again the image PS may be regarded as an object with respect to the reflector OB, and thus we shall have the image denoted by RT; in like manner, the image QR may be regarded as an object with respect to the reflector OA, and thus we shall have the image denoted by SV. Also RT may be regarded as an object with respect to the reflector OA, and thus we shall have the image denoted by VT; in like manner SV may be regarded as an object with respect to OB, and thus we shall have the image denoted by TV, which is coincident with that just observed. No more images will be obtained, for VT is *behind* both reflectors.

449. It will be perceived that the five images in conjunction with the object form a symmetrical figure. In our diagram the figure has six angles; those at Q, S, and T are all equal; and also those at P, V, and R are all equal. If the reflectors had been inclined at an angle of 90° there would have been three distinct images, one image being formed in a duplicate manner, as is the case with

VT in the diagram. The five images in the diagram will not be all simultaneously visible to an eye, *whatever* may be the position of it; we need only refer to Art. 216, where it is shewn that in the simple case of reflection from a single plane reflector the eye must not be outside certain boundaries if it is to see an assigned point of the image.

450. In the construction of the Kaleidoscope the parts are so adjusted as to put the eye in as favourable a position as possible for seeing the object and the images simultaneously. The reflectors are placed inside a tube, parallel to its axis; and the eye brought near to the line in which the reflectors meet, or would meet if produced. The object consists of small fragments of coloured glass, which are placed in a cell formed by two circular pieces of white glass, fixed near each other but leaving room for the fragments to move about and arrange themselves in various ways.

XLII. VELOCITY OF LIGHT.

451. We have explained in Chapter XVIII. how the velocity of light is inferred from astronomical facts; we shall now give an account of two experimental methods by which, in recent times, this velocity has been determined.

452. *Fizeau's experiment.* Suppose a ray of light to proceed from an origin and to fall at right angles on a plane reflector; it will be reflected back over its original course, and so may be received by an eye placed behind the origin. Let a wheel rotate in a plane perpendicular to the course of the ray, and let the rim be furnished with teeth, all of the same breadth, with spaces, also of the same breadth as the teeth, between them: place the wheel so that while it rotates the teeth will in succession cross the course of the ray of light. Let the rotation be at first very slow; the observer receives no light when a tooth is on the course of the ray, but does receive light when the space between two consecutive teeth is on the course: thus there is alternately brightness and darkness. Let the velocity of rotation be gradually increased; then the observer will

see persistent brightness when the velocity is such that the impression made by the passage of the ray remains until it is renewed by the reception of another ray: see Art. 422. Let the velocity of rotation be still further increased; it is found that by proper adjustment any ray which has passed between two teeth will on being reflected back just meet the hinder tooth and consequently be stopped: then nothing is seen. The rate of rotation of the wheel being known, and also the number of teeth, we can calculate the time which light takes in passing from the wheel to the reflector and back again; and thus the velocity of light becomes known. If the rotation be still further increased the ray will sometimes get through and sometimes be stopped; then by the persistence of impressions on the retina the observer sees a continuous light though of much less brightness than if the wheel were removed. If the rotation of the wheel is just twice as rapid as that which produced the eclipse, all the light which goes through an opening between two teeth will return through the next opening; and the brightness will be as great as it can possibly be while the wheel is interposed. If the rotation of the wheel is just three times as rapid as that which produced the first eclipse there will be an eclipse again, so that another determination of the velocity of light can be made: and so on.

453. We have explained the principle of Fizeau's method; a few remarks must be made as to the details. We have spoken of a *ray* of light, but a single ray would be too faint to produce an adequate impression, and therefore a *beam* of parallel rays is used. Let there be two tubes, each with a convex lens at one end; let the tubes be put on a common axis, at a considerable distance apart, with the lenses facing each other. Then let an origin of light be placed in one tube at the distance of the focal length from the lens; the rays proceeding from the origin, after passing through the lens constitute a parallel beam, which passes on to the lens of the other tube and after refraction is brought to a focus. At this focus the plane reflector is placed, at right angles to the axis of the tube; the rays are reflected back to the lens, become parallel by refraction through it, and fall again on the lens of the first

tube: they are then refracted to the point from which they started. In practice the origin of light is placed *outside* the first tube, and by means of a piece of plane glass inclined at an angle of 45° to the axis of the tube, the rays are sent along the tube in the same way as if the origin really had been at the point supposed within the tube: when the rays return they pass *through* the plane glass, and so reach the eye which is on the axis of the tube ready to receive them. In Fizeau's experiment the distance between the two lenses was nearly five miles and a half. The process is remarkable as the first which gave a direct proof that light takes a finite time to travel from one place to another; but it is not susceptible of extreme accuracy. The result of Fizeau's experiments gave about 196000 miles per second nearly for the velocity of light.

454. *Foucault's experiment.* This was first performed in 1850 very soon after that of Fizeau. *AB* denotes a

plane reflector which can turn about an axis in its own plane through *C*, perpendicular to the plane of the paper. *D* is a fixed concave spherical reflector, having its axis on *DC*. A ray of light *PC* falls on the plane reflector, and when this, in the course of its rotation, is in a suitable position the ray is reflected along *CD*, then it returns to *C* and is reflected by the plane reflector back towards its origin. The plane reflector *ACB* is supposed to turn round in the direction of the hands of a watch, so that the dotted line denotes the position into which it comes after moving through a small angle. Suppose that the reflector rotates very slowly, then since the ray is sent backwards once in every rotation the light received at P will be intermittent; but when the reflector turns round about 30 times in a second the light becomes continuous by reason of the persistence of impressions on the retina. Let the rate of rotation be gradually increased until the reflector turns round 700 or 800 times in a second; then while the ray goes from C to D and back again to C, the plane reflector turns through an angle which is appreciable though very small. Hence the ray is sent back, not along CP, but along such a direction as Cp. The line joining P and p may be supposed perpendicular to CP; the distance Pp can be measured, and from this the velocity of light can be deduced.

455. We proceed to notice some details. Instead of a single ray PC a beam of parallel rays is used. A lens is placed having its axis coincident with PC, at the distance of its focal length from C, so that the beam of parallel rays is brought to a focus at C; the centre of D is at C, so that rays going from C to D are reflected accurately back to C. A piece of plane glass, denoted by HK, inclined at an angle of 45^0 to PC, is placed near P, so that the rays pass through it as they go to C; and when they return they are reflected by this glass to an eye at E. A fine wire is stretched across the beam of light near P; and it is this which corresponds to the point p of the diagram, and is observed by the eye. Finally the eye in observing is aided by a powerful magnifying glass. The experiment has been repeated with extreme care, and the result is held to be the most accurate value of the velocity of light yet obtained: that value is 298 millions of metres per second, or about 185000 miles per second.

456. A very important result is obtained by a slight modification of the experiment. Let the ray instead of passing in *air* from C to D, and back again, be made to pass through a tube filled with *water:* then it is found that the distance Pp is increased. Thus we see that light moves more slowly in water than in air; this is a very valuable fact with respect to the theories of light, which we now proceed to explain.

XLIII. THEORIES OF LIGHT.

457. The science of Optics is often divided into two parts which are called respectively Geometrical Optics and Physical Optics. The former treats of such results as follow from the ordinary laws of reflection and refraction; this part we have already considered: the latter treats of the properties of light as they are exhibited in interference, the colours of thin plates, diffraction, double refraction, and polarisation; to this part we now proceed. Hitherto we have been able to trace the course of light under various circumstances, and to explain that course as the consequence of simple experimental laws; but the facts to which we are now coming are of such a curious kind that they cannot be adequately described without some notion of the nature of light itself.

458. Only two theories respecting the nature of light have been received with attention by philosophers, namely the *corpuscular theory* and the *wave theory*. According to the corpuscular theory light consists of particles of matter shot forth by luminous bodies and entering into our eyes; and according to the wave theory light consists of waves excited by luminous bodies in an elastic medium which pervades all space. Thus in the corpuscular theory light is supposed to resemble odours; just as a rose is believed to emit minute particles which reach the nostrils and affect us with the special scent of the flower, so a star is held to emit minute particles which reach the eyes and

affect us with the special visible appearance of the star. In the wave theory light is supposed to resemble sound; just as a bell when struck communicates vibrations to the air which pass on and reach our ears, so light is held to consist in vibrations communicated by luminous bodies to a medium in some degree resembling the air, but far more attenuated; this medium is called the *ethereal medium*, or briefly the *ether*. The corpuscular theory has the advantage of offering the most elementary mode of conceiving the facts of reflection and refraction; but it is quite unable to deal with the varied phenomena which we shall notice in the following Chapters of this book. On the other hand the wave theory, although it gives a somewhat more complicated representation of the elementary facts, is very successful in application to those of a higher kind. There still remain however some phenomena, the explanation of which according to this theory is attended with difficulty; perhaps the aberration of light is the most conspicuous of these.

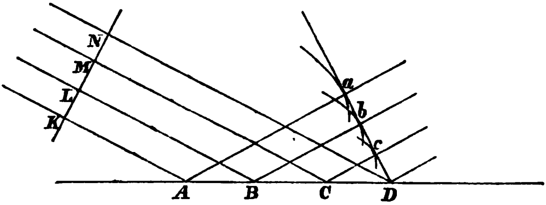
459. The corpuscular theory is sometimes called the *Newtonian* theory: Newton himself however does not appear to have held it very firmly, though he adopted its language in describing some of his observations. He urged as an objection against the wave theory that light ought, according to that theory, to pass round an obstacle instead of being, as is apparently the case, entirely intercepted; and he referred in support of this objection to the fact that sounds seem to pass unobstructed round obstacles. This objection, once held to be very powerful, is now met in a twofold manner. In the first place it is found that although sounds do pass round obstacles, yet they do so with some loss of intensity; thus what may be called an *acoustic shadow* is really produced by the interposition of an obstacle between the origin of sound and the ear. In the second place it is found that the rays of light do in some degree bend round obstacles, and give rise to the remarkable phenomena which we shall notice under the head of *diffraction*. The first person who presented the wave theory of light in a definite shape was Huygens,

though Hooke may have had some vague notions respecting it. The work in which Huygens expounded his views was published in 1690, under the title of *Traité de la Lumière:* it is one of the most remarkable productions in the whole range of Natural Philosophy. The supposed inclination of Newton for the corpuscular theory, and the certain adoption by Huygens of the wave theory, have been ingeniously connected with the nature of their respective studies in the following remarks: "Newton was accustomed to view the universe as composed of bodies in motion—each of these bodies or planets revolving round its own axis—while it moved with great velocity in its orbit round the sun as the centre of attraction: and accordingly it is not surprising that he should regard light as composed of corpuscular atoms, shot forth with extraordinary velocity and each moving round its own axis. Huygens on the other hand had his attention very much directed to the pendulum and its oscillations; and it accordingly need not surprise us that he should regard light as a phenomenon of oscillation similar to that which had occupied his attention."

460. The corpuscular theory, supported as it apparently was by the great name of Newton, was accepted by the eminent mathematical philosophers of the eighteenth century. In the early part of the present century Dr Thomas Young attempted to revive the wave theory in England; but he did not succeed in gaining any attention. A few years later Fresnel and Arago in France were led by various experiments to adopt this theory, and Fresnel also supported it with considerable mathematical skill. Then it was received by British philosophers with great favour, and especially recommended at Cambridge by the authority of Airy and Whewell. Sir D. Brewster, who has been called the *Father of modern experimental Optics*, was probably the last very distinguished investigator who resisted the wave theory; and the tone of his opposition was much mitigated in the later years of his life.

461. According to the corpuscular theory of light the laws of reflection at a plane surface are very easily explained, by recurring to the analogy of perfectly elastic balls impinging on a hard plane: see Vol. I., Art. 284. We

proceed to treat this on the wave theory. Suppose *KA* and *ND* to be two parallel rays of a pencil of light falling on a plane reflecting surface *ABCD*. Let *KLMN* be at right angles to the direction of the rays; we imagine the light to consist of a plane wave, like that which we have considered in Art. 56 in the case of sound. Then *KLMN* denotes what is called the *front* of the *wave*. The wave

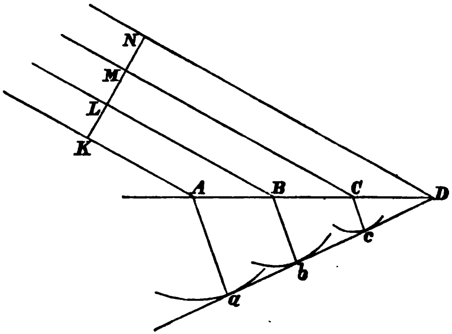


reaches the reflecting surface first at *A*, and then successively at other points in *ABCD*. As soon as the wave reaches *A* that point becomes the origin of a disturbance in the ether, which spreads out in the form of a sphere having its centre at *A*, and which may be called an *undulation*. Let the circle described round *A* as a centre denote the boundary which the undulation has reached during the interval between the arrival of the original plane wave at *A* and *D* respectively; then the radius of this circle is equal to the excess of *ND* over *KA*; because so long as we keep to the same medium, the velocity with which motion is propagated through the ether is supposed to be the same in every direction. Similarly a circle described from the centre *B*, with radius equal to the excess of *ND* over *LB* will denote the boundary which the undulation from *B* has reached during the interval between the arrival of the original plane wave at *B* and *D* respectively. Similarly let a circle be described from *C* as a centre with radius equal to the excess of *ND* over *MC*; this will denote the boundary of the undulation from *C*. Then it can be shewn, by Geometry, that a certain straight line drawn from *D* will touch all the circles thus described; the points of contact are denoted by *a*, *b*, *c* in the diagram: for the sake of clearness the entire circles are not drawn, but only portions of them in the neighbourhood of *a, b, c* respectively. This straight line will also touch *all the other circles* that could be drawn from points between *A* and *D* as centres, with radii determined by the same law as we have already used for the three circles considered. The straight line *abcD* is inclined to *AD* at the same angle as *KLMN*, but in the opposite direction. The circles represent undulations of light proceeding in the form of spheres from the various points of *AD* as centres, and the straight line *abcD* represents a plane which touches all these spheres. Now it is partly assumed and partly demonstrated, that the undulations of light counteract each other *except along the plane which touches all the spheres;* along this plane they corroborate each other, and thus form a plane reflected *wave* corresponding to the plane incident *wave.* Each *ray* of light is perpendicular to the surface of the *wave;* so that at any point of the reflected wave the ray of light is denoted by the radius of the sphere touched at that point: thus to the incident *ray KA* there is a corresponding reflected *ray Aa,* which makes the same angle with the surface as *KA* does, and the two rays are in the same plane which is perpendicular to the reflecting surface.

462. The reader on first studying this subject will probably find the preceding Article difficult, on account of the number of points which it involves. The geometrical principles though very simple may be new to him, and may require some consideration; but the mechanical principles employed form the most important part of the process, and these must be considered as assumptions which may appear reasonably probable, though they cannot be demonstrated at this stage. These are, first, the principle that every point of the reflecting surface becomes, when the plane wave reaches it, the centre of an undulation which spreads out with uniform velocity in a spherical form; and second, the principle that these undulations by their interference just counteract each other and leave the ether at rest at any instant except along a certain plane. The second principle is often called by the name of Huygens; the reader will gain help in understanding it from Chapter XI. which treats on *interference* with respect to sound.

463. The refraction of light on the corpuscular theory is not assisted by any simple illustration like that of elastic balls in the case of reflection. Nevertheless suppositions may be made of a reasonable character by which the problem can be reduced to the calculation of the motion of a particle acted on by certain forces; and the result is that the laws of refraction, as known by experiment, are deduced. But there is one fatal objection to this theoretical investigation; it makes the velocity of the particle *greater* in the medium into which the light is refracted than in a vacuum; this however is contrary to fact, as shewn by various experiments: one of these we have already noticed in Art. 456. We proceed then to the explanation of refraction on the wave theory; it will be found that this is consistent with the fact noticed in Art. 456.

464. Suppose *KA* and *ND* to be two parallel rays of a pencil of light falling on a plane refracting surface



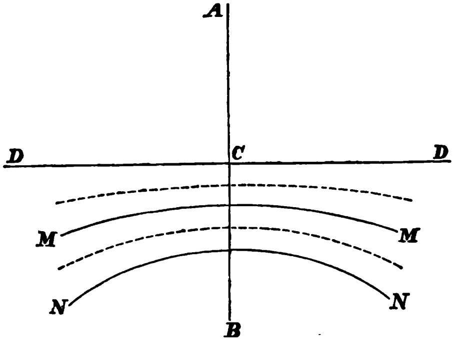
ABCD. Let *KLMN* be at right angles to the direction of the rays; then this will denote the plane front of the wave. The wave reaches the refracting surface first at *A*, and then successively at other points in *ABCD*. As soon as the wave reaches *A*, that point becomes the origin of an undulation in the ether which spreads out in all directions in the medium in the form of a sphere having its centre at A. We suppose, however, that the velocity with which motion is propagated in the medium is less than that in vacuum. Describe a circle with centre A, and with radius equal to the space a wave would move over in the medium in the same time as it would describe the excess of ND over KA in vacuum. Let circles be described from B and C, and from all other points of AD, according to the same law. Then a straight line $abcD$ will touch all these circles, representing a plane which will touch all the spheres which the undulations form. The undulations by their interference will counteract each other at all points at any instant except along this plane, which therefore represents the plane refracted wave. At any point of the refracted wave the *ray* is denoted by the radius of the sphere touched at that point; thus to the incident ray KA there is a corresponding refracted ray Aa. Finally, it may be shewn by Geometry that the sine of the angle which KA makes with the normal to the refracting surface at A bears the same proportion to the sine of the angle which Aa makes with the normal, as the velocity of light in vacuum is to the velocity of light in the medium; that is, as the excess of ND over KA bears to Aa.

465. In Arts. 461 and 464 we have for the sake of clearness to the beginner distinguished between the words *wave* and *undulation*. We consider *undulations* as proceeding from various points and by their combination giving rise to a *plane* wave. But the words *wave* and *undulation* are in general used by writers on Optics as identical in meaning; thus *undulatory* theory is used as equivalent to *wave* theory. What we have called *undulations* are sometimes called *partial* waves, if there is any risk of confounding them with the single aggregate wave which they make up by interference.

466. In Arts. 461 and 464 the incident wave of light was supposed to be a *plane* wave; we might however have supposed it to be a portion of a *spherical* wave, and then the reflected or refracted wave would have been spherical. Whenever a *wave* of light is mentioned the reader will be

prepared to think, if necessary, of the *ray* of light which corresponds to any point of the surface of the wave: if the wave is a plane wave the ray at any point is perpendicular to the plane surface of the wave there: if the wave is spherical the ray coincides with the direction of the radius of the surface at that point.

467. A most valuable part of the wave theory for the purpose of explaining various phenomena is the principle of *interference*, to which we have already adverted. In the application of this principle to sound we obtain the following result: two equal sounds *may* so combine as to give a sound of twice the intensity of either, and on the other hand, they *may* so combine as just to counteract each other and produce silence; see Art. 105. Now let us apply

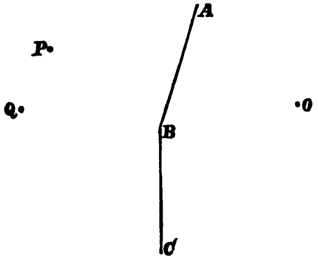


this principle to light. Suppose that an origin of light is situated at a certain distance from the plane of the paper, exactly over A; and let there be another origin of light at the same distance from the plane of the paper, exactly over B. Let the two origins be exactly alike; denote the former by P, and the latter by Q. Let C be midway between A and B; and draw the straight line DCD at right angles to ACB. Then any point D in this straight line is equally distant from P and Q, and the illumination there is double what it would be from either origin alone. Again, it can be shewn by Geometry, that a curve MM may be drawn having this property; at every point of the curve the distance from *P* exceeds the distance from *Q* by the same quantity: let this quantity be equal to the *length of a wave of light*, and then, as in the case of sound, the lights will again combine so as to give an effect equal to the sum of the single effects. But between *MM* and *DD* a curve may be drawn, indicated by the dotted line, at every point of which the distance from *P* exceeds the distance from *Q* by *half* the length of a wave of light, and along this curve the two lights will counteract each other and produce darkness. Similarly beyond *MM* a curve *NN* may be drawn at every point of which the distance from *P* exceeds the distance from *Q* by *twice* the length of a wave of light, and along this curve we have, as along *MM*, the sum of the two illuminations. Between *MM* and *NN* a curve may be drawn, indicated by the dotted line, at every point of which the distance from *P* exceeds the distance from *Q* by three halves of the length of a wave of light; and along this curve there will be darkness. Similarly other curves in succession can be drawn corresponding to an excess of the distance from *P* over the distance from *Q* of three, four, five, or any whole number of lengths of a wave of light; and all these curves will be bright with the sum of the two illuminations. And intermediate curves can be drawn in which the excess of distance is an *odd* number of half wave-lengths: and all these curves will be dark. For points intermediate between a bright curve and a dark curve the brightness will be intermediate. A similar set of curves can be drawn on the other side of *DD*; for these the distance from *Q* always exceeds the distance from *P*: the curves are called *hyperbolas* by mathematicians.

468. There is some difficulty in verifying these statements by experiments. The length of a wave of light is a very small quantity, and it is found by careful mathematical investigation that the bright curves and the dark curves run very close together, unless the distance between the two origins of light is extremely small. Moreover the length of a wave is different for the rays of different colours, as in sound the length of a wave is different for notes of different pitch; and in consequence

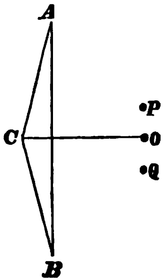
of this the bright curves and the dark curves do not occupy the same position for all the various coloured rays which compose white light. Hence the phenomena are most simple when homogeneous light is used; and then we have, as the theory indicates, merely alternations of brightness and darkness; the change produced when ordinary light is used will be understood after reading the next Chapter. Various methods have been devised by which the theoretical conclusions of Art. 467 are illustrated by experiment: of these we will now describe two.

469. Let AB and BC denote two plane reflectors, placed so as to be nearly in the same plane, that is, so as to



form a very obtuse angle. Let O denote an origin of light; then the light which falls on AB will after reflection proceed as if it came from a point P as far behind AB as O is before it: in like manner the light which falls on BC will after reflection proceed as if it came from a point Q as far behind BC as O is before it. The virtual origins P and Q are very near each other if the angle ABC is very obtuse, and as they are precisely alike they are in the circumstances in which the real origins of Art. 467 were assumed to be. Thus the light proceeding from O consists after reflection at AB and BC of two interfering streams; and the phenomena of Art. 467 can be observed: it is only necessary to place a white plane surface before the mirrors at a convenient distance to receive the light from P and Q, and at the same time to prevent the light from O reaching this surface by interposing some opaque screen. The origin of light O is conveniently obtained by bringing parallel rays to a focus through a convex lens, and allowing them to diverge from this point.

470. Again, let ABC denote a prism of glass having the angle at C very obtuse, and let an origin of light be placed at some point O on the straight line drawn from C perpendicular to AB and produced. Rays falling from O on AB, and refracted by AB and AC will proceed as if they came from a point P; and rays falling from O on AB, and refracted by AB and BC will proceed as if they came from a point Q. The two virtual origins P and Q will be very near each other if the angle ACB is very obtuse; and thus they are situated in the manner required in Art. 467.



471. In the experiments of the preceding two Articles it is easy to shew that the alternate bright and dark bands which are observed do really arise from the interference of the pencils proceeding from the two virtual origins; for if one of these pencils is intercepted the whole system vanishes. Moreover a very important result can be established by the aid of these experiments. In Art. 467 the line DCD is such that every point of it is equally distant from the two origins, the rays being supposed to travel from both origins through a vacuum. Now let a plate of some material, as glass, be interposed in the path of the rays which come from one origin, say P. If light travels more *slowly* in glass than in air the effect is the same as if the distance of points in DCD from P were *greater* than the distance from Q; so that the line corresponding to equal distances is, as it were, moved from its position in the diagram, and brought nearer to A. In this manner it is found, as the result, that the bands are shifted a little *towards* A. But if light travels more *rapidly* in glass than in a vacuum the result will be that the bands are shifted a little *from* A. Experiments shew distinctly that the bands are shifted *towards* A; and hence it follows that

light travels more *slowly* in glass than in a vacuum. The same result holds for any other transparent medium: thus the conclusion of Art. 456 is confirmed by another mode of investigation.

472. The reader will doubtless find the wave theory of light difficult at first: it cannot be denied that the demands on our belief are great. The vast ethereal ocean "which bathes the shores of the farthest star" is a hard postulate: unseen, unfelt, unknown in itself, it seems to exist only in scientific faith as the necessary vehicle for the endless phenomena of light. The theory however appeals with much more force to the mathematician who alone can fully appreciate the close agreement between calculation and experiment in many cases, and can understand how the bold conjectures of profound theoretical investigation have been verified by trial: but even he will admit that difficulties exist in the fundamental mechanical principles of the subject. However at the present day this is the only theory which can be said to exist; for no person now regards light as consisting of streams of particles rushing through space. The judgment of Sir J. Herschel on the wave theory, pronounced many years since, and before some of the most remarkable triumphs of the theory, has been frequently quoted: "a theory which, if not founded in nature, is certainly one of the happiest fictions that the genius of man has yet invented to group together natural phenomena, as well as the most fortunate in the support it has unexpectedly received from whole classes of new phenomena, which at their first discovery seemed in irreconcileable opposition to it. It is, in fact, in all its applications and details one succession of *felicities*, insomuch that we may almost be induced to say, if it be not true, it deserves to be so."

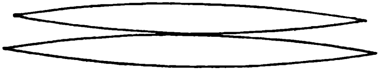
473. A curious property of the course of a ray of light between two points may be noticed here. In the diagram of Art. 461, suppose that a particle is to move from the point L to the point b with a certain uniform velocity, touching at the reflecting surface on the route; then we may ask what course must the particle take so as to get over the route in the shortest time. It may be shewn by Geometry that the particle must move in the course

which a ray of light would take; that is, the route must consist of two straight lines *LB* and *Bb*, which make equal angles with the reflecting surface at the point where they meet it. It will be easy to verify, by drawing a few diagrams, that the whole route in this case is shorter than it would be in any other, and so the time is less. Now a rather more difficult question may be proposed. In the diagram of Art. 464 suppose that a particle is to pass from the point *L* to the point *b*, moving in vacuum with a certain uniform velocity, and in the medium with a certain uniform velocity, less than the former; then we may ask what course must the particle take so as to get over the route in the shortest time. It may be shewn by Geometry that in this case also the particle must move in the course which a ray of light would take; that is, the route must consist of two straight lines *LB* and *Bb*, so situated that the sine of the angle which *LB* makes with the normal to the surface at *B* must have the same proportion to the sine of the angle which *Bb* makes with the normal, as the velocity in vacuum bears to the velocity in the medium. As an illustration let *L* denote a point on land, *b* a ship at anchor, and *AD* the shore; suppose that a person can walk at the rate of 4 miles an hour, and row at the rate of 3 miles an hour: then the question is, what course must he take to pass by walking and rowing from *L* to *b* in the shortest time. The point *B* must be such that the sine of the angle which *LB* makes with the normal to the surface at *B* is $\frac{4}{3}$ times the sine of the angle which *bB* makes. The property of light which this Article illustrates is known under the statement that light passes from one point to another in the *shortest time consistent with the circumstances.*

XLIV. NEWTON'S RINGS.

474. Every one has noticed the beautiful colours which may be seen on very thin films of transparent bodies, such as soap-bubbles; these phenomena were carefully studied by Newton, and from his mode of producing them have since been called *Newton's rings.*

475. Let a convex lens in which the surfaces have very large radii, be placed on another convex lens in



which the surfaces have very large radii; or instead of one of the lenses a plate of glass may be used. The two pieces of glass must be kept in close contact, which can be easily effected by means of three clamping screws placed round the circumference. Thus a thin stratum of air is comprised between the two contiguous glass surfaces, and the colours exhibited by this thin stratum can be examined. In one of Newton's experiments the lower glass was plane, and the upper glass was an equi-convex lens in which the radius of each surface was 92 inches.

476. Suppose first that a person looks *down* on the stratum of air, so that it is seen by reflected light. Then seven rings are visible, having a common centre at the point of contact of the two glasses; each ring exhibits various colours something like those of the solar spectrum. The colours in the first three rings are very distinct, but they become more faint and diluted in the other rings, and almost entirely disappear in the seventh ring. Next suppose that a person looks *through* the stratum, so that it is seen by transmitted light. Then five rings are visible, having a common centre at the point of contact of the glasses; but they are much fainter than those seen by reflection. By comparing the two sets of rings it is found that the colour seen by reflection at any point of the stratum is the *complementary* colour to that seen by transmission at the same point.

477. Newton determined carefully the thickness of the stratum of air at the point corresponding to the various coloured rings; to do this he measured the diameter of a ring, and calculated the corresponding thickness: this is an easy problem of geometry, since the radii of the surfaces in contact are known. Thus for a specimen of his results consider the *second* ring, as seen by reflection; the eye is supposed to be over the point of

contact of the glasses, so that the rays of light pass in a direction very nearly perpendicular to the stratum: then the colours succeed in the following order, violet, indigo, blue, green, yellow, orange, bright red, scarlet; and the corresponding thickness of the stratum of air varies from about eleven-millionths of an inch to nineteen-millionths.

478. Newton modified the experiment by putting water instead of air between the lenses; he found that the rings then became smaller and the colours fainter. On measuring the thickness of water at which any assigned ring was formed he found that it was about equal to the corresponding thickness of air *divided by the index of refraction for water*, that is, divided by $\frac{4}{3}$, that is, multiplied by $\frac{3}{4}$. Thus the thickness of water at which any ring is formed is $\frac{3}{4}$ of the thickness of air at which that ring is formed. In like manner, taking with Newton $\frac{31}{20}$ as the index of refraction for glass, the thickness of a stratum of glass at which any ring is formed is $\frac{20}{31}$ of the thickness of air at which that ring is formed.

479. When the light is reflected or transmitted obliquely instead of perpendicularly the rings increase in size, the same colour consequently now requiring a greater thickness of the air or other medium to produce it.

480. We have stated the general appearance of the rings as seen by common light; let us now suppose that they are seen by homogeneous light, for example, yellow light. In this case we have alternately dark rings and bright yellow rings; and many more rings are visible than when common light is used. Newton determined the thickness of the stratum of air at the successive brightest parts of the bright rings and darkest parts of the dark rings. Take for the present $\frac{1}{178000}$ of an inch as the unit

of length. Then the thickness of the air is 1 unit, 3 units, 5 units, 7 units, at the brightest parts of the first, second, third, fourth, bright rings; and so on. Also the thickness of the air is 2 units, 4 units, 6 units, 8 units, at the darkest parts of the first, second, third, fourth, dark rings; and so on. The central point is always dark. If instead of yellow light we take homogeneous light of any other colour the general appearances are the same, but the size of the rings is different. The rings are largest for red light, and diminish as the index of refraction increases; thus taking the diameter of any ring for the extreme red to be 1, the diameters of the corresponding ring for the other colours are as follows: orange ·924, yellow ·885, green ·825, blue ·763, indigo ·711, violet ·681, extreme ·630.

481. When the laws of the rings as seen in homogeneous light are mastered, we can easily understand the succession of colours obtained by viewing the rings in common light. As common light is composed of all the different coloured rays we have now the various systems of dark and bright coloured rings existing simultaneously. If the size of the rings were the *same* for all the colours the bright coloured rings would by their combination produce *white* light: but as the size is not the same the rings do not coincide and do not result in white light. Consider the first bright ring; this is smallest for the violet rays, and greatest for the red rays; hence when these rings exist together, the union of the two will produce a whitish ring tinged with violet round the inner boundary where the red is feeble, and with red round the outer boundary where the violet is absent. If we also take into account the rings of the other colours, we see that at intermediate positions various tints may appear, arising from the fact that one or other of the colours is there at its brightest stage, and so predominates over the rest. It is obvious too why fewer rings are seen when common light is used than when homogeneous light is used. For as soon as we arrive at a part which is at a sensible distance from the point of contact of the lenses, the thickness of the stratum of air changes rapidly as we proceed, and the rings of any assigned colour crowd

closely together; hence when rings of all colours are combined there is no appreciable breadth over which any single colour is decidedly predominant; thus by the mixture of many colours we obtain sensibly white light.

482. We must now turn to the bearing of the phenomena on the theory of light. Newton explained the existence of the rings by supposing that every particle of light as it moved was alternately in a state or *fit* for reflection and for transmission. Thus let us consider the second surface of a film of air; then in the middle point of a dark ring, as seen from above, the particles of light falling on the stratum of air are in the most decided stage of a fit of transmission, and so pass entirely through the stratum and are lost to the eye. At the middle point of a bright ring the particles of light are in the most decided stage of a fit of reflection and are entirely reflected. At intermediate points the state of the particles varies between these two extremes, some are transmitted and some are reflected. One mode of conceiving how these fits can exist is to suppose that a particle of light revolves round an axis at right angles to the direction of the ray, and that it is of the nature of a magnet, with an attractive and a repulsive pole. Then by virtue of the rotation these poles will be presented alternately to a surface on which the light falls, giving rise to refraction or reflection according as the attractive or repulsive pole is nearest to the surface at the time the particle reaches it. But it is not necessary to devote much space to a theory which is now completely disregarded; it fails to give any adequate account of the blackness of the dark rings, for it supposes some rays always to be reflected at the first surface of the film, so that there should always be some illumination even at the points where experiment shews that there is none. In fact there is nothing in this theory which can supply the place of the principle of *interference*, by virtue of which two lights may give rise to actual darkness. A double fit in Newton's theory corresponds to the length of a wave in the wave theory; for in the space of the double fit the particle returns again to its former state.

483. Let us now consider how the wave theory applies to the phenomena of Newton's rings. Here darkness is

supposed to be produced by the interference at the same point of two waves which differ by half a wave length, or by some odd multiple of half a wave length; just as silence arises in a similar state of things according to the theory of sound. Consider, for the case of yellow light, the darkest part of any dark ring. Light reflected from the upper surface of the stratum of air is combined with light which has passed through the stratum, been reflected at the lower surface, and has then come back to the eye; as the result is darkness it indicates a difference of path traversed by the two portions of light equivalent to some odd multiple of half a wave length. At the next dark ring we have a similar combination, and we may infer that the difference of path corresponds to the next odd multiple of half a wave length. Now the *difference* of the thickness of the stratum of air in the two cases is about $\frac{1}{89000}$ of an inch, and as the ray which goes through the stratum and returns, traverses this distance *twice*, the conclusion is that $\frac{2}{89000}$ of an inch is about the length of a wave for yellow light.

484. We can now determine how many vibrations a particle of ether makes in a second; the calculation is like that relating to sound given in Art. 58. In one vibration light just travels over a space equal to the length of a wave; and thus the number of vibrations in a second is equal to the velocity of light per second, divided by the length of a wave. Thus taking the velocity of light at 186000 miles per second, we should multiply this number by 5280, and the product by 12, to reduce it to inches, and then divide by $\frac{2}{89000}$. The following numbers are extracted from a table given by Sir J. Herschel in his treatise on *Light;* the table is calculated on the supposition that the velocity of light is 192000 miles per second, which was the value formerly adopted. The first column states the colour of the rays, the second column the length of a wave in air in parts of an inch, the third column the number of wave lengths in an inch, and the fourth column the number of vibrations per second expressed in *millions of millions.*

Red	·0000256	39180	477
Orange	·0000240	41610	506
Yellow	·0000227	44000	535
Green	·0000211	47460	577
Blue	·0000196	51110	622
Indigo	·0000185	54070	658
Violet	·0000174	57490	699

Sir J. Herschel remarks: "That man should be able to measure, with certainty, such minute portions of space and time, is not a little wonderful; for it may be observed, whatever theory of light we adopt, these periods and these spaces have a *real existence*, being, in fact, deduced by Newton from direct measurements, and involving nothing hypothetical but the names here given them."

485. It will be remembered that the table of the preceding Article applies to light in *air;* it is easy to see what changes must be made to accommodate the table for any other medium, as for example *water*. By Art. 478 all the numbers in the second column must be multiplied by $\frac{3}{4}$; and hence it will follow that all the numbers in the third column must be multiplied by $\frac{4}{3}$. The numbers in the fourth column will remain unaltered; they are obtained by dividing the velocity of light in the medium by the length of a wave in the medium; and for water both dividend and divisor become $\frac{3}{4}$ of what they were for air, and so the quotient is unaffected.

486. From the table it follows that the sensibility of the eye to colour has a much narrower range than that of the ear to pitch. For the number of vibrations for the violet ray is about $\frac{3}{2}$ of the number for the red ray; this number falls short of the number 2, which measures the interval of an octave in music, and corresponds to a fifth: see Art. 116. If however the solar spectrum be considered in its full extent, so as to include the ultra red

and the ultra violet portions, the whole range corresponds to about four octaves.

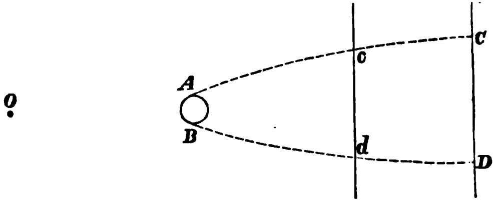
487. When Newton's rings are seen by reflected light the point of contact is *dark*, as we have stated in Art. 480: we must again advert to this because it is a difficult point in the theory. The fact is that where the stratum of air is practically non-existent, by reason of excessive thinness, there is darkness by the combination of rays; and this seems strange. It would be impossible to discuss the difficulty in an elementary work like the present, so we must be content with a brief notice. It appears that there is a difference in the nature of reflection, according as the medium containing the incident and reflected rays is *rarer* or *denser* than the medium on the other side of the reflecting surface. Thus when light is reflected at the *lower* surface of the stratum of air the act of reflection itself is accompanied by a change equivalent to a difference in path of half a wave length. At the middle point of the first bright ring the thickness of the air is half of $\frac{1}{89000}$ of an inch, so that the double thickness corresponds to $\frac{1}{89000}$ of an inch, that is to half a wave length; and the act of reflection at the lower surface of the air is equivalent to the difference of half a wave length; so that on the whole there is a difference of a wave length, and thus we obtain brightness. A very striking confirmation of the correctness of the theory respecting the darkness of the central spot in Newton's rings has been obtained. Suppose the two glasses employed to be of different refractive power, and the stratum between them to be of intermediate refractive power; then the reflection takes place under like circumstances at the upper and lower surfaces of the stratum: that is in *both* cases the incident and reflected rays are in the *rarer*, or in both cases in the *denser*, of the adjacent media. The central spot will then be *white* instead of black. This has been verified by taking two lenses, one of crown glass and one of flint glass, and placing between them a stratum of oil of sassafras.

XLV. INFLECTION OR DIFFRACTION OF LIGHT.

488. Let us suppose light to proceed from a point and to be partially intercepted by an obstacle with sharp edges; then a shadow of the obstacle will fall on a wall or screen suitably situated. The geometrical boundary of the shadow will be determined by supposing straight lines drawn from the origin of light to all parts of the edges of the obstacle and produced to the screen. Now it might have been supposed that this boundary would be clear and well-defined, having on one side uniform brightness and on the other uniform darkness; but this is found by careful observation not to be the case. Within the shadow the light fades away gradually instead of suddenly, and outside the shadow the illumination is not precisely the same as it would be if the obstacle were removed. In this experiment light instead of moving in exact straight lines seems *bent* out of its course by the edges of the obstacle; other phenomena of a similar kind have been observed, and all of them are grouped together under the title of *inflection or diffraction of light.* To observe these phenomena light proceeding from a *point* should be admitted into a dark room; the point for origin is conveniently obtained by the use of a convex lens, as in Art. 469. The phenomena of diffraction were first studied by Grimaldi, a Jesuit of Bologna, and soon afterwards by Newton. In modern times Young and Fresnel have paid great attention to them, and the wave theory has been successfully applied to explain the results of experiment.

489. In sun light the shadows of all bodies are found to be encircled by three parallel coloured fringes; the nearest to the shadow is the broadest and most luminous; the furthest from the shadow is the narrowest, and so faint as to be not easily visible. The colours occur in the following order from the shadow: first violet, indigo, pale blue, green, yellow, red; next blue, yellow, red; and lastly pale blue, pale yellow, and red. Suppose the origin of the light, and the body, to remain fixed, but the screen on which the shadow is received to be moved from or towards the body;

then the fringes become narrower as the screen approaches the body. Thus let *O* represent the origin of light, and



AB an object; then *C* and *D* may denote points on one of the fringes outside the shadow. Let the screen be moved nearer to *AB*; then *c* and *d* may represent the corresponding places of the points on the fringe. It is found that *A*, *c*, and *C* do not lie on a straight line, but on a curve of the kind which mathematicians call an *hyperbola.* Similarly *B*, *d*, and *D* also lie on an hyperbola. The dotted lines are intended to give a notion of these curves. The fringes increase in size when the body is brought nearer to the origin of light, other circumstances remaining the same.

490. If homogeneous light be used the fringes will be of the same colour as the light, and will be broadest for red light and narrowest for violet light. Hence, as in Art. 481, we see how, by the simultaneous existence of these, the coloured fringes visible by ordinary light are produced.

491. In the shadows of small bodies, or of long and narrow bodies, there are also fringes alternately bright and dark *within* the boundary of the geometrical shadow. They are produced by the interference of the rays of light which pass on both sides of the body close to it, and they disappear when the rays which pass on one side are intercepted by a screen.

492. A striking case is that of the shadow formed by a very small circular disc. Rays passing round the edge of the circle meet at the centre of the shadow where they combine to produce an illumination. Poisson cal-

culated that the brightness at this point would be the same as if the disc were removed, and the result has been confirmed by experiment; it is stated that Arago exhibited this to Napoleon, who could scarcely believe that the disc had not been perforated.

493. Interesting phenomena are also seen when, instead of obstacles of various forms, light is allowed to pass through apertures of various forms. One case, studied by Newton, is that where the aperture is the interval between two knife edges set at a very acute angle; fringes are observed to border the shadows, and a dark line occurs in the middle of the bright space. Another case is that of a circular aperture, such as may be formed in a piece of lead or tinfoil by a common pin. Instead of receiving the light on a screen for the purpose of examining it in these diffraction experiments, we may receive it on an eye lens, as we should if it came from an object which we wished to magnify. In the case last mentioned, that of light passing through a small circular aperture, the luminous point is seen surrounded by rings; and the colours vary with the distance of the lens from the aperture. When two apertures are used instead of one, two systems of rings will be seen, one surrounding each centre, and besides these other sets of fringes are visible, which are rectilinear when the apertures are equal.

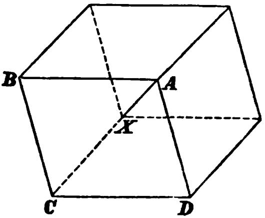
494. The phenomena of diffraction have been explained according to the wave theory of light, and a very close agreement is obtained between the results of experiment and those of calculation; thus furnishing strong evidence of the truth of the theory. In this and the preceding Chapter we have noticed various curious facts: namely those of interference, of the colours of thin plates, and of diffraction. Now consider for a moment only homogeneous light, say yellow light; then the theoretical explanation of all these facts depends mainly on *one numerical quantity*, namely the length of a wave of yellow light: and it is a severe test of the truth of a theory that it can account accurately for many and various phenomena by the use of a single fundamental number.

XLVI. DOUBLE REFRACTION. UNIAXAL CRYSTALS.

495. We have hitherto supposed that a single ray of light, at least of a definite colour, when falling on a transparent body is refracted according to the *law of sines.* This is found to be the case when the transparent body is of precisely the same nature throughout, so that it has the same composition, the same density, and the same temperature, in every direction in which a ray can pass. There are numerous transparent bodies which fulfil these conditions, such as gases, fluids, various kinds of crystals, and artificial substances, such as glass, which are produced by fusion followed by slow and gradual cooling. On the other hand there are numerous transparent bodies for which the simple law of sines does not hold; when a ray of light enters such a body it is split into two rays, and the law of sines does not hold for both these rays; sometimes it does not hold for either ray. The phenomena are grouped together under the title of *double refraction,* and the bodies which affect light in this manner are called *doubly refracting substances.* These bodies include various crystals; animal structures such as horn, shells, and the crystalline lenses of eyes; and artificial substances such as resins, gums, jellies, and glass quickly and unequally cooled.

496. In treating of double refraction in the present Chapter we will consider especially one crystal in which the property was first discovered, and which exhibits it very clearly. This crystal is called by various names as *Iceland spar, calcareous spar, calc spar, calcite,* and *carbonate of lime:* it is found in many countries, but most plentifully in Iceland. It occurs in the form of crystals of various shapes, and often in large masses; but by cleaving it can always be easily brought into the form represented by the diagram which is called a *rhomb of Iceland spar.* This is bounded by six equal four-sided figures, of the kind which in geometry is called a *rhombus:* al

the sides of a rhombus are equal, and the opposite sides

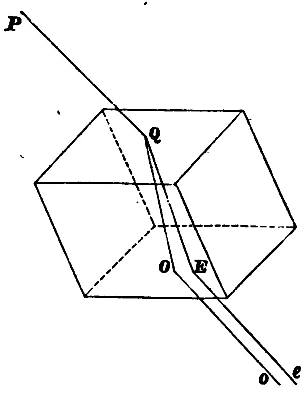


are parallel. In the present case let *ABCD* represent one face of the rhomb; then the angle *BAD* is 101° 55′, and the angle *ADC* is 78° 5′. The acute angle between a face and any of the other faces which it meets is 74° 55′. The straight line *AX*, which joins the opposite *obtuse* angles of the solid, is called the *axis* of the crystal: it is equally inclined to all the six faces, namely at an angle of 45° 23′. Iceland spar is very transparent, and in general colourless. The faces represented in the diagram may be called the *natural* faces of the crystal; but with care and patience a crystal may be cut by a plane in any direction, and polished.

497. It is important to understand the sense in which the word *axis* will be used in this Chapter. Geometrically considered the axis of the crystal is the fixed straight line *AX*, but optically considered *any* straight line parallel to *AX* may be called an axis: thus optically considered the axis is a fixed *direction*, not a fixed *straight line*. If we suppose the crystal represented by the diagram to be divided by planes parallel to its faces into numerous smaller crystals, all of exactly the same shape as the original, then each crystal will have its own geometrical axis; and when the small crystals are fitted together in the places they first occupied, any two axes are either parallel or in the same straight line. In the present Chapter we consider only crystals, which, like Iceland spar, have a *single* direction for the axis, and which are therefore called *uniaxal* crystals.

498. We now proceed to the phenomena of double refraction. Take a rhomb of Iceland spar, like that denoted by the diagram, with smooth and polished faces, and so large that one of its edges is at least an inch long. Make a distinct black dot on a sheet of paper, place the rhomb on the paper over the dot, and look through the rhomb; then *two* dots will be seen: thus rays have proceeded through the rhomb in two ways, and have formed two images. If we keep the eye and the paper fixed, and move the rhomb round, always in contact with the paper, it will be seen that one image retains its place while the other appears to revolve round it.

499. Let PQ represent a ray incident at Q on one face of the rhomb; this ray is separated by the rhomb into two. It is found that one of the two rays obeys throughout its course the ordinary laws of refraction; that is, the incident ray and the refracted ray are always



in the same plane with the normal to the surface at the point of incidence, and the sine of the angle of incidence bears a fixed proportion to the sine of the angle of refraction: this fixed proportion may be taken roughly as equal to $\frac{5}{3}$, more accurately as 1·658. In the diagram QO denotes the course of this ray in the crystal, and it emerges along the line Oo which is parallel to PQ: this is called the *ordinary* ray. The other ray does not obey the ordinary laws of refraction; it does not in general when refracted remain in the *plane of incidence*, that is in the plane which contains the incident ray and the normal to the surface at the point of incidence; all that is immediately obvious is that it proceeds in some straight line, as QE, through the crystal, and emerges in a direction Ee parallel to PQ: this is called the *extraordinary* ray. Huygens discovered the law which determines the course of the extraordinary ray, and published it in the book mentioned in Art. 459.

500. The law thus propounded by Huygens can be readily understood by a mathematician, but it is difficult to explain in a manner suitable for an elementary work like the present; we shall however endeavour to give some notion of it. Geometry brings to our notice a certain solid figure called an *oblate ellipsoid of revolution;* this name may be conveniently abbreviated to *oblatum.* The earth is known to have such a figure; it is popularly described as a sphere somewhat flattened at the poles, and an orange is taken as an illustration of a body possessing such a figure. A more exact notion may be obtained without passing out of the science of Optics. Let a sphere be placed on a table, and illuminated by a bright point placed in any position, not exactly over the sphere; the shadow of the sphere on the table will be bounded by a curve which is called in Geometry an *ellipse.* Let such a curve be traced on card board and cut out. The longest straight line that can be drawn within the curve is called the *major axis;* a straight line drawn in the curve at right angles to the major axis and passing through the middle point of it is called the *minor axis.* Let the curve be set in rapid rotation round a fixed straight line coincident with the minor axis; then by reason of the persistence of impressions on the retina the mind obtains a notion of a solid figure: this is accurately an *oblatum.* It will be found that by varying the position of the bright point, which

forms the shadow of the globe, we may obtain ellipses which are more or less drawn out; and consequently one oblatum may be much more flattened out than another. The axis of the oblatum is the same as the minor axis of the ellipse in our mode of producing the figure of the solid.

501. When a disturbance is excited in the ether at any point, whether of a vacuum or of a singly refracting substance, the disturbance takes the form of a spherical undulation, proceeding from this point as a centre, which spreads out with the same velocity in all directions. When a disturbance is excited in the ether at any point of Iceland spar it gives rise to two undulations proceeding from this point as centre; one of these undulations takes the form of a *sphere*, and spreads out with the same velocity in all directions; the other takes the form of an *oblatum*, so that the velocity is not the same in all directions, being least along the axis, and greatest in directions at right angles to this. The sphere and the oblatum which exist at any instant, touch each other at the ends of the axis of the oblatum, all the rest of the surface of the oblatum being outside that of the sphere; and this axis coincides in position with the axis of the crystal at the point where the disturbance was excited. The undulation in the form of a sphere gives rise to the ordinary refracted ray, and the undulation in the form of an oblatum gives rise to the extraordinary refracted ray.

502. Suppose that we wish to determine the refracted rays which correspond to a ray incident on a face of the crystal. For the ordinary ray we proceed as in Art. 464; the radius of the sphere described from A will then be three-fifths of the excess of ND over KA. But for the extraordinary ray we must substitute an *oblatum* in place of the *sphere;* the oblatum must have its axis coincident with the axis of the crystal, which will not necessarily be either in the plane of the paper or perpendicular to it: and it is found as a consequence of this that the straight line Aa is not necessarily in the plane of the paper. The dimensions of the oblatum are determined by a calculation on which we do not enter here; the minor axis of the ellipse, from which we suppose it produced, is to be equal

to the diameter of the sphere used for the ordinary ray; the major axis of the ellipse is to be in the same proportion to the minor axis as 67 is to 60, or roughly as 9 is to 8.

503. It will require some familiarity with mathematics to trace accurately all the consequences which follow from the theory of Huygens; we must content ourselves with an attempt to state various special facts with precision. The reader may however if he pleases omit the next six Articles.

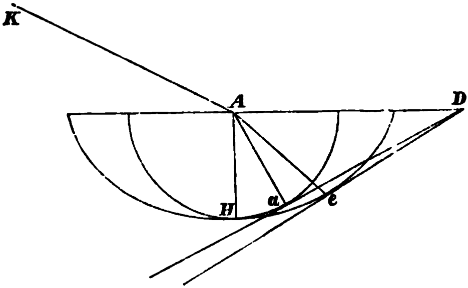
504. The plane which contains the ray incident at any point, and the normal to the surface at that point, may be called the *plane of incidence.* The extraordinary ray in general deviates from the plane of incidence; but there are two cases in which it remains in this plane, namely when the axis of the crystal is in this plane, and when the axis of the crystal is perpendicular to this plane: in these two cases all is symmetrical on the two sides of the plane of incidence, and there is no reason for deviation towards one side of it rather than towards the other.

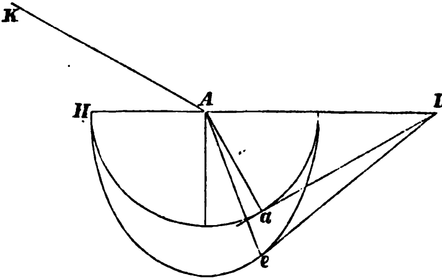
505. There is not *double* refraction along the axis of the crystal; that is, whatever may be the situation of the face of the crystal through which the ray enters, if the ray refracted according to the ordinary law goes along the axis of the crystal so also does the extraordinary ray, and there is no separation. The face of the crystal through which the ray enters may be one of the *natural* faces, or one obtained by cutting the crystal in any manner.

506. Let us suppose that the axis of the crystal is at right angles to the plane of incidence, then it may also be supposed to be in the face of the crystal through which the ray enters: in this case the extraordinary ray, like the ordinary, obeys the usual laws of refraction, but with a different index of refraction. For, as stated in Art. 504, the refracted ray now remains in the plane of incidence; and we have now the process of Art. 464, taking an oblatum instead of a sphere: since the axis of the oblatum is at right angles to the plane of the paper, the section of the oblatum by the plane of the paper is a circle as in the diagram of Art. 464. The radius of the circle is larger than the radius for the case of the ordinary ray,

in the proportion of 9 to 8; and the index of refraction is found to be about $\frac{3}{2}$ instead of the $\frac{5}{3}$ which belongs to the ordinary ray. Thus the extraordinary refracted ray makes a larger angle with the normal to the surface at the point of incidence than the ordinary refracted ray. If the axis of the crystal is in the face of the crystal, and the incident ray is perpendicular to the face, there is no separation of the ray into two parts.

507. The two diagrams which are now given furnish constructions like that of Art. 464 for determining the refracted rays which correspond to an incident ray KA. For

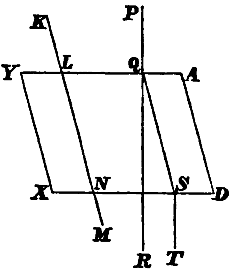




the sake of simplicity we draw the circle and the ellipse, corresponding to one point only, namely the point A; the reader can imagine curves of the proper size drawn corresponding to the other points in AD. The axis of the crystal is supposed to lie in the plane of the paper, which is the plane of incidence in both cases: in the upper diagram the direction of the axis is AH, which is at right angles to the plane refracting surface; in the lower diagram the direction of the axis is AH, which is in the plane refracting surface. The circle denotes the intersection of the undulation which is in the form of a sphere by the plane of incidence; the ellipse outside the circle denotes the intersection of the undulation which is in the form of an oblatum by the plane of incidence. Da touches the circle at a, and thus determines Aa the direction of the ordinary refracted ray; De touches the ellipse at e, and thus determines Ae the direction of the extraordinary refracted ray. It will be seen that in the lower diagram Aa is further from the normal to the surface at A than Ae is, whereas in the upper diagram Ae is further from the normal to the surface at A than Aa is. Imagine that KA and AD remain fixed, and that the circle and the ellipse of the upper diagram turn together in the plane of the paper, round A as a centre, in the manner of the hands of a watch; then in some position intermediate between that from which we start and that corresponding to the lower diagram, the two rays denoted by Aa and Ae will come together; *so that there will be no separation of the rays although they do not pass down the axis.* The matter is not of importance, but it is frequently stated or implied that the two refracted rays never coincide except in the cases already mentioned in Arts. 505 and 506. In the case here noticed the two rays though going in the same direction have different velocities.

508. Consider the ray denoted by QO in Art. 499. When it reaches the surface at O, although most of it may emerge, there will in general be a part reflected; this reflected ray will be *double*, because two undulations start from O, namely one in the form of a sphere, and the other in the form of an oblatum. In like manner *two* rays will be reflected from E, corresponding to the incident ray QE.

509. Let $ADXY$ represent a section of the crystal drawn in Art. 496, through AD and AX; and suppose a ray incident on AY in the plane of the section. Huygens examined this case with special attention: the angle ADX is about 71°. Suppose for instance that a ray PQ is incident at right angles; then the ordinary ray goes on along QR, while the extraordinary ray remains in the plane of incidence but makes with QR the angle SQR of about 6° 40′, and emerges along ST which is parallel to PQ. Again, let a ray KL, incident at L, make the angle KLY about 73° 20′; then the corresponding extraordinary ray LNM is not bent at L or at M, but makes one straight line with KL. This fact that a ray might fall *obliquely* on a plate, and pass through it without any deviation, seems to have struck Huygens as very remarkable. The student will more readily appreciate the fact that, on the principles of Huygens, such a case must exist, by drawing various diagrams, first taking the angle KLY very small, and then gradually increasing this angle until it is nearly two right angles: it will soon appear obvious that there must be some intermediate angle for which there is no change of direction at L for the extraordinary ray.



510. We have described the phenomena of double refraction as they are exhibited by Iceland spar, and it was formerly supposed that all crystals with a single axis resembled this; but in 1814 Biot discovered another class. In this class the second undulation starting from any point of the crystal, instead of taking the form of an *oblatum*, takes the form of an *oblong ellipsoid of revolution;* this name may be conveniently abbreviated to *oblongum*. A lemon may be taken as a rough illustration of the form; but we may obtain an exact notion of it thus: let the ellipse of Art. 500 be put in rapid rotation round a fixed straight line coincident with its *major* axis, and the mind will thus obtain the notion of the solid. The axis of the

oblongum is the same as the major axis of the ellipse in our mode of producing the solid. The sphere and the oblongum, which exist at any instant touch each other at the ends of the axis of the oblongum, all the rest of the surface of the oblongum being inside that of the sphere; and this axis coincides in position with the axis of the crystal at the point where the disturbance was excited.

511. Crystals of the class to which Iceland spar belongs have been called *negative* crystals, and those of the other class *positive* crystals; the majority of crystals are negative. The following list supplies examples of each class:

Negative	*Positive*
Iceland spar	Zircon
Sapphire	Quartz
Emerald	Ice
Ruby.	Tortoise shell.

512. The terms *positive* and *negative* are not very well chosen, and errors are sometimes made in attempting to distinguish in popular language between the two classes of crystals. The essential distinction is that with respect to the nature of the extraordinary undulation which arises when a disturbance is excited at any point of a crystal; for negative crystals this is an oblatum which includes the sphere corresponding to the ordinary undulation; and for positive crystals this is an oblongum which is included by the sphere corresponding to the ordinary undulation. Thus the extraordinary undulation in the former case is outside, and in the latter case is inside, the ordinary undulation; so that in the former case the extraordinary undulation travels with *greater* velocity than the ordinary, and in the latter case with *less;* but in the direction of the axis of the crystal the velocities are equal. Again, in the diagrams of Art. 507 it is obvious that the perpendicular from A on De is greater than that from A on Da; this we may put into words by saying that in negative crystals the extraordinary refracted plane *wave* travels faster than the ordinary one; the contrary holds for positive crystals. Also in the diagrams of Art. 507 we see that Ae is always greater than Aa; this we may put into words by saying that in negative crystals the extraordinary refracted plane *ray* travels faster than the ordinary one; the contrary holds for positive crystals. For both kinds of crystals the velo-

cities of the two *waves* are equal in one particular direction, and so are the velocities of the *rays;* namely along the axis of the crystal.

513. The difference between positive and negative crystals is very clear in the particular case in which the axis of the crystal is at right angles to the plane of incidence. Then the ordinary ray and the extraordinary ray both obey the usual laws of refraction: in *negative* crystals the index of refraction is *greater* for the ordinary than for the extraordinary ray; in positive crystals the contrary holds.

514. The reader will have to distinguish carefully between a wave and a ray, and between the velocity of a wave and the velocity of a ray. In both the diagrams of Art. 507 the plane front of the extraordinary refracted *wave* is less bent from the plane front of the incident wave than the plane front of the ordinary refracted wave is; and this is always true for negative crystals. But we must not infer that the extraordinary *ray* is always less bent from the course of the incident ray than the ordinary ray is; in the lower diagram the extraordinary ray is more bent, but in the upper diagram it is less bent.

XLVII. DOUBLE REFRACTION. BIAXAL CRYSTALS.

515. The matters considered in the three preceding Chapters, speaking generally, were known as facts by Newton and his contemporaries; in the remaining part of the present work which is devoted to Optics we shall be occupied with discoveries almost all of which have been made during the present century.

516. Some crystals are found which exhibit the phenomena of double refraction under more complicated laws than those which have hitherto been described. Instead of *one* optic axis, on which the phenomena depend, there are now *two*; and crystals which thus exhibit them are called *biaxal* crystals. According to Brewster all those crystals which in their simplest form have one eminent line round which the figure is symmetrical, have *one* optic axis coincident with that line; but the greater number of crystals have *two* optic axes.

517. In biaxal crystals neither of the two rays into which an incident ray is separated follows in general the usual laws of refraction. In the case of a uniaxal crystal we stated that an undulation excited in the crystal is propagated in a double form, namely a sphere, and an oblatum or an oblongum which touches the sphere at the ends of its axes. Now theory points out that for biaxal crystals instead of this form of undulation we have another of a very singular character, consisting of two intersecting surfaces, so that one is partly within and partly without the other: the combination of these two surfaces is called *Fresnel's wave surface*, which we shall abbreviate into *Fresnel's surface*. The directions of the refracted rays which correspond to any incident ray must be determined by a construction like that of Art. 464, substituting Fresnel's surface instead of a sphere. If light passes through a plate of the crystal the emergent rays will both be parallel to the incident ray as in Art. 499. We proceed to state some results obtained by theory which will be useful to the student for reference, though he may not fully comprehend them at first.

518. Theory points out the existence of remarkable straight lines connected with Fresnel's surface. There are two diameters such that at either end of them a plane can be found which is at right angles to the diameter and touches the surface all round the circumference of a certain circle; these two straight lines coincide in direction with the *optic axes* of the crystal. There are two other diameters such that each passes through two points at which the two portions of Fresnel's surface intersect; these are called *ray axes*. The two optic axes and the two ray axes lie all in the same plane. The optic axes and the ray axes were sometimes confounded in the early history of the subject; the former presented themselves most obviously to experimentalists, while the latter seem to have been first distinctly revealed by theory.

519. When we speak of a *plane wave* moving in a certain direction we mean that the front of the wave is always at right angles to that direction. Now in a uniaxal crystal the velocity of a plane *wave* which passes in the direction of the axis is the same whether it be the ordinary wave or the extraordinary wave; or in other words, we

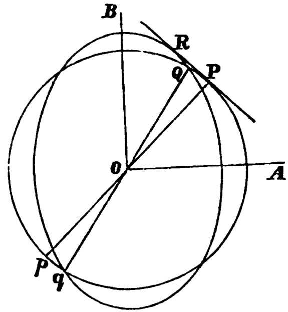
may say that there is no separation of the plane wave into two. In like manner in a biaxal crystal there is no separation of a plane wave into two along the direction of an *optic axis*. Again, in a uniaxal crystal the velocity of a *ray* which passes in the direction of the axis is the same whether it be regarded as an ordinary ray or an extraordinary ray. In like manner in a biaxal crystal the velocity of a ray which passes along a *ray axis* is the same in whatever way the ray may have entered the crystal so as to pass in this direction. We cannot however say that if a ray is so incident as to give rise to a ray refracted along the ray axis then the incident ray will not separate into two: in this respect the biaxal crystal differs from the uniaxal crystal.

520. Theory has obtained the following results. The difference of the reciprocals of the squares of the velocities of the two *rays* which have the same direction varies as the product of the sines of the angles between the direction and the two *optic* axes. The difference of the squares of the velocities of two plane *waves* which move in the same direction varies as the product of the sines of the angles between the normal to the front of the wave and the two *ray axes*. These two statements are quite distinct though they have been sometimes confounded.

521. The uniaxal crystal may to some extent be regarded as a particular case of the biaxal crystal; in this case the two optic axes and the two ray axes *all coincide* with the single axis of the uniaxal crystal. The two statements of the preceding Article then take this form: in a uniaxal crystal the difference of the reciprocals of the squares of the velocities of the two rays which have the same direction varies as the square of the sine of the angle between that direction and the optic axis; also the difference of the squares of the velocities of two plane waves which move in the same direction varies as the square of the sine of the angle between that direction and the optic axis.

522. Suppose that at any point of a biaxal crystal the direction of the optic axes is determined; then there are three other straight lines connected with these to which we will now advert; we will call them *principal* lines.

One of the principal lines bisects the angle between the optic axes; another is at right angles to the first; both of these are in the plane of the optic axes: the third principal line is at right angles to the plane of the optic axes. A section of Fresnel's surface by a plane through the first two principal lines cuts the surface in two curves, one a circle and the other an ellipse; these intersect at four points of which Q denotes one; QOq is a ray axis. PR is a straight line which touches the circle at P and the ellipse at R; and POp is an optic axis. The other ray axis and the other optic axis can obviously be drawn. OA and OB are two of the three principal lines at O; the former coincides in direction with the minor axis of the ellipse, and the latter with the major axis. The third principal line will be at right angles to the plane of the paper; for convenience we will call it OC. Then a section of Fresnel's surface through OA and OC will give an ellipse and a circle, the latter falling within the former; and a section through OB and OC will give an ellipse and a circle, the latter falling without the former. In all the three sections the ellipse and the circle have the same centre, namely O; and the axes of the ellipse fall on the two principal lines which are in the plane of the section.



523. Let the plane of incidence of a ray coincide with the plane containing two of the three *principal lines*, so that the third principal line will be in the refracting surface; then the refracted rays will be both in the plane of incidence. One of them will follow the ordinary *law of sines;* the other will proceed in a manner which we have already to some extent illustrated in Art. 507; the ellipse which corresponds to the extraordinary ray will not however *touch* the circle which corresponds to the ordinary ray, but fall within it, or fall without it, or cut it, according as OA, OB, or OC is the principal line which is in the reflecting surface.

524. It is possible for a ray to pass into a biaxal crystal without separation into two rays. Suppose for example that a principal line is at right angles to the plane refracting surface, and that a ray is incident at right angles to the surface; then there is no separation into two rays so far as direction is concerned; the two rays continue in the same straight line at right angles to the surface though they move with different velocities. Other cases of the coincidence in direction of the two refracted rays may be found also in the manner of Art. 507.

525. Sir W. R. Hamilton by studying Fresnel's surface was led to the conclusion that if a ray of light passed along a *ray axis* of a biaxal crystal it would, on emergence, not proceed in the form of a single ray, but spread out in the form of a *hollow cone* of rays. The ground of the conclusion was the geometrical fact that at the point of the surface denoted by the letter Q in Art. 522 an assemblage of straight lines might be drawn all touching the surface and forming not a plane as is usually the case, but a cone. For instance, in the plane of the paper in Art. 522 two such straight lines might be drawn, one to touch the ellipse at Q and one to touch the circle there; two straight lines drawn from Q at right angles to these would be two of the rays of the hollow cone of rays. Again, Sir W. R. Hamilton was also led to the conclusion that if a ray were incident on a crystal in a certain direction it would on entering the crystal spread out into a hollow cone of rays, and on emergence from the crystal into a hollow cylinder of rays. The ground of this conclusion was the geometrical fact that a plane touching Fresnel's surface at the end of an optic axis will touch it along the circumference of a certain circle. In the diagram of Art. 522 we have a straight line touching the surface at P and at R; thus P and R are points on the circumference of the circle just mentioned, and PR is a diameter of the circle. The ray must be so incident on the crystal that OP shall be normal to the corresponding *wave* within the crystal; OP is one of the rays which form the hollow cone, and the straight line joining OR would be another. The phenomena are now called briefly *conical refraction;* the former is called *external conical refraction,* and the latter

internal conical refraction. The prediction that such results would be obtained is to be ranked among the boldest ever uttered by mathematical genius; it has been confirmed by experiment, especially in the case of the crystal called *arragonite.* When a crystal is *uniaxal* the phenomena of conical refraction disappear; they are replaced by the single fact that there is never any separation of a ray which passes along the optic axis: see Art. 505. Such facts as those of conical refraction, and that stated in Art. 492, deduced from the wave theory and verified by trial, are justly considered to furnish strong evidence of the truth of that theory.

526. The following list gives the names of some biaxal crystals, with the angle in each case between the optic axes, according to Brewster:

Talc	$7^0\ 24'$	Brazilian Topaz	$49^0\ 51'$
Mother of Pearl	$11^0\ 28'$	Sugar	50^0
Arragonite	$18^0\ 18'$	Carbonate of Soda	$70^0\ \ 1'$
Spermaceti	$37^0\ 40'$	Tartaric Acid	79^0

527. In uniaxal crystals the position of the optic axis is the same for light of all colours; but in biaxal crystals this is not always the case. Sir J. Herschel found that in *Rochelle salt* the inclination of the optic axes is greater for violet light than for red light. Brewster found that *Glauberite* had two optic axes, inclined at an angle of 5^0 for red light, and one optic axis for violet light. In all cases the optic axes for light of all colours lie in the same plane.

528. The doubly refracting power may be imparted permanently or transiently to some substances which do not naturally possess it; Brewster gives the following example. Let a circular cylinder of glass be brought to a red heat, and then rolled along a plate of metal until it is cool: it will become a member of the positive uniaxal kind of doubly refracting bodies, the axis of the cylinder being the optic axis. But this axis differs from that of a positive crystal, for it is in a fixed *position* and not merely in a fixed *direction:* the optic axis of the glass is the axis of the cylinder, and not any straight line parallel to this. If instead of making the cylinder of glass red hot it had

been placed in a vessel and surrounded with boiling oil or boiling water, it would have acquired the same doubly refracting power when the heat had reached the axis, but only transiently: the power disappears when the cylinder is heated uniformly. If the cylinder were heated uniformly in boiling oil, or at a fire, so as not to soften the glass, and were then placed in a cold fluid, it would acquire the doubly refractive power of a *negative* crystal, when the cooling reached the axis, but only transiently: the power disappears when the cylinder is cooled throughout. If in these cases the cylinder is not circular but elliptical, it will have *two* axes of double refraction. Similar results may be obtained by compressing and by bending soft solids, such as animal jellies and isinglass.

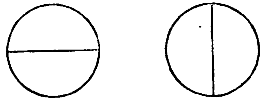
XLVIII. POLARISATION BY DOUBLE REFRACTION.

529. We have hitherto had no occasion to consider any thing with respect to a ray of light except the course which it takes from one point of space to another. If two parallel rays of the same colour were moving in the same medium we should have had no reason to suspect any difference between them; but we are now about to discuss a class of facts from which it follows that there might be great difference between two such rays. To use a rude illustration we may suppose that we have two rods of wood of the same size and weight, and painted of the same colour; then there will be no obvious distinction between the rods. But suppose we cut through them both, and thus expose to view a section of them; we know that the appearances may be very different, arising from the different way in which the *grain* of the wood runs: this may be disposed in curve lines, or in parallel straight lines, or in intersecting straight lines.

530. Let a ray of sun light be admitted through a small aperture into a dark room; then let it be reflected from any smooth body or transmitted through any transparent body. If we keep to the same *body*, and let the ray fall at the *same* angle the result is absolutely the same whether the surface of the reflecting or refracting body be

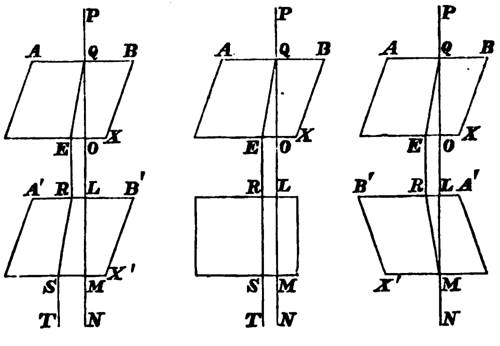
placed above or below the beam, to the right or to the left of it. The same property belongs to the light of a candle or of any self-luminous body. Such light is called *common light.* But if we examine the two rays of light which arise from a single ray by the action of a doubly refracting crystal, we shall find that this is not the case; each ray has properties with respect to a fixed direction, and these fixed directions for the two rays are nearly or exactly at right angles. This property of light which had passed through a doubly refracting crystal, was known to Huygens and Newton, and led Newton to propose the question: "Have not the Rays of Light several sides, endued with several original Properties?" In modern times it has been found that the property can be given to light in various ways; and the whole body of facts and laws related to the subject is entitled the *Polarisation of Light.* In the present Chapter we treat of polarisation as produced by double refraction.

531. Suppose a ray of light has been separated into two by a doubly refracting crystal. We may represent a section of one refracted ray by a circle with a diameter drawn in it; and then the section of the other refracted ray may be represented by a



circle with a diameter drawn at right angles to that of the former circle. Each ray is said to be *polarised.* The *plane of polarisation* of a polarised ray is a plane containing that ray, and having a fixed position, with respect to which the ray has various definite properties: these properties we shall explain as we proceed. Suppose a ray of light is incident on a uniaxal crystal; we know that by refraction two rays are obtained, one called the *ordinary* ray, and the other the *extraordinary* ray. It is found by experiment that the plane of polarisation of the ordinary ray is a plane which contains that ray and the direction of the optic axis of the crystal; and that the plane of polarisation of the extraordinary ray is, nearly or exactly, at right angles to that of the ordinary ray. It will be sufficient for our purpose to say that the planes are at right angles, without in future adding *nearly or exactly.*

532. Let ABX be a section of a rhomb of Iceland spar made by a plane which contains the optic axis AX, and is perpendicular to the natural face AB of the crystal, and therefore also to the face which is parallel to this. Let any ray PQ be incident perpendicular to AB; then this ray will be separated into two by the action of the crystal, namely the ordinary ray QO which undergoes no deviation, and the extraordinary ray QE, which remains in the plane of incidence but is refracted according to the theory of Huygens. Both rays on reaching the second surface of the crystal emerge parallel to the original direction PQ, as indicated by the straight lines OL and ER of the diagram. Take a second crystal exactly like the former and place it so as to receive the rays coming from the first, and then the appearances will be different ac-



cording to the position of the second crystal with respect to the first. Three cases are represented in the diagrams; in all cases the faces of the two crystals *through which the light passes* are all parallel.

I. The left-hand diagram. The second crystal is supposed to be placed in a situation precisely like that of the first. The optic axis $A'X'$ is parallel to the optic axis AX, and any straight line is parallel to the corresponding straight line, as for instance $A'B'$ to AB. In this case the ordinary ray *QO* pursues its course *LMN* through the second crystal without deviation. The extraordinary ray is refracted at *R* along *RS* which is parallel to *QE*, and emerges along *ST* which is parallel to *PQ*. The courses of the rays in the second crystal are the same as they would be if the second crystal were moved up to the first so as to form one piece in its natural state of twice the thickness of the first piece. The distance between *MN* and *ST* is twice the distance between *OL* and *ER*, as would naturally be the case from passing through a double thickness of crystal.

II. The middle diagram. Let the second crystal be now turned through a right angle, keeping the face upon which the rays fall always parallel to itself. Then the section *A'B'X'* which in the left-hand diagram was in the plane of the paper now comes to be at right angles to that plane. The ray *QE*, which was the extraordinary ray in passing through the first crystal, now undergoes no deviation, but passes through the second crystal precisely like an ordinary ray. The ray *QO*, which was the ordinary ray in passing through the first crystal, now undergoes extraordinary refraction by the second crystal; this takes place in a plane at right angles to the plane of the paper, and so it cannot be represented in the plane of the paper: thus the diagram gives the appearance of rays which are parallel in their course through the second crystal, whereas after entering that crystal they are not parallel until they emerge from it.

III. The right-hand diagram. Let the second crystal be turned onwards, through another right angle. The plane *A'B'X'*, which was originally in the plane of the paper, comes again into that plane or parallel to it; but it is turned in the opposite way, as indicated in the diagram. In this case the ordinary ray *QO* pursues its course *LMN* through the second crystal without deviation. The extraordinary ray *QE* undergoes extraordinary refraction again: but, owing to the difference in the position of the optic axis of the crystal, the deviation is now in the contrary direction to that in the first crystal, and so the ray is brought to the *same* point *M* as the former ray, and coincides with that ray along *MN* at emergence.

The reader must notice the important fact which is incidentally brought out: the ordinary ray obtained from the first crystal *cannot be separated* into two rays by the action of the second crystal which is placed as in any of these three Cases, that is exactly in the same position as the first crystal, or turned through a right angle, or turned through two right angles. The same thing is true of the extraordinary ray obtained from the first crystal.

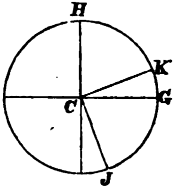
533. Let us now advert to positions of the second crystal *intermediate* between those considered in the preceding Article, and in this Article we will take positions intermediate between those of Case I. and Case II. The first crystal remains fixed and the second crystal is turned gradually round through a right angle. The second crystal will in general separate each ray coming from the first crystal into two; so that from the original single ray we obtain *four* rays; these vary in *brightness* as the second crystal is turned round, and as we have seen, they reduce to *two* rays at the beginning which is represented in Case I., and at the end which is represented in Case II. In order to trace the changes in the emergent light let us suppose that the incident light consists of a beam of parallel rays coming through a small round hole near the surface AB, and that the eye is placed beyond the crystals so as to see an image of this hole. We will denote the four images in the following way: Y is that formed by light which undergoes ordinary refraction by both crystals, Z is that formed by light which undergoes extraordinary refraction by the first crystal and ordinary refraction by the second, y is that formed by light which undergoes ordinary refraction by the first crystal and extraordinary refraction by the second crystal, and z is that formed by light which undergoes extraordinary refraction by both crystals. The place of Y is fixed throughout, and the place of Z is fixed throughout; y appears to describe a quadrant of a circle round Y as a centre; and z appears to describe a quadrant of a circle round Z as a centre. At the beginning of the experiment we are considering Y and z alone are visible, about equally bright; the point midway between them is the situation which Z occupies when it is visible. As the second crystal is

turned round Y and z become gradually fainter, and at last disappear; while on the other hand y and Z appear, at first being very faint, and gradually increase in brightness, until at last they are as bright as Y and z were at first. The images Y and z are always equally bright; and the images y and Z are always equally bright. When the second crystal has been turned through half a right angle all four images are equally bright, and occupy the four corners of a rhombus which has two opposite angles each equal to half a right angle, and two other angles each equal to a right angle and a half. The line which joins Y and y, and also the line which joins Z and z, is parallel to the plane containing $A'B'$ and $A'X'$.

534. Let us take in the next place positions intermediate between those of Case II. and Case III. of Art. 532. The first crystal remains fixed, as before, and the second crystal is turned gradually round through a second right angle. Y and Z remain fixed as before, y describes another quadrant of a circle round Y as a centre, and z another quadrant of a circle round Z as a centre; y and Z become gradually fainter, and at last disappear; while on the other hand Y and z gradually increase in brightness: at the end Y and z coincide, giving a single image which is twice as bright as either was in the first situation at the beginning of Art. 532. When the second crystal has been turned through half a right angle from the position it occupied at the end of Art. 533 the four images are equally bright, and occupy the four corners of such a rhombus as was mentioned before. The line which joins Y and y, and also the line which joins Z and z, is parallel to the plane containing $A'B'$ and $A'X'$.

535. The rays which form the images Y and Z are polarised in a plane parallel to that containing $A'B'$ and $A'X'$; the rays which form the images y and z are polarised in a plane at right angles to this. It is easy to state the laws which connect the brightness of an image with the direction of the plane of polarisation of the light which forms it. A ray of light which falls on the first crystal is separated into two rays of equal brightness, the ordinary ray which is polarised in a plane containing the ray and

the optic axis, and the extraordinary ray which is polarised in a plane at right angles to this: these polarised rays then fall on the second crystal. Let *C* be the centre of a circle, and let the radius *CK* denote the direction of the plane of polarisation of a ray which falls on a crystal; let *CG* represent the direction of a plane which passes through this ray and through the axis of the crystal; and let *CH* be at right angles to *CG*. Then the incident ray is separated into two rays, one polarised in the plane denoted by *CG*, and the other polarised in the plane denoted by *CH*. Let the brightness of the incident ray be represented by 1; then it is found that the brightness of the ray polarised in the plane *CG* is represented by the square of the sine of the angle *HCK*; and the brightness of the ray polarised in the plane *CH* is represented by the square of the sine of the angle *GCK*. For example, suppose that in Art. 533 the second crystal has been turned through an angle of 30°; and consider the ray which has undergone ordinary refraction by the first crystal: thus if *CK* denote the plane of polarisation of this ray the angle *GCK* is 30°, and the angle *HCK* is 60°. The ray polarised in the plane *CG* is that corresponding to the image denoted by *Y*; the brightness of this image is therefore represented by the square of the sine of 60°. Now by the Table of Art. 161 the sine of 60° is ·866; the square of this number is very nearly ·75, which therefore represents the brightness of the image *Y*. The ray polarised in the plane *CH* is that corresponding to the image denoted by *y*; the brightness of this image is therefore represented by the square of the sine of 30°. Now by the Table of Art. 161 the sine of 30° is ·5; the square of this number is ·25, which therefore represents the brightness of the image *y*. The sum of ·75 and ·25 is 1; so that the two images *together* correspond in brightness to the brightness of the incident light. In like manner as we have estimated the brightness of *Y* and *y* it may be shewn that the brightness of *z* is the same as that of *Y*, and the brightness of *Z* the same as that of *y*. For the plane of polarisation of the ray which corresponds to the images



z and Z will be denoted by the radius CJ which is at right angles to CK; the brightness of z is represented by the square of the sine of the angle JCG, and the angle JCG is equal to the angle HCK: and the brightness of Z is represented by the square of the sine of the angle between CJ and HC produced through C, and this angle is equal to the angle GCK.

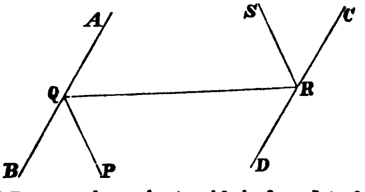
536. When a ray of common light falls on a doubly refracting crystal it is, as we have said, separated into two rays of equal intensity, polarised in planes at right angles to each other. Experiment and theory shew that a ray of common light may in general be supposed equivalent to *two co-existing rays* of polarised light, each of half the brightness of the ray of common light, and having their planes of polarisation at right angles. Thus the action of a doubly refracting crystal amounts to giving definite positions to the planes of polarisation, which in the ray of common light may be considered as arbitrary; and also to separating the two components of the common ray which originally coincided.

XLIX. POLARISATION BY REFLECTION AND REFRACTION.

537. In the year 1810 M. Malus, a French engineer officer, discovered that light could be polarised by reflection. He was looking through a piece of Iceland spar at the light of the setting sun reflected from the windows of the Luxembourg palace in Paris, and on turning the crystal round he was surprised to observe a remarkable difference in the intensity of the two images: that formed by the more refracted rays was alternately brighter and fainter than that formed by the less refracted rays, at each fourth part of the entire revolution. The observation being followed up led to the result that if light is incident at an appropriate angle on the surface of any body, transparent or opaque, except metals, the reflected ray will be *polarised in the plane of reflection*. That is, the reflected ray will have, with respect to this plane, precisely the same properties which the ordinary ray in Iceland spar has with respect to the plane containing that ray and the optic axis

of the crystal. The appropriate angle of incidence is found to be 56^0 for glass, and $52^0\ 45'$ for water.

538. Let a ray of light PQ be incident on a plate of glass AB at an angle of 56^0, and let QR be the reflected



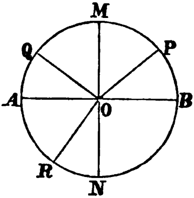
ray; then QR, as we have just said, is found to be polarised in the plane PQR. Suppose the ray QR to fall at the *same angle of incidence*, on a second plate of glass, CD. If CD is parallel to AB the reflected ray RS is in the same plane as PQ and QR, and it will be clearly visible to an eye suitably placed. Let the reflector CD be turned gradually round RQ, *keeping always at the same inclination to it;* then the plane of the second reflection, namely that containing QR and RS, gradually increases its inclination to the plane of the first reflection, namely that containing PQ and QR. During this rotation of CD the brightness of RS gradually diminishes, until the reflector comes into such a position that the plane of the second reflection would be at right angles to the plane of the first reflection; and then the ray RS becomes so faint that it may be said practically to vanish. Thus we have a new property and test of polarised light, namely that it will not be reflected from a plate of glass when it is incident on the plate at an angle of 56^0, and the plane of incidence is at right angles to the plane of polarisation.

539. The angle at which a ray must be incident on a reflecting substance in order that the reflected ray may be polarised is called the *polarising angle*, or the *angle of polarisation*, for that substance. Brewster discovered the law which determines the value of this angle for any assigned reflecting surface, which is briefly expressed thus: *the index of refraction is the tangent of the angle of polarisation.* We may however express the law in such a

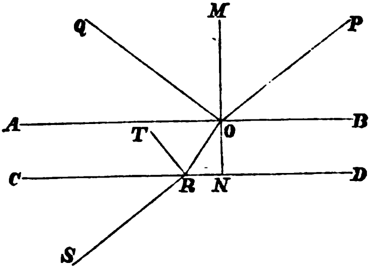
way as to avoid the introduction of the word *tangent*, which involves rather more mathematical knowledge than we assume: *divide the sine of the angle of polarisation by the sine of the complement of the angle and the quotient is the index of refraction.* Take for example glass; the sine of 56° is ·829; the complement of 56° is 34°, and the sine of 34° is ·559; divide ·829 by ·559, and the quotient is about 1·5.

540. The refractive index is different for the differently coloured rays of which common light consists; hence we cannot completely polarise a ray of common light by reflection, and therefore such a ray cannot be *wholly* extinguished by a second reflection in a plane at right angles to the plane of the first reflection. If the angles of incidence be adjusted to suit the index of refraction of *red* rays, the faint light which comes from the second reflector will have a violet tint; and in like manner if the angles of incidence be adjusted to suit the index of refraction of *violet* rays, the faint light which comes from the second reflector will have a red tint. It is found that when the angles of incidence are adjusted to suit the most luminous rays of the solar spectrum, the faint light which comes from the second reflector has a *purplish* tint.

541. Let *AB* denote a reflecting surface on which a ray *PO* is incident; let *OQ* denote the reflected ray, and *OR* the course of that part of the ray which is refracted: let *MON* be the normal at *O* to the surface *AB*. Then if *POM* is the angle of polarisation the angle *QOR will be a right angle.* For by Brewster's law the sine of *POM* divided by the sine of the complement of *POM* is equal to the index of refraction; and by the law of refraction the sine of the angle *POM* divided by the sine of the angle *RON* is equal to the index of refraction. Hence it follows that the angle *RON* must be equal to the complement of the angle *POM*, that is, to the complement of the angle *QOM*. Hence the angles *RON* and *QOM* are together equal to a right angle; and therefore the angle *QOR* is a right angle.



542. Let a ray PO fall on the surface AB of a *plate* at the polarising angle; then the ray OR which is refracted



into the plate and falls on the second surface CD at R, will be incident *at the proper polarising angle for this case.* For let OQ denote the ray reflected at O: then, by Art. 541, the angle QOR is a right angle. Let RT denote the ray reflected at R, and RS that refracted; then the angle TRC is equal to the angle ORN, which is equal to the angle ROA by Art. 155. Again, RS is parallel to PO by Art. 265; therefore the angle SRC is equal to the angle POB, which is equal to the angle QOA. Hence the sum of the angles TRC and SRC is equal to the sum of the angles ROA and QOA, that is to a right angle. Hence, by Art. 541, the ray denoted by RT is polarised. The ray denoted by RT after being refracted at the surface AB would emerge parallel to OQ; and thus in conjunction with OQ would give a polarised beam of greater intensity than OQ alone. In this way, by using several parallel plates, one behind the other, and placed so that a ray falls on the first plate at the polarising angle, we obtain a polarised beam of parallel rays consisting of portions which have been reflected at the different plane surfaces and have made their way back through the first plate: such a *pile of plates* is very useful for furnishing a bright polarised light.

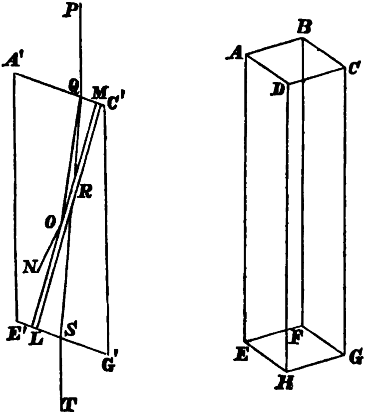
543. Polarised light may also be obtained by *refraction.* Let a ray of light fall at any angle on a transparent

medium; then some of the light is reflected and some refracted. The reflected light is found to be not entirely polarised, unless the light is incident at the polarising angle; it is generally held to consist of a mixture of common light and of light polarised in the plane of reflection; the amount of the polarised light increases as the angle of incidence approaches the polarising angle either from above or below. Such a mixture of common light and of polarised is called *partially polarised light*. In like manner the refracted light is found to be partially polarised, the plane of polarisation being at right angles to that for the reflected light. Moreover it was discovered by Arago that the quantity of the polarised light in the refracted light is exactly equal to the quantity in the light simultaneously reflected. By sending light through a pile of parallel plates in sufficient number the light finally emergent appears to be completely polarised.

544. Polarised light may be conveniently obtained by the aid of *tourmaline*. If a slice of this crystal be taken parallel to the axis, and a ray of light allowed to fall on it, only an extraordinary ray emerges, which is polarised in a plane at right angles to that containing the incident ray and the axis: the ordinary ray is absorbed by the crystal. Take two such slices, hold them in contact, and keep one fixed while the other is turned round. When the two slices are in the position which they would naturally occupy as parts of the same crystal, light will pass freely through the two; as one piece is turned round the amount of light which will pass through the two diminishes, until at last when one piece has been turned through a right angle no ray will pass through the two.

545. Thus we have various methods of procuring polarised light, but the method by double refraction is the best, because by it we get from a given beam of light a stronger polarised beam than in any other way. "Through a thickness of *three* inches of Iceland spar we can obtain two separate beams of polarised light *one third* of an inch in diameter, and each of these beams contains half the light of the original beam, excepting the small quantity of light lost by reflexion and absorption." Brewster's *Optics*.

546. *Nicol's Prism* is a convenient arrangement by which we give to Iceland spar a property equivalent to that which tourmaline possesses naturally, namely, that of suppressing one of the two rays into which an incident ray is separated. Let *ABCDEFGH* represent a piece of Iceland spar in its natural state; the optic axis lies in the plane *ACGE*. The face *ABCD* is inclined to the edge



AE at an angle of 71°. By grinding and polishing this face is removed, and another is produced which is inclined at an angle of 68° to *AE*, and is at right angles to the plane *ACGE*. A similar operation is performed with respect to the plane *EFGH*. The crystal must then be cut into two by a plane perpendicular to *ACGE*, and the two parts carefully fitted together again, with a thin layer of Canada balsam between them. Let *A'C'G'E'* be the figure which thus replaces *ACGE*, so that the angle *C'A'E'* is 68°. Let *LM* denote the layer of Canada balsam; this may pass through *C'*, or it may pass through *E'*; all that is essential is that the angle between *A'C'* and *LM*, produced if necessary, should be a right angle. Let *PQ* denote a ray incident on $A'C'$, in a direction parallel to $A'E'$; this will be separated into the ordinary ray QO and the extraordinary ray QR. Now if there were no deviation at Q the ray QO would fall on the Canada balsam at the angle just suited for *total reflection*. Owing to the deviation at Q the ray QO falls on the Canada balsam at a somewhat greater angle, and is of course totally reflected; thus it passes away in the direction ON. The extraordinary ray however goes through the Canada balsam, and finally emerges in the direction ST, which is parallel to PQ. Thus we get rid of the ordinary ray, and retain the extraordinary ray, which is polarised in a plane at right angles to $A'C'G'E'$. This contrivance was invented by the late Mr Nicol of Edinburgh; it was at first rather neglected by English philosophers, but the utility of it was soon recognised abroad, and it is now universally appreciated. Practically in making the instrument it is in general found necessary to take two pieces of Iceland spar and grind each away, as it is difficult to cut one piece into two.

547. The explanation given by the wave theory of Newton's rings makes them depend on the interference of light reflected from the two surfaces of a thin film. The truth of this explanation is confirmed by using light polarised in the plane of reflection in order to view the rings, instead of common light. It is found that the brightness of the light in the rings varies with the angle of incidence, and that the *whole system vanishes* in two cases, namely, when the angle of incidence at either surface of the film is equal to the corresponding polarising angle.

L. INTERFERENCE OF POLARISED RAYS.

548. The principle of the interference of light is applied, as we have seen in Chapter XLIII., to explain various interesting phenomena exhibited by common light. It becomes an obvious enquiry to ascertain if there is any peculiarity with respect to the interference of *polarised* light; and the experimental investigations of Arago and Fresnel have established five laws which we shall now give.

I. Two rays derived from the same source, and polarised in the same plane, are capable of interfering with each other, like rays of common light.

II. Two rays derived from the same source, and polarised in planes at right angles to each other, are incapable of interfering with each other. When the angle between the planes of polarisation is intermediate between zero and a right angle, fringes of interference are formed with an intermediate brightness; the fringes being most bright when the angle is zero, and vanishing when the angle is a right angle.

III. Two rays originally polarised in planes at right angles to each other, may be brought into the same plane of polarisation without acquiring the property of interfering.

IV. Two rays polarised in planes at right angles to each other, and afterwards brought to similar states of polarisation, interfere like rays of common light, provided they belong to a pencil originally polarised in one plane.

V. In the phenomena of interference produced by rays which have undergone double refraction a difference of half a wave length must in some cases be allowed, as one of the rays is retarded by that amount owing to some unknown cause.

549. It would be beyond our limits to enter at length into the experimental evidence of the preceding five laws. We may state generally that the first law follows from the fact that all the interference experiments succeed with light polarised in one plane as well as with common light. To establish the second law light is brought nearly to a point by the aid of a convex lens, and is then allowed to pass through two parallel narrow slits in a thin sheet of copper. Two portions of a plate cut from a good crystal of tourmaline, in the manner stated in Art. 544, are placed before the slits so as to polarise the rays which pass through them. It is found that the usual phenomena of interference are visible when the optic axes of the two pieces of tourmaline are parallel, and are invisible when these axes are at right angles; in intermediate positions of the axes the phenomena are seen with intermediate degrees of brightness. To establish the third law place a

rhomb of Iceland spar in the manner denoted by the upper three diagrams of Art. 532, so as to receive the rays from the tourmalines; and let the plane ABX of the Iceland spar be inclined at an angle of 45° to each of the planes of polarisation of the rays from the tourmalines. Each of the two rays emerging from the tourmalines will now be separated by the Iceland spar into two equal polarised rays. The plane of polarisation of two of the four resulting rays will be the plane ABX of the crystal; the plane of polarisation of the other two will be at right angles to this. Thus the former two rays might have been expected to interfere, and so also might the latter two; but no phenomena of interference are seen, and so the law is established. The experimental evidence of the truth of the fourth and fifth laws is rather more complicated, and we must omit it: the fifth law resembles a fact to which we have already drawn attention in Art. 487.

550. From the laws respecting the interference of polarised rays a remarkable result is deduced by theory, namely, that the oscillations of each particle of ether in the waves which produce light are at *right angles* to the direction in which the light is propagated. This forms a striking difference between sound and light. Suppose that a plane wave of sound is transmitted through a straight tube, as in Art. 56, then the oscillations of the particles of air are in a direction parallel to the tube, that is at right angles to the plane wave. But if a plane wave of light is transmitted through a tube the oscillations of the particles of the ether are in directions at right angles to the length of the tube, and therefore in the plane front of the wave or parallel to it. In the case of waves of common light the oscillations are at right angles to the direction of propagation of the wave, but are not otherwise limited; in the case of polarised light it is believed that the oscillations are also at right angles to a fixed plane, probably the plane of polarisation.

LI. COLOURED RINGS.

551. The systems of rings formed round the axes of doubly refracting crystals were discovered by Brewster; they are among the most splendid phenomena of optical science. Let a plate of Iceland spar be taken perpendicular to the optic axis, and placed between two slices of tourmaline cut as in Art. 544. Let the optic axes of the two tourmalines be at right angles to each other; then by allowing light to pass through the apparatus to the eye we see a system of coloured rings, resembling Newton's rings, intersected by a rectangular black cross. Next put the optic axes of the two tourmalines parallel to each other; then the black cross is replaced by a bright one, and in the system of rings the colour of any ring is complementary to what it was before. When the optic axes of the two tourmalines are inclined at an angle between zero and a right angle there is a double cross, or eight rayed star, the arms of which have a brightness intermediate between the darkness of the cross in the first situation and the brightness of the cross in the second situation. If in any assigned situation of the two tourmalines the Iceland spar is turned round no change is made.

552. In this experiment the tourmaline furthest from the eye is called the *polarising plate* or the *polariser;* its use is to furnish a beam of polarised light, and it may be replaced by any of the other contrivances for the same end, such as a reflecting or refracting plate of glass, or a *pile* of such plates. The tourmaline nearest the eye is called the *analysing plate* or the *analyser;* its use is to analyse or separate into parts the light which has passed through the Iceland spar, or any such crystal; instead of the tourmaline we may use a reflecting plate, as of glass, which can be turned round so as to keep the light always inclined to the plate at the polarising angle, but to allow the plane of reflection to take different inclinations to the plane of polarisation as determined by the polariser. Or instead of the analyser we may use a thick rhomb of Iceland spar which will produce a great separation of images; and then two systems of rings will be seen together.

553. Instead of common light we may use homogeneous light, as red for example; then the rings will be alternately dark and red. The rings are largest for red light, least for violet light, and of intermediate sizes for the other colours. By the simultaneous existence of all these rings the coloured system for common light is produced as in other cases: see Art. 481. If the piece of Iceland spar be made thinner the rings become larger.

554. Other uniaxal crystals, when cut into plates perpendicular to the optic axis, exhibit rings of a similar kind. The rings formed by positive crystals, as zircon and ice, do not to the eye differ from those formed by negative crystals; yet they possess different properties. For if we combine a plate of Iceland spar with a plate of one of the positive crystals which taken singly would produce rings of the *same* size, then on looking through the combination, placed between the tourmalines, we find that the rings *disappear*. Generally if we combine two plates of positive crystals, or two plates of negative crystals, we obtain such a system of rings as would be produced by one of the crystals alone of a thickness corresponding to the *sum* of the two thicknesses; but if we combine a plate of a positive crystal with a plate of a negative crystal, we obtain such a system of rings as would be produced by one of the crystals alone of a thickness corresponding to the *difference* of the two thicknesses. Thus, for instance, if one plate is of Iceland spar, and the other of zircon, the result is the same as if we diminish the thickness of the Iceland spar by the thickness of a plate of this substance which would produce rings of the same size as the plate of zircon.

555. In biaxal crystals the phenomena are somewhat different; the rings tend to take an *oval* form. If the angle between the two optic axes is considerable the systems of rings corresponding to both axes cannot be seen at once; but when this angle is small they can. A crystal called by the names *nitre*, *saltpetre*, and *nitrate of potash*, has its optical axes inclined at an angle of 5°, and the two systems of rings can be seen well together. A plate of the crystal is taken, which has its surfaces at right angles to the straight line bisecting the angle formed by the optic axes; the plate should not exceed one sixth or one eighth of an inch in thickness. The system of rings is then very

elaborate. The smaller rings are nearly circular, arranged in two sets, one concentrically round one pole, and the other concentrically round another pole. As the rings increase in size they become elongated towards the point midway between the poles; then they become a single curve like the figure 8 crossing at the midway point; then a single curve nipped so as almost to meet at the middle; afterwards curves slightly flattened taking an oval shape. There is besides sometimes a rectangular cross having one arm passing through the poles; and sometimes, instead of the cross, curves called *brushes* which pass through the pole. The appearances change according to the positions of the optic axes of the polariser, and also they change when the plate of nitre is turned round. Thus the cross and the brushes appear sometimes dark and sometimes bright.

556. The wave theory of light gives satisfactory explanations of these phenomena of coloured rings, shewing that they all depend on interference; but this matter is too difficult for an elementary work.

557. In viewing these rings the eye is supposed to be placed very near to the crystal which is used; the rays which enter the eye are thus inclined at various angles to the plate of the crystal and therefore traverse different thicknesses of the plate. It is to this circumstance, as theory shews, that the *rings* are due. There are however other phenomena observed when light is allowed to pass through a crystal in such a manner that the rays are parallel, and so traverse equal thicknesses; these were discovered before the coloured rings which we have described in the present Chapter. These phenomena are conveniently exhibited by the aid of a crystal called *gypsum* or *selenite*, which is a *crystallised sulphate of lime;* the advantage of this substance is that it cleaves with great facility. Take a thin film of this substance, place it between the polarising and analysing tourmalines, and let parallel rays pass through the whole apparatus to the eye. If the plate be of a certain thickness the eye will see a uniform red tint; as the film is turned round the tint gradually changes in brightness, being twice in its most vivid condition, and twice zero, in the course of a complete revolution. For other thicknesses the tint will be of other colours, which will in like manner

pass through variations of brightness when the film is turned round. Mica is another crystal which is convenient for this experiment.

558. If two slices of tourmaline, cut as in Art. 544, are placed with their optic axes at right angles to each other, then, as we have already stated, no light can pass through the pair. But, as we have just seen, if a film of selenite or of mica be interposed between the tourmalines light can pass through the system. The effect thus produced by the interposed crystal was formerly called *depolarisation;* but that term is now rarely used. Dr Whewell says: "The effect which the mica thus produced was termed *depolarization;*—not a very happy term, since the effect is not the destruction of the polarization, but the combination of a new polarizing influence with the former. The word *dipolarization*, which has since been proposed, is a much more appropriate expression."

LII. ROTATION OF THE PLANE OF POLARISATION.

559. When a ray of polarised light passes through a medium its plane of polarisation usually remains fixed in direction, so that the plane is the same when the ray leaves the medium as when it enters the medium. But there are media which have the power of changing the plane of polarisation of a ray, turning it, as it were, through an angle which is proportional to the thickness of the medium traversed. Thus it might happen that in passing through a plate of such a medium, of the thickness of one hundredth of an inch, the plane of polarisation was turned through an angle of 1^0; then in passing through a plate of that medium of twice the thickness, the plane would be turned through an angle of 2^0, and it would be turned through a right angle by passing through a plate of the medium $\frac{90}{100}$ of an inch thick, that is $\frac{9}{10}$ of an inch thick. Phenomena connected with such a change of the plane of polarisation are collected under the title of *rotating polarisation* or *rotation of the plane of polarisation.*

560. *Quartz*, which is also called *rock crystal*, is a uniaxal crystal; a plate cut off by planes perpendicular to the axis will exhibit the rotation of the plane of polarisation very clearly. If the plate is $\frac{1}{25}$ of an inch thick the plane of polarisation of a red ray which traverses the plate is turned through an angle of $17\frac{1}{2}^0$, and the plane of polarisation of a violet ray through 45^0.

561. On trying the experiment with various specimens of quartz it is found that one crystal may turn the plane of polarisation in *one* direction, and another crystal may turn it in the *opposite* direction. Hence quartz in respect to this property is divided into the two classes of *right-handed* and *left-handed*.

562. Certain liquids have the power of producing rotation of the plane of polarisation; the power is much feebler than in the case of quartz, but this can be compensated by sending the light through a greater thickness of the medium. The fluid is put into a tube and the ray of light passes along it. This use of polarised light gives a very delicate instrument of analysis. For example, various kinds of sugar may appear identical to the taste, and even to the ordinary tests of chemistry; but yet solutions of them in water may be distinguished by their different degrees of power in rotating the plane of polarisation, or by the fact that they turn this plane in contrary directions.

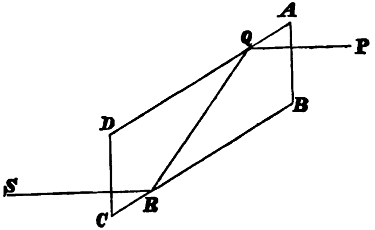
563. We proceed to notice briefly the extension which the theory of light has received in consequence of the facts mentioned in this Chapter. The polarised light which we have hitherto considered may be called *plane polarised light* to distinguish it from another kind now to be considered. In a plane polarised ray every particle of ether is believed to oscillate in a straight line which is perpendicular to the direction of propagation of the corresponding wave of light, and which is also perpendicular to a certain fixed plane through the normal to the front of the wave. It has been doubted whether this fixed plane is the plane of polarisation of the ray, or a plane at right angles to it; but the prevalent opinion is that the direction of oscillation is perpendicular to the plane of polarisation. A ray of common light seems to be practically equivalent to the

combination of two rays of polarised light, the planes of polarisation being at right angles to each other, but not necessarily fixed. Now theory indicates the existence of a kind of light in which each particle of ether moves in a curve lying in the plane front of the wave; this curve may be a circle, and then the light is called *circularly polarised light;* or the curve may be an ellipse, and then the light is called *elliptically polarised light.* It may seem to the reader that there is a wide difference between motion in a straight line and motion in a circle or an ellipse, and that consequently it is somewhat arbitrary to include such motions of the particles of ether under the common title of *polarised light;* but mechanical principles shew that there is a connection between these motions which are apparently so unlike. The reader must not confound *circular* polarisation with *rotatory* polarisation; certain *experimental* facts of a curious but not difficult character are collected under the title of *rotatory* polarisation; and to account for these facts the *theory*, which is somewhat abstruse, of *circular* and *elliptical* polarisation has been devised. We cannot however explain in an elementary way the application of the theory to the facts.

564. The various kinds of light, thus shewn by theory to exist, may be discriminated by experiment. Let a ray of light fall on a plate of glass at the polarising angle, and let the plate be turned gradually round, keeping the angle of inclination of the incident ray the same, as was supposed for the reflector *CD* in Art. 538. Then if the reflected ray vanishes in any position of the reflector the incident ray is *plane* polarised, the plane of polarisation being at right angles to the plane of incidence on the reflector in this position. If the reflected ray never vanishes, but still passes through decided fluctuations of brightness the incident ray is *elliptically* polarised. If the reflected ray passes through no fluctuations of brightness the incident ray is either *common light* or *circularly polarised;* this experiment will not distinguish between the two. Again, if *plane polarised light* passes through a plate of Iceland spar cut perpendicularly to its axis, and is then analysed, a system of rings is seen as described in Chapter LI.; and when the analysing tourmaline is turned round the colour

of the rings and the brightness of the cross *vary*. If instead of the original plane polarised light we use *circularly polarised light* then *no change occurs* in the rings and cross as the analysing tourmaline is turned round. If instead of the original plane polarised light we use *common light* then no rings are seen.

565. Circularly polarised light and elliptically polarised light may be obtained by the aid of a certain piece of glass called *Fresnel's Rhomb*. The parallelogram *ABCD* repre-



sents a section of the rhomb by the plane of the paper, two faces of the rhomb being parallel to the plane of the paper, and therefore exactly like this figure. The angles at *A* and *C* are each about $54\frac{1}{2}°$. A ray of light incident on *AB* at right angles will pass into the rhomb without deviation; then it will be totally reflected first at *AD* and next at *BC*; and it will finally pass out through *DC* at right angles: the course of this ray may be represented by *PQRS*. If the incident ray is polarised in the plane of the paper so also is the emergent ray; if the incident ray is polarised in a plane perpendicular to the plane of the paper so also is the emergent ray. If the incident ray is polarised in a plane inclined at an angle of 45° to the plane of the paper the emergent ray is *circularly polarised*. If the incident ray is polarised in a plane inclined at any other angle between 0° and 90° the emergent ray is *elliptically* polarised.

HEAT.

LIII. INTRODUCTION.

566. The important influence of heat on all departments of life is obvious from familiar experience. We know that the sun is the source from which is derived the warmth necessary to enable the earth year after year to produce the grain and the fruits which are necessary for human subsistence. We see that the supply of heat from this source is adjusted to the wants and constitution of man; and it is difficult to believe that the race could long survive any great change, either in the way of increase or decrease with respect to the average temperature of the world. The benefit we derive from other sources of heat is also most obvious; the perpetual presence of fires in our stoves is requisite for ordinary comfort, while the numerous large chimneys in our manufacturing towns tell of the operation of heat in supplying the endless wants of modern civilized existence. These are instances of the *practical* importance of heat; with respect to its influence in scientific theories and explanations something has been learned from the former volume of this work, and more will appear as we proceed.

567. Opinions have varied as to the nature of heat; but only two of these need be noticed. According to one opinion heat is a subtle fluid which can pass from one body to another, and can penetrate into the interior of bodies. According to another opinion, now generally held, heat consists in the oscillatory motions both of the particles of bodies, and also of a very subtle medium which pervades all space; this medium is believed to be that which is the vehicle of light, and is called *ether* or the *ethereal medium:* see Chapter XLIII. It is not necessary, however, for us to discuss the difficult subject of the nature of heat; it will be sufficient to describe under appropriate classification the principal facts which have been ascertained with respect to the operation of heat. It has been sometimes proposed to use the term *caloric* for the cause of the sensations we feel, and the results we observe, and which we associate with the word *heat;* but this word *caloric* has been usually applied in connection with the *first* of the two opinions on the nature of heat which we have described, and is now rarely used.

568. One of the most obvious effects of heat is to produce a change in the *size* of a body exposed to it. Bodies, whether solid, liquid, or gaseous, become in general larger as they become hotter, and smaller as they become colder. The effect is most distinct with respect to gaseous bodies: let a bladder containing some air be carefully closed, and put near a fire; then the visible swelling of the bladder shews that the air inside, as it becomes hotter, occupies more room under the same pressure than it did in its cooler state. Liquids under the influence of heat do not expand so obviously as gaseous bodies do, but still examples of such expansion occur familiarly; thus if a vessel full of cold water be placed over a fire the water becomes at last extremely hot, and the increased bulk is shewn by the fact that some water runs over. Solid bodies expand less than liquids, when under the influence of heat, and thus direct evidence of this expansion may not be very familiar, but still the fact is abundantly established by experience: for example, the iron rim, or tire, which surrounds a wooden wheel with spokes, is made hot before it is put on; thus it expands so that it can pass readily

over the wood, and then as it cools it contracts, and compresses all the other parts of the wheel stringently together.

569. By the operation of heat the size of a body is altered; but there is a still more remarkable change, as is well known, which is produced when sufficient heat is applied, namely, the passage from the solid state to the liquid state, or from the liquid state to the gaseous state. Thus *ice* by the application of heat is converted into *water*, and *water* by the application of heat is converted into the gaseous body which is known as *steam*. We naturally endeavour to trace a connection between the heat supplied to a body, and the size of the body, or the state in which the body occurs, out of the three with which we are familiar. The word *temperature* is used to denote the amount of *sensible* heat in a body, that is of heat which will be evident to our senses assisted in any way that is convenient; and we have to contrive some mode of registering and comparing the temperatures of various bodies under various circumstances. The *thermometer* is an instrument for such purposes; this estimates temperature by the amount of the expansion of mercury in a glass vessel.

570. The thermometer is a common instrument, and we have already described it in Arts. 559...562 of the first volume; so that we need not repeat the details of the construction. There are two very important points indicated by marks on the stem of the instrument; the first is that to which the mercury reaches when the instrument is put into melting snow, and the second is that to which the mercury reaches when the instrument is put into the vapour of water boiling under the standard pressure of the atmosphere: the former point is called the *freezing point*, and the latter the *boiling point*. The space on the instrument between the marks which correspond to these two points may be divided into any number of equal parts which is convenient; these parts are called *degrees*. In the Centigrade thermometer the space is divided into 100 degrees, and 0 is put at the freezing point, and 100 at the boiling point; this is the instrument commonly used for scientific purposes. In Fahrenheit's thermometer the space is divided into 180 degrees, and 32 is put at the

freezing point, and 212 at the boiling point: this is the instrument in popular use in England, and it is often called the *common thermometer*. In Reaumur's thermometer the space is divided into 80 equal parts, and 0 is put at the freezing point, and 80 at the boiling point. A small circle placed over a number is used as an abbreviation of the word *degrees;* the capital letter C placed after a number indicates the Centigrade scale, and the capital letter F indicates Fahrenheit's scale. Thus 40^0 C. denotes the temperature of 40 degrees in the Centigrade scale, and 40^0 F. denotes the temperature of 40 degrees in Fahrenheit's scale. We shall employ both scales; generally Fahrenheit's scale when we state ordinary facts, and the Centigrade scale when we state scientific laws.

571. It may be a question how far the indications of a mercurial thermometer correspond with the actual amount of heat present in a body; but this point cannot be discussed now. There are, however, two important facts to be noticed. If the thermometer is put into melting snow, at any time or place, it is found that the mercury always occupies the same space in the instrument; that is, it reaches just to the point marked as the *freezing point*. If the thermometer is put into the vapour of water, boiling at any time or place, provided the pressure of the atmosphere has its standard value, it is found that the mercury always occupies the same space in the instrument; that is, it reaches just to the point marked as the *boiling point*. Thus as the volume is always the same when the instrument is put into melting snow, it is natural to assume that the state with regard to heat will then also be always the same; and a similar inference holds when the instrument is put into the vapour of water boiling under the standard pressure.

LIV. EXPANSION OF SOLIDS.

572. The fact that solid bodies expand when exposed to the influence of heat may be taken as well known; in experimental lectures however it is usually illustrated in such a manner as the following. A cylindrical rod, and a

flat plate, both of brass, are provided; the plate has a notch cut along one edge, such that it is of the same length as the cylinder when the plate and the cylinder are of the same temperature. The cylinder then just fits into the notch at first, but when the cylinder has been made very hot in a fire it is found to be too long for the notch. Again, the plate has a circular hole bored through it, into which the cylinder will just fit at first; but when the cylinder has been made very hot it is found to be too large to fit into the hole.

573. When a solid body is exposed to the influence of heat, its length, its breadth, and its depth all increase; and thus its volume is increased. The rate of expansion of the length, breadth, and depth, is in general the same, and is called the *linear* expansion; the rate of expansion of the volume is called the *cubical* expansion. Now it is a fact that the linear expansion is always a small quantity, at least for such moderate changes of temperature as come under practical observation; and in consequence of this the linear and the cubical expansions are connected by the following simple rule: *the rate of cubical expansion is three times the rate of linear expansion.* We proceed to explain and establish this rule.

574. Suppose that for a given rise of temperature the length of a solid is increased by *one thousandth part* of itself; then in general the breadth will be increased by one thousandth part of itself, and also the depth by one thousandth part of itself. Thus if we take a cubic inch of the solid in its original state, the length, the breadth, and the depth will each become 1·001 inches under the influence of the given rise of temperature. Hence the volume of the solid instead of being 1 cubic inch becomes in cubic inches $1·001 \times 1·001 \times 1·001$, that is 1·003003001. This number is somewhat greater than 1·003, but the excess is so small as to be practically insensible. We may therefore take it as 1·003, so that the volume is increased by *three thousandths* of its first amount. Hence in this case the increase of the volume is denoted by *three times* that fraction which denotes the increase of the length: so that the rule is established for this case. Now when we come to our

numerical results it will be seen that the increase of length corresponding to a rise of 1° C. in temperature is far less than one thousandth part; so that if we estimate the rate of expansion by the amount of expansion which corresponds to a rise of 1° C. in temperature the rule will be practically quite trustworthy.

575. Experiments have been carefully made by which the amount of expansion of various solid bodies corresponding to the rise of the temperature through an assigned range has been determined. For instance, an iron bar is taken and surrounded with ice; the ice is removed without disturbing the iron bar, and boiling water is substituted: the result is that the iron bar becomes longer, and the exact amount of the extension must be determined by some method by which minute lengths can be measured. One such method is by counting the number of turns which must be given to a fine screw, the dimensions of which are known, in order to push the iron bar in the direction of its length until one end has been moved through a space equal to the extension, so as to come again into its original position. Two general results have thus been established with respect to the expansion of solid bodies.

I. Let any solid body be gradually heated so that its temperature rises from that of the freezing point to that of the boiling point, and then let it be gradually cooled down again to the temperature of the freezing point: it is found that the magnitude of the body at an assigned temperature is the same in the former case as in the latter. Thus the contraction which accompanies the fall of temperature exactly corresponds to the expansion which accompanies the rise. The condition that the body is to be *gradually* cooled down must be carefully regarded in order to ensure the return to the original volume when the original temperature recurs: this is to be specially noticed with respect to glass and some metals.

II. If glass or any metallic body is gradually heated so that its temperature rises from that of the freezing point to that of the boiling point, the amount of the expansion is proportional to the number of degrees which indicate the rise of temperature.

576. Thus suppose we ascertain the whole expansion in passing from the temperature of the freezing point to that of the boiling point, and divide it by 180, then we obtain the amount of the expansion when the temperature rises one degree Fahrenheit at any point of the scale, as for instance in the rise from 80^0 to 81^0, or in the rise from 100^0 to 101^0: at least this is strictly true for such bodies as are specified in the second result of the preceding Article.

577. We have stated in Art. 573, that under the influence of heat the length, the breadth, and the depth of a body expand generally at the same rate; this however is not universally the case; for instance, it is not the case with such crystals as have the property of *double refraction* with respect to light. Thus if Iceland spar have its temperature raised it expands in the direction of the axis, but contracts in directions at right angles to the axis; the expansion however is greater than the contraction, and so on the whole the volume is increased. Exceptions have also been found to the general law that a solid body always expands when the temperature rises; the iodide of silver is such an exception, and so is also a certain composition called *Rose's fusible metal*, which consists of two parts by weight of bismuth, with one of lead, and one of tin. Rose's fusible metal expands until the temperature reaches a certain point, and then contracts. If a strip of Indian rubber is stretched and then heated it will shrink in length.

578. We will now give some numerical results. In the following Table the first column contains the name of the substance; the second column states the expansion in length which a bar of the length unity receives as the temperature rises from that of the freezing point to that of the boiling point; the numbers in the third column are obtained by dividing those in the second column by 180, so that they state the linear expansion corresponding to a rise in temperature of 1^0 F.: these numbers are frequently called *coefficients of linear expansion*. If it be desired to express the coefficients with respect to a degree of the *Centigrade* Thermometer the numbers in the second column should be divided by 100 instead of by 180.

Linear Expansions.

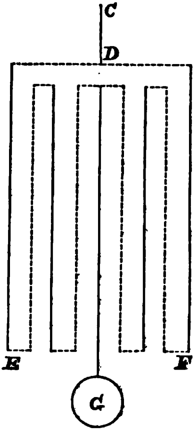
	For 180° F.	For 1° F.
Glass	·000861,	·0000048.
Platinum	·000884,	·0000049.
Untempered Steel	·001079,	·0000060.
Cast Iron	·001125,	·0000063.
Wrought Iron	·001220,	·0000068.
Tempered Steel	·001240,	·0000069.
Gold	·001466,	·0000081.
Copper	·001718,	·0000095.
Brass	·001878,	·0000104.
Silver	·001915,	·0000106.
Tin	·002173,	·0000120.
Lead	·002858,	·0000159.
Zinc	·002942,	·0000163.

It will be observed that the expansion of platinum is nearly the same as that of glass; hence a platinum wire may be fused into glass and no fracture is caused when the temperature is lowered, for then both substances contract in the same degree.

579. The fact that bodies expand under the influence of heat has to be carefully regarded in various circumstances of science and of practical life. As an example of the former kind we may take the case of a real pendulum: see Vol. I. Art. 338. The time in which such a pendulum will make a small oscillation depends on the distances of its various parts from the axis about which the oscillation takes place; but the pendulum changes its volume owing to variation of temperature, and consequently the time of a small oscillation will in general change when the temperature changes, becoming longer as the pendulum becomes longer. But for the accurate measure of duration it is necessary that the pendulum should always perform its small oscillations in the *same* time, and hence pendulums must be constructed which will answer this end. Such pendulums are called *compensation pendulums;* because they are so adjusted that the effects which changes of temperature produce in the various parts shall just *compensate each other.*

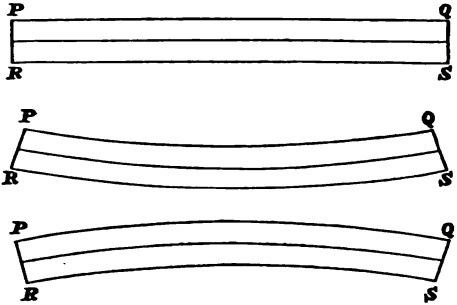
580. *Graham's Compensation Pendulum.* This pendulum consists of a rod suspended from a horizontal axis at one end, *C*, and having at the other end a glass vessel, *DE*, containing mercury. When the temperature rises the rod and the glass vessel expand, and this tends to bring the mercury further from the axis round which the motion takes place; but the mercury itself also expands, and at a greater rate than the rod and the glass vessel, and this tends to bring the mercury nearer to the axis. It is possible so to adjust the quantity of mercury that the two opposing tendencies may counteract each other, and leave the *centre of oscillation* in the same position for all such temperatures as will practically occur: see Vol. I. Art. 338.

581. *Harrison's Compensation Pendulum.* This pendulum consists of five rods of steel and four rods of brass placed alternately and united by horizontal rods of brass. In the diagram the dotted lines denote brass rods, and the full lines denote steel rods. *C* is the position of the horizontal axis about which the pendulum can oscillate, the motion being supposed to take place in the plane of the paper. *CD* is a rod fixed at *D* to the framework of the nine rods. *G* is a weight attached to the middle rod of the nine. It will be seen from the diagram that when the steel rods expand the effect is to move *G further* from the fixed axis *C*; and when the vertical brass rods expand the effect is to move *G nearer* to the fixed axis *C*. Now the rate of expansion of steel is about two-thirds of that of brass; hence the lengths may be so adjusted that the effect produced by the five steel rods shall just counteract that produced

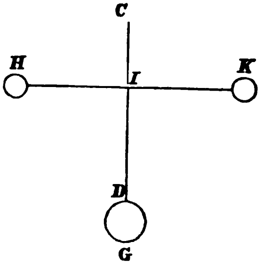


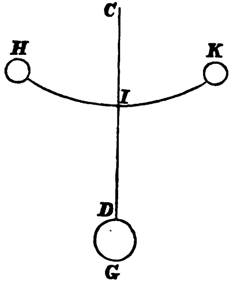
by the four vertical brass rods of the nine: for the five steel rods count as *three* in measurement, and the four brass rods as *two*. Thus the *centre of oscillation* remains in the same position for all such temperatures as will practically occur. This pendulum is sometimes called the *gridiron pendulum*.

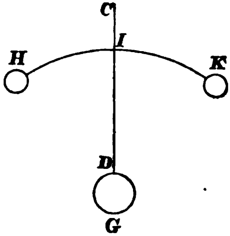
582. The adjustment of a pendulum so as to render it independent of changes of temperature may also be



effected by the aid of *compensating strips*. Let *PQ* and *RS* in the upper diagram denote two bars of the same length which are held firmly together by screws; suppose the upper bar of iron and the lower bar of copper. If the temperature be raised, since copper is more expansible than iron, the compound bar takes the form of the middle diagram, being curved with its concavity upwards; and if the temperature be lowered the compound bar takes the form of the lower diagram, being curved with its concavity downwards, for now the copper shrinks more than the iron. Let *CD* represent a pendulum rod which can oscillate round a fixed horizontal axis at *C*, the motion being supposed to take place in the plane of the paper; let *G* represent a weight attached to the end *D* of the rod. Let two equal weights denoted by *H* and *K* be attached to the ends of a *compensating strip*, and let the strip be firmly fixed to the rod in the manner indicated by the upper diagram: the strip is to be placed so that the more expansible metal may be on the under side. If the temperature rises the system takes the form represented by the lower left-hand diagram. *G* goes further

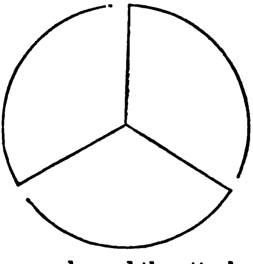






from the fixed axis by reason of the expansion of *CD*, but *H* and *K* are brought nearer to the axis by reason of the bending of the compensating strip. If the temperature sinks the system takes the form represented by the lower right-hand diagram; *G* comes nearer to the axis by reason of the contraction of *CD*, but *H* and *K* are brought further from the axis by reason of the bending of the compensating strip. The lengths and weights of the parts of the system may be so adjusted that the contrary effects shall compensate each other, and thus leave the *centre of oscillation* in the same position for all such temperatures as will practically occur.

583. The *Balance Wheel* of a chronometer furnishes another example of the precautions taken to counteract the influence of temperature. If the rim were one unbroken circle the wheel would expand under the influence of heat, and contract under the influence of cold; and the motion would be rendered slower in the former case, and quicker in the latter. To obviate this the rim is divided into three equal pieces, each fastened to one spoke; and each piece, like a compensating strip, consists of two layers of metal, the more expansible metal being outside. Consider one spoke and the attached piece. Suppose the temperature *rises*, then the spoke expands, and this tends to take the matter of the rim further *from* the centre. But on the other hand, the rim bends *towards* the centre, because the more expansible metal is outside. Suppose the temperature *sinks*, then the spoke contracts, but on the other hand the rim bends *from* the centre. Thus on the whole by suitable adjustment the opposing effects will just counteract each other.



584. In all structures where metal is employed it is necessary to pay attention to the fact that the metal swells or shrinks in bulk according as the temperature rises or falls; because forces of great power are thus brought into action. Suppose that the temperature of a bar of iron is 15° above the temperature of the surrounding medium, and that the ends of the bar are then tightly screwed to walls or other obstacles; it is found that when the bar has cooled down to the temperature of the surrounding medium it draws the obstacles together with a force of one ton for every square inch in the section of the bar. In the system of pipes by which water is conveyed through a town it is necessary to provide at certain intervals some contrivances by which parts of the system may be capable of yielding within a certain limit

to the influence of a rise or fall of temperature; for example, one tube may be allowed to slide within another like the joints of a common telescope. In iron bridges similar precautions must be taken. The great tube which forms a bridge over the Menai Straits is about 1800 feet long; and it is found necessary to allow for a change of length of about a foot: to effect this, the ends of the tube rest on various cast iron rollers and balls of gun metal about 6 inches in diameter.

585. A very useful application of the expansibility of iron was made many years since in Paris. The roof of the large gallery of the *Conservatoire des Arts et Metiers* pressed the sides of the building outwards so as to endanger its stability. Strong rods of iron were placed parallel to each other across the gallery, of such a length as to project externally beyond the walls, and nuts were screwed on at the ends so as to be in contact with the walls; and thus no further giving way could occur: but by an additional process the walls were restored to their original positions. Every alternate rod was heated by lamps beneath; the rods expanded, and consequently the nuts were removed through a small distance from the walls: the nuts were then again screwed home, and the rods, as they cooled, contracted and drew the walls slightly towards each other. While these rods were contracting the other rods formerly omitted were heated, and thus expanded, and the nuts at their ends again screwed home. The operations were repeated until the walls were finally restored to vertical positions. This simple and effective process has been adopted at other places for a similar object, as at the cathedral of Armagh.

LV. EXPANSION OF LIQUIDS.

586. Liquids like solids expand when their temperature rises, and contract when it sinks. The rate at which

the change of volume takes place, that is what we have called in Art. 573 the *cubical expansion*, may be ascertained in the following manner. We will suppose that mercury is the liquid on which we have to experiment. Take a tube bent twice at right angles, and open at both ends. Put mercury into the tube; then when the temperature is the same throughout the mercury will stand at the same level in the two vertical branches; as for instance at AB and CD respectively. Let each of these vertical branches be surrounded by a metal cylinder of larger dimensions; then by introducing substances of a suitable temperature between a vertical branch and its surrounding cylinder, we can bring the mercury in that branch to any temperature we please. Thus, for example, we may envelope the left-hand vertical branch with melting ice, and the right-hand vertical branch with boiling water. Now we know, from the principles of Hydrostatics, that when there is equilibrium the heights of the mercury in the two vertical branches above the common level surface are inversely as the specific gravities of the two portions of the mercury: see Vol. I. Art. 420. The common level surface in the present case may be taken to be the level of the horizontal connecting tube, which for this experiment is made extremely fine. The right-hand vertical column would be of exactly the same height as the left-hand vertical column if there were no difference of temperature, and when there is a difference of temperature the difference in heights is due to the diminution of density produced by expansion, and enables us to determine the amount of that expansion.

587. The cubical expansion of a liquid produced by a rise of temperature may be found by any method which determines the *specific gravity* both at the lower and the higher temperature. For example, plunge a solid of known weight into a liquid at any temperature, say 32° F., and thus determine the *weight lost* by the solid. Let the temperature of the liquid be raised to any standard that may be convenient, say 180° F.; and let the weight lost

when the solid is immersed be again determined. The actual expansion of the solid is supposed to be known, and we can easily allow for it; thus we can determine the weight which would have been lost in the second case supposing that the volume of the *solid* had remained unchanged. The weights lost in the two cases are then in the same proportion as the specific gravities of the liquids in the two cases; thus the proportion of the specific gravities becomes known, and then the amount of expansion of the liquid for the change of temperature can be determined.

588. The following example will illustrate the method. Let the solid be such that if its volume at 32^0 F. is 1 cubic inch, its volume at 180^0 F. will be 1·004 of a cubic inch; let the weight lost when the solid is plunged into a liquid at 32^0 F. be 1000 grains, and when the solid is plunged into the same liquid at 180^0 F. be 970 grains. As the volume of the solid has expanded from 1 to 1·004 it is clear that if the volume had remained unchanged the weight lost in the second case would have been $\frac{970}{1 \cdot 004}$ grains; and thus the proportion of the specific gravity of the liquid at 32^0 F. to the specific gravity of the liquid at 180^0 F. is $1000 \div \frac{970}{1 \cdot 004}$, that is $\frac{1 \cdot 004 \times 1000}{970}$, that is about 1·035. Thus the specific gravity at 32^0 F. is 1·035 times the specific gravity at 180^0 F.; and therefore if any quantity of the liquid be taken at 32^0 F., and the temperature raised to 180^0 F., the volume becomes 1·035 times what it was at first.

589. The facts relating to the expansion of liquids are usually stated with respect to *cubical* expansion, and not, as in the case of solids, with respect to *linear* expansion. It is convenient to remember that speaking roughly mercury increases in volume by about *one fiftieth* part as the temperature rises from that of the freezing point to that of the boiling point of water. The expansion of mercury has been very carefully determined, and we will state the results which have been obtained: by the *coefficient of expansion* we now mean the increase of *volume* corresponding to a rise in temperature of 1^0 C. It is found

then that the coefficient of expansion for mercury is $\frac{1}{5550}$ between 0° C. and 100° C.; it is $\frac{1}{5425}$ between 100° C. and 200° C.; and it is $\frac{1}{5300}$ between 200° C. and 300° C. Thus the expansion of mercury is not quite uniform; the rate of expansion increases somewhat as the temperature rises; and this is found to hold with respect to other liquids, so that in general the rate of expansion increases as the temperature approaches that at which the liquid boils. The expansion of mercury is found to be sensibly uniform in the range of temperature from 36° C. below zero up to 100° C.

590. The process of Art. 586 has this advantage, that the result is not affected by the change of volume of the glass tubes which accompanies the change of temperature. The coefficient of the expansion of mercury thus determined is called, for greater precision, the coefficient of *absolute* expansion. But if mercury be put into a vessel, and then the whole exposed to heat, both the mercury and the vessel expand; the mercury then does not *seem* to expand so much as it would if the vessel remained unchanged. The expansion thus observed, which is the *difference* between the real expansions of the mercury and the vessel, is called the *apparent* expansion; it may be determined by experiment. For example, take a cylindrical glass vessel to which a very fine tube is attached; weigh this, then fill it with mercury and weigh it again: hence by subtraction we find the weight of the mercury alone. Next raise the temperature of the whole to any assigned standard; in consequence of this the mercury expands, and some of it is forced out through the fine tube: this quantity must be very carefully collected and weighed, and then we can determine the *apparent* expansion of the mercury for the known change of temperature. Suppose, for instance, that the mercury weighed 1000 grains when the whole was at the temperature 0° C., and that when the temperature was raised to 100° C. a portion of the mercury was forced out which weighed 16 grains. Thus we infer that if mercury which would

weigh 984 grains at 0° C. is put into the glass vessel it will expand and fill the vessel when the temperature is 100° C. and that the amount of expansion is equal to the volume of 16 grains of mercury. Thus the fraction $\frac{16}{984}$ represents the *apparent* expansion of mercury corresponding to this rise of temperature; and dividing the fraction by 100 we obtain the coefficient of the *apparent* expansion of mercury. It is found that this coefficient is about $\frac{1}{6480}$; it varies however slightly according to the nature of the glass and the form of the vessel made of it. The difference between this and the $\frac{1}{5550}$ of Art. 589 represents the coefficient of cubical expansion of the glass vessel; the difference is very nearly $\frac{1}{38700}$, that is, about ·0000258.

591. The following Table gives the amount of the *apparent* expansions of volume of various liquids when the temperature is raised from 0° C. to 100° C.; the amount of *absolute* expansion may be found by adding that for glass through the same range, namely ·00258:

Water	·04666,
Water saturated with salt	·05,
Sulphuric acid	·06,
Oil of Turpentine	·07,
Nitric acid	·11,
Alcohol	·116.

592. The change in volume of water produced by a change of temperature has been carefully observed, and it is found that water has its greatest density, and therefore its least bulk, at about the temperature 4° C., which corresponds to 39·2° F. If the temperature of the water either rises above or sinks below this point the water expands. At 0° C. the water freezes; thus through the range from 0° C. to 4° C. water *contracts* as it becomes warmer, thus offering an exception to the general law that heat expands bodies.

LVI. EXPANSION OF GASES.

593. The expansion of gases is subject to a simpler law than the expansion of solids and liquids in two respects: first the rate of expansion continues uniform through a wider range of temperature, and secondly, the numerical coefficient is very nearly the same for all gases. It is found that if the pressure to which a gas is exposed remains constant, whatever that pressure may be, the gas expands to the extent of ·3665 of its original volume while the temperature rises from 0^0 C. to 100^0 C.; and moreover that this expansion is spread uniformly over the whole range of temperature. Thus if the temperature of a gas rise 1^0 C. between 0^0 C. and 100^0 C., the volume increases by ·003665 of the volume at 0^0 C. provided the pressure remains the same throughout. This fact or law is sometimes called by the name of Gay Lussac who established it, and sometimes by the name of Charles who had previously studied the subject.

594. The facts as stated in the preceding Article are sufficiently correct for all practical purposes, though very close experimental investigation has shewn that they are not absolutely exact. The experiments have been conducted in two ways with respect to air. In one way the pressure was not really kept constant, but the volume was; and the volume which the air would have occupied under a constant pressure was deduced by *Boyle's Law*, according to which the volume varies inversely as the pressure: see Vol. I. Art. 497. In this way the number ·3665 was obtained. In another way of experimenting the pressure was kept constant, and the volume allowed to change; in this way, instead of ·3665, the number ·3670 was obtained. Moreover it is found that some gases have for their coefficient of expansion a number slightly different from the ·003665 which applies to air: for example, the coefficient is ·0036678 for hydrogen, ·0036682 for nitrogen, and ·0036896 for carbonic acid. It appears also that the coefficient is not really constant for all pressures, but increases somewhat as the pressure increases. Thus on the whole it is inferred that Charles's law is not absolutely

exact, but that the less the density of a gas becomes the more nearly does this law hold; the gases for which it holds most closely are those which have never been liquified.

595. The expansion or contraction of air according as the temperature rises or falls is the cause of various familiar results. Thus in a chimney there is a constant current of air upwards which is called a *draft;* for the warm air being lighter, bulk for bulk, than the cold air, ascends by the principles of Hydrostatics; and then other air passes into the fire from the surrounding space, thus maintaining the combustion and the upward current. When the fire is first lighted the chimney may be so cold that the current does not readily establish itself, and then the smoke escapes into the room instead of finding its way upwards; but in a short time the air in the chimney becomes warmer and lighter than the air in the room, and then the usual action takes place. The flame of a lamp is supplied with air in the same manner as the current of air is maintained in a chimney; the heat of the flame expands the air above it, so that this air becomes lighter and ascends; then, to fill up the place of this, fresh air comes in from the surrounding space, and passes close by the flame, thus supporting the combustion: the process is aided by the glass chimney which surrounds the flame. Since hot air is lighter than cold air, the air near the ceiling of a room is generally at a somewhat higher temperature than that near the floor; this statement may be verified by the aid of a good thermometer. So also a fire-balloon rises; for this is a bag of silk or paper which is filled with hot air, and is lighter than an equal bulk of the atmosphere.

596. We know from Hydrostatics the relation which subsists between the volume of a gas and the pressure to which it is subjected, when the temperature remains the same; this relation is called *Boyle's Law:* see Art. 594. We know from Art. 593 the relation which subsists between the volume of a gas and the temperature when the pressure remains constant. Hence we are able to solve various problems relating to the volume of gases, when both the temperature and the pressure are changed, as we will now illustrate by examples.

(1) A mass of air at the temperature 0^0 C., under the pressure of the atmosphere denoted by 30 inches of the barometer, occupies 100 cubic inches: find the volume which it will occupy at the temperature 40^0 C. under the pressure of 28 inches. First suppose the pressure to change from 30 inches to 28 inches while the temperature remains the same; then the volume becomes $100 \times \frac{30}{28}$ cubic inches. Next suppose the temperature to rise from 0^0 C. to 40^0 C.; then the volume increases in the proportion of $1 + 40 \times \cdot 003665$ to 1, that is in the proportion of 1·1466 to 1. Hence finally the required volume is $100 \times \frac{30}{28} \times 1 \cdot 1466$ cubic inches, that is, 122·85 cubic inches. It will be easily seen that we shall arrive at the same result if we suppose the temperature to change first, and the pressure afterwards.

(2) A mass of air at the temperature 40^0 C., under the pressure of 28 inches occupies 122·85 cubic inches: find the volume which it will occupy at the temperature 0^0 C., under the pressure of 30 inches. Proceeding in the same way as before we obtain $122 \cdot 85 \times \frac{28}{30} \times \frac{1}{1 \cdot 1466}$ cubic inches, that is, 100 cubic inches.

(3) A mass of air at the temperature 40^0 C., under the pressure of 29 inches occupies 100 cubic inches: find the volume which it will occupy at the temperature 60^0 C. under the pressure of 30 inches. First suppose the pressure to change from 29 inches to 30 inches, while the temperature remains the same; then the volume becomes $100 \times \frac{29}{30}$ cubic inches; next, precisely as in Example (2), we see that at the temperature 0^0 C. the air will occupy $100 \times \frac{29}{30} \times \frac{1}{1 \cdot 1466}$ cubic inches. Then to find the volume at the temperature 60^0 C. we must multiply this by $1 + 60 \times \cdot 003665$, that is, by 1·2199. Hence the required volume in cubic inches is $100 \times \frac{29}{30} \times \frac{1 \cdot 2199}{1 \cdot 1466}$, that is, about 102·84.

597. The decimal ·003665 may be represented very closely by $\frac{1}{273}$. Hence if the temperature of a gas is raised, while the volume remains constant, the pressure which it supports must increase by $\frac{1}{273}$ part for every degree from 0° C. upwards; thus at the temperature of 273° C. the pressure would be doubled: at least this will be the case if we assume that the law which has been established for temperatures between 0° C. and 100° C. holds for temperatures between 100° C. and 273° C. If the law also held good for all temperatures below 0° C., then at the temperature 273° C. *below* 0° C. the gas would have lost all its elastic force. This temperature of 273° C. below 0° C. is much lower than any which can be practically reached, but still it is convenient for some purposes to refer to this as an ideal starting point, and it is called the *absolute zero of temperature.* The corresponding point on Fahrenheit's scale is 491° below the freezing point, that is, 459° below zero. It is from this number that the Rule is derived which is given in Vol. I. Art. 498; though there for simplicity 450 has been used instead of 459. Temperature when reckoned from the absolute zero of temperature is called *absolute temperature;* we shall not have occasion to attend further to this subject, but may just state that mathematicians find the notion of absolute temperature convenient, as they are able by the aid of it to combine Boyle's Law and Charles's Law in this single statement: *the product of the volume of any gas into the pressure which it supports is always proportional to the absolute temperature.*

LVII. THERMOMETER AND PYROMETER.

598. We have already described the construction of the mercury thermometer and have explained the three modes of graduating it which are in use: we shall now add a few remarks on this thermometer, and also notice some others.

599. The Centigrade scale is that which is most commonly employed in works on science; it has the obvious advantage of taking for zero a very important point, namely that which corresponds to the temperature of freezing water: and the division of the interval between this and the boiling point into a *hundred* equal parts is convenient for numerical calculations. Fahrenheit's scale was introduced at an early period, and has been widely diffused; but it has otherwise little to recommend it: we may notice the remark made by Professor Maxwell that "mercury expands almost exactly one ten-thousandth of its volume at 142° F. for every degree of Fahrenheit's scale, and that the coldest temperature which we can get by mixing snow and salt is near the zero of Fahrenheit's scale."

600. The thermometer gives really very little information respecting the state of a body and the amount of heat which the body contains. Thus the thermometer does not lead us to suspect that about thirty times the quantity of heat is required to raise the temperature of a pound of water one degree as would suffice to raise the temperature of a pound of mercury one degree; yet such is the fact, as we shall see hereafter. Nor does the thermometer instruct us directly respecting *latent heat;* we know however that heat may enter into a body and remain insensible so far as the indications of the thermometer are concerned: this has been noticed in Vol. I. Art. 479, and will be further considered hereafter.

601. Since the temperature at which water boils depends on the pressure of the atmosphere, it is necessary to be precise on this point in order that we may really know what is meant by the *boiling point.* This was formerly not strictly defined for Fahrenheit's thermometer; but it has been recently recommended by the Government Commissioners for constructing standard weights and measures that the following definition should be adopted: the temperature of the boiling point shall be that of water boiling at London under the pressure of 29·905 inches of mercury, the density of the mercury being that which it has at the freezing point, that is at 32° F. The boiling point of the

Centigrade thermometer is defined to be the temperature of water boiling at Paris under the pressure of 760 millimetres of mercury; this is equivalent in length to 29·922 inches. But gravity is somewhat less at Paris than at London; and thus the pressure of 29·922 inches at Paris is equivalent to that of 29·914 inches at London. Hence the boiling point of the Centigrade thermometer is not quite the same temperature as the boiling point of Fahrenheit's thermometer, defined in the manner stated above, but is a trifle higher.

602. We estimate a change of temperature by the change of volume which accompanies it. Now liquids in general are more convenient than solids or gases for the purpose of a thermometer, since solids expand too little, and gases expand too much, by the application of a moderate degree of heat. Mercury has the advantage of affording a large range of temperature; it freezes about 40^0 C. below the zero of the centigrade thermometer, and boils at 350^0 C. The mercury thermometer cannot however be depended on very close to these limits; for, as is the case with other liquids, the expansion or contraction of mercury is not quite uniform when the temperature is such that the mercury is about to change its state for that of a solid or a vapour. Mercury has also the advantage of being very susceptible to the influence of heat; that is a comparatively small amount of heat is required to raise the temperature of an assigned quantity of mercury one degree.

603. When the freezing point and the boiling point are marked on the tube of a thermometer the graduations can be immediately recorded if we are sure that the tube is of uniform bore: but as this may be doubtful it is necessary to test it before the bulb and the tube are filled with mercury. Accordingly a small quantity of mercury is introduced into the tube and moved gently along it: if this occupies the same length, as for example an inch, at every part of the tube, we may infer that the bore is uniform. But if the mercury occupies different lengths at different parts of the tube we conclude that the bore is not uniform: but even then we may make use of the tube by

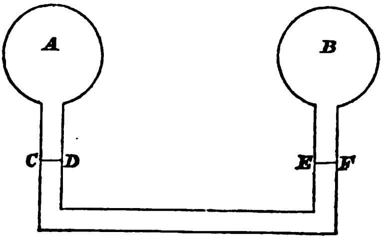
marking off lengths along it which the same small quantity of mercury will successively occupy.

604. The mercury thermometer will not measure temperatures which are more than 40° C. below 0° C., as mercury freezes at that temperature. For lower temperatures a thermometer is used in which instead of mercury coloured alcohol is used. Alcohol boils at about 78° C., but it does not become solid at any known degree of cold. An alcohol thermometer is graduated by putting it, together with a mercury thermometer, into a bath which can be raised successively to various temperatures; then the alcohol thermometer is marked with the same degrees as the mercury thermometer exhibits: by this means we can obtain the graduation of the alcohol thermometer through the range which it has in common with the mercury thermometer, and it must be continued below this by proportion.

605. *Weight Thermometer.* A thermometer is sometimes used called the *Weight Thermometer*. Instead of the *long* closed stem of the common thermometer this has a *short open* stem; thus when the mercury expands some of it runs out: this is carefully collected in a cup, and the amount of it is a measure of the excess of the temperature above that for which the instrument just remains full.

606. *Air Thermometer.* Any contrivance by which we can observe the volume of an assigned mass of air under a constant pressure may be called an *air thermometer*: for the volume would be known by Art. 593 if the temperature were given, and so on the other hand if the volume is known the temperature can be inferred. An instrument was invented by Leslie for detecting the slight differences in the temperatures of neighbouring objects, which is called *Leslie's air thermometer*, and sometimes the *differential thermometer*. A and B are hollow glass globes containing air; they are connected by a tube bent twice at right angles. A coloured liquid is placed in this tube so as to occupy part of it, and when the air is at the same temperature in A as in B, this liquid will stand at the same level in the two uprights, which we denote by CD and EF. Suppose however that the globe A is brought

into contact with a body warmer than itself; then the air in A expands, and so its elastic force increases, and consequently the level CD sinks and EF rises. The instrument



may be graduated by bringing A and B into contact with bodies which have a known difference of temperature, and marking the level of the liquid in either of the upright tubes. *Rumford's thermoscope* is substantially the same instrument as this.

607. *Breguet's Metallic Thermometer.* This is a very delicate thermometer which depends on the different rates of expansion of three different metals. Three strips are taken, one of platinum, one of gold, and one of silver; these are pressed together in a rolling mill so as to form a metallic ribbon, and then coiled like a screw. One end is attached to the top of a fixed upright rod, and the other end is attached to a light copper needle. The silver is the most expansible metal, and forms the inner face of the coil; the platinum is the least expansible and forms the outer face of the coil; the gold is intermediate in expansibility between the other two, and occupies the middle place in the coil. When the temperature rises the coil unwinds, and the needle moves round in the direction opposite to that in which the hands of a watch move; when the temperature sinks the coil twists up, and the needle moves round in the same direction as the hands of a watch move. The graduation is effected by comparison with a mercury thermometer.

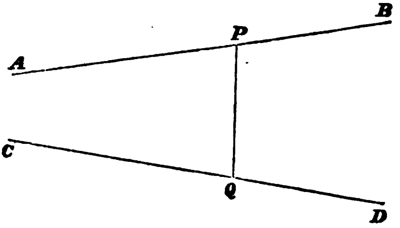
608. The *Thermopile* is an instrument which is now much used in scientific investigations for indicating minute

differences of temperature. The theory of the instrument cannot be understood without some knowledge of electricity and magnetism; but the main fact is, that if bars of two different metals are soldered together and heat is applied to the junction a current of electricity is generated. The current is conveyed through a wire which is coiled round a space enclosing a magnetic needle; and the strength of the current is shewn by the extent to which the needle is moved aside from the position it would otherwise occupy: from the strength of the current the amount of the heating is inferred. Properly speaking the name *thermo-electric pile* is given to the part of the instrument which excites the current of electricity, and the name *galvanometer* to the part which measures it; then the whole is called the *thermo multiplier:* but for shortness the whole is usually called the *thermopile.*

609. *Register Thermometers.* These are thermometers which are designed to leave a record of the highest or the lowest temperature which has occurred during an interval when the instrument was not constantly observed; thus, for example, it may be proposed to have a record of the highest temperature during the course of a day, or of the lowest temperature during the course of a night. To record the *highest* temperature a mercury thermometer is used which has a small piece of iron in the tube. The thermometer is placed with its tube horizontal, and then as the mercury expands the piece of iron is pushed on to the point marking the highest temperature; as the temperature declines the mercury contracts and leaves the piece of iron behind in the place to which it had been pushed. To record the *lowest* temperature an alcohol thermometer is used which has a small glass cylinder in the tube. The thermometer is placed with its tube horizontal, and when the temperature declines the glass cylinder is carried by the alcohol as it shrinks to the point marking the lowest temperature; when the temperature rises the alcohol passes between the cylinder and the tube, leaving the former in the place to which it had retired.

610. *Pyrometers.* The name *thermometer* is given to instruments by which temperatures lying within moderate

limits are estimated; the name *pyrometer* is given to instruments designed to estimate very high temperatures. Various contrivances for this purpose have been suggested, but they do not seem to yield very accurate results. We will describe one which is called *Wedgewood's Pyrometer.* AB and CD are bars of brass about two feet long, fixed so



that the distance of the ends B and D apart is ·5 of an inch, and the distance of the ends A and C apart is ·3 of an inch. Cylinders are formed of a certain kind of clay which are just big enough to fit the end BD at the temperature 212° F. These clay cylinders have the curious property of shrinking when exposed to heat, the more so as the heat is increased; hence when such a cylinder PQ after being exposed to a strong heat is placed between the fixed brass bars we may judge of the degree of heat by noticing the point at which the cylinder will remain in contact with both the bars. But there is no mode of graduating the instrument with precision; and even if it were accurately graduated with respect to one clay cylinder there would be no security that the constitution of a second clay cylinder would be the same as that of the first, or that the same graduation would serve for both. The exceptional property which the clay possesses of *contracting* as the temperature rises seems due to the fact that the clay at a low temperature contains much moisture, which is expelled by strong heat; and thus as some of the ingredients disappear the cylinder may become smaller, though the solid parts may be really expanded under the influence of heat.

LVIII. MELTING OR FUSING.

611. The only effect of heat to which we have hitherto paid attention is that of changing the volume of substances to which it is applied; but we shall now consider the important transformations of state which are produced by it, namely from the solid to the liquid state, and from the liquid to the gaseous. In the present Chapter we shall confine ourselves to the first of these transformations.

612. It is a general principle that solid bodies can be made to take the liquid form by raising the temperature to a suitable height: this temperature is believed to be always the same for the same body, though different experimentalists may obtain values of it which differ slightly. Thus ice passes into the liquid state at the temperature 0^0 C., tin at 230^0 C., and lead at 330^0 C. This transformation of state is called *melting* or *fusing*; the term fusing is usually applied to bodies, as metals, which require a very high temperature in order to make them become liquid. The term *liquefaction* is also used; but this may express either the change of a solid to a liquid, or the change of a gaseous body to a liquid. Some substances however do not *fuse*, however high the temperature to which they may be exposed, but become decomposed; such are paper and wood. Many substances were formerly considered to be incapable of fusion, and were called *refractory;* but the number is gradually diminishing as means are found of producing higher temperatures.

613. *While the process of melting goes on the temperature does not rise.* Suppose a mass of ice at the temperature 0^0 to be placed in a vessel, and the vessel immersed in a bath of hot water. The ice gradually melts, obviously owing to the heat received from the hot water; for a thermometer placed in the hot water shews that the

temperature of it gradually declines, so that we may be certain heat passes into the ice. But a thermometer in contact with the melting ice does not indicate any rise of temperature *until* the ice is all melted: after this the resulting water gradually rises in temperature. In like manner we may put sulphur in a glass vessel and heat it; the temperature of the sulphur will gradually rise until it reaches about 110° C.; then the sulphur begins to melt and the temperature does not rise until all the sulphur is melted.

614. *Latent heat.* Thus while a solid is fusing heat may be supplied which does not affect the thermometer, so that it may be said to be not sensible: it is briefly called *latent heat.* There are various ways of presenting the fact which these words are intended to state; the following is very simple and decisive. Let a pound of water at 0° C. be mixed with a pound of water at 80° C.; the mixture will be found to have the temperature of 40° C.: the hotter pound has given up heat to the colder pound, and the temperature of one is raised as much as that of the other is lowered. But suppose a pound of *ice* at 0° C. to be mixed with a pound of water at 80° C.; the ice will melt, and then the temperature of the whole mass of water will be 0° C.; thus while the hot water has lost 80° of temperature, the ice has been converted into water without any elevation of temperature. Hence we see that heat may be considered to have become *latent* in the melted ice; and that the amount of it is enough to raise the temperature of a weight of water equal to that of the ice from 0° C. to 80° C. We have used 80° C. for simplicity, but the number should be more exactly stated as 79°·25 C., for which 79° C. is usually taken: the corresponding number in Fahrenheit's scale is 142°.

615. In like manner other solids in fusing absorb heat which becomes latent, though the quantity is not so great as in the case of water. The following Table exhibits for various substances the amount of latent heat in Centigrade degrees; the temperature at which fusion takes place is also recorded in Centigrade degrees:

	Latent heat.	Temperature at fusion.
Water	79·25	0
Silver	21	1000
Tin	14	228
Sulphur	9 4	114
Lead	5·4	330
Phosphorus	5	44
Mercury	2·8	40 *below* 0° C.

616. Now let us consider the opposite process to fusion, namely, that which is called solidification or congelation. Let the temperature of water be gradually lowered until it reaches 0° C.; then the water begins to freeze: and as in melting a large quantity of heat is absorbed and becomes latent, so in freezing a large quantity of heat is dismissed; this may be verified by experiment. Let a quantity of water at 30° C. be placed in a vessel, and immersed in a large bath of mercury which is 10° C. below 0° C.; and let one thermometer be put in the vessel of water, and one in the vessel of mercury. These thermometers will shew that the temperature of the water gradually declines, while that of the mercury rises, until the water is at the temperature 0° C.; the water then begins to freeze, and its temperature remains unchanged until the whole is frozen, but the temperature of the mercury *continues to rise:* thus we see that heat is still passing from the freezing water though its temperature does not decline.

617. The number which expresses the *latent heat* when ice melts is much greater than the corresponding number for other substances; thus it may be said that in a certain sense water is the most difficult of all substances to freeze, and this extreme difficulty is a fact of great importance in the constitution of nature. Suppose water to cool down gradually until it arrives at the temperaturo 0° C.; then small portions of ice are formed, and the heat dismissed from them passes into the surrounding water and elevates its temperature slightly, thus retarding for a short time the freezing of more water. Similarly from the freezing of another portion heat is obtained to elevate the temperature of the rest; and so on. Thus the freezing of a large mass of water, like a lake or a river, is rendered

a much more gradual process than it would otherwise be. It is obvious that were it not for the strong influence exerted by the principle of latent heat, the sudden changes of temperature which occur would occasion much inconvenience to man that is now avoided. A thaw after a long frost would produce almost a deluge by the sudden conversion of masses of ice into water; and on the other hand a frost without warning might almost instantaneously congeal lakes and reservoirs. We shall see hereafter that the principle of *latent heat* applies also when liquids are converted into gaseous bodies; and the following remarks on the principle in both its applications are given by Dr Whewell in his *Bridgewater Treatise:* "The consequences of this property of *latent heat* are very important. It is on this account that the changes now spoken of necessarily occupy a considerable time. Each part in succession must have a proper degree of heat applied to it. If it were otherwise, thaw and evaporation must be instantaneous: at the first touch of warmth all the snow which lies on the ground would rush like an inundation into the water-courses. The hut of the Esquimaux would vanish like a house in a pantomime: the icy floor of the river would be gone without giving any warning to the skater or the traveller: and when, in heating our water, we reached the boiling point, the whole fluid would *flash into steam* (*to use the expression of the engineers*) and dissipate itself in the atmosphere, or settle as dew on the neighbouring objects."

618. Although we say that water freezes at 0^0 C., yet water may in various ways have its temperature reduced considerably below that point without freezing. Thus, if a vessel of water be covered and kept perfectly still the temperature may be reduced to 5^0 C. below 0^0 C. without the freezing of the water. But then if any motion be given to the water, or if a solid be dropped into it, part of the water freezes, and the temperature of the whole rises to 0^0 C. The change of temperature arises from the fact that the water which freezes parts with its latent heat. In this case the ice would be found to constitute about one-sixteenth of the whole original mass of water. For the latent heat dismissed would be enough to raise

the ice alone through 80^0 C., as we know from Art. 614; so that if the whole mass is raised through 5^0 C. the inference is that the weight of the whole mass is $\frac{80}{5}$ times the weight of the ice, that is, 16 times the weight of the ice.

619. The freezing point of water is slightly lowered when the pressure on the surface of the water is increased; the amount is found to be one-seventieth part of a degree of Fahrenheit for every additional pressure equal to that of the atmosphere. It is known that when two pieces of thawing ice are placed in contact they freeze together: this fact is expressed by the word *regelation.* This has been connected by some writers with the fact of the lowering of the freezing point of water under pressure in a manner which we shall briefly indicate. It will be seen in the next Article that water in freezing undergoes an increase of bulk. It is found that if pressure is applied to a vessel which contains any substance, partly in the liquid and partly in the solid state, that resembles water in having its freezing point lowered by pressure, then some of the solid portion is melted and the temperature of the whole sinks to the freezing point which corresponds to the increased pressure. So when two pieces of ice at the thawing point are held together, the pressure causes some melting to take place at the surfaces in contact; the water thus obtained passes out of the way; the temperature declines; and then as soon as the pressure is diminished the two parts are frozen together at a temperature slightly below 0^0 C.

620. *Change of volume.* In the act of congelation there is usually a sudden and considerable change of volume. We have already said that water expands as it cools from 4^0 C. to 0^0 C.; but this is a gradual process and small in extent: when, however, the water freezes there is a sudden expansion to the extent of nearly one-thirteenth of the bulk, so that 13 cubic inches of water become 14 cubic inches of ice. This sudden expansion is effected with great force, and thus often leads to troublesome accidents: if water is left in decanters in

cold weather the decanters may be broken when the water freezes; and the pipes which convey water through houses are frequently burst during severe frosts. On the other hand important advantages are secured by this property of water; ice formed in lakes and rivers rises to the surface, and thus serves as a screen to protect the rest of the water from the cold, and so preserves it unfrozen. Moreover the ice, being at the surface, is exposed to the warmth of the sun and air, and thus will not remain long when the frost disappears.

621. Most metals undergo a sudden *contraction* in passing from the liquid to the solid state; there are however three exceptions, cast iron, bismuth, and antimony, which *expand* at this change of state. Money cannot be *cast* from gold, silver, or copper, because of their contraction on becoming solid; and therefore it must be *stamped.* Mercury in freezing undergoes a sudden and marked contraction; the earliest observers of the freezing of mercury were unaware of this, and so they estimated the temperature according to the average contraction with which they were familiar, and consequently supposed the cold to be much more intense than it really was. But it is now ascertained by careful experiment that the freezing point of mercury is 40° C. below 0° C.

622. *Freezing Mixtures.* By mixing two or more special substances together a very intense degree of cold can be procured. The simplest example is that in which common salt is mixed with pounded ice. Suppose equal weights of these two substances to be rapidly mixed together at the temperature of the freezing point; then it is found that the temperature will sink as low as 32° F. below the freezing point: snow may be used instead of pounded ice. This was the extreme cold which was attainable in Fahrenheit's days, and he took it for the starting point of the scale in his thermometer. At the present day, however, chemists are able to reach a much lower temperature: for instance, by mixing three parts by weight of snow with two parts by weight of dilute sulphuric acid at the temperature of 32° F. a temperature is obtained which is 23° F. below 0° F. The action of salt on ice may be seen in the common practice of throwing

salt on the ice or frozen snow on the pavement in order to clean it; the salt combines quickly with the ice or snow and forms a liquid brine, which either runs off into the gutter or can be easily removed: the temperature of the pavement may be much lowered during the process, but this is of no importance with respect to the object in view.

623. When a freezing mixture has been obtained we may by its aid cool down to a very low temperature the materials used for composing another freezing mixture, and thus reach a still lower temperature. For example, if equal weights of snow and dilute sulphuric acid be mixed at the temperature of 20° F. below 0° F., the temperature of the mixture will sink to 60° F. below 0° F.

LIX. BOILING.

624. We have seen that, if the temperature of ice is gradually raised, the ice melts and assumes the form of water when the temperature is 0° C.; we proceed to consider what takes place when the temperature is still further raised. The facts are to some extent familiar to us: we know that if the water is placed in a vessel exposed to the atmosphere, and heat applied, the temperature gradually rises until it reaches 100° C.; the water then passes off in the form of steam at that temperature, and the temperature of the water continues *stationary*.

625. It has been found that water boils under the same pressure at slightly different temperatures in vessels of different materials; the temperature is somewhat higher in a glass vessel than in a metal vessel: it appears however that the temperature of the *steam* is the same whatever may be the nature of the vessel. The fact that the temperature continues *stationary* while water is boiled in a vessel open to the atmosphere, explains why vessels composed of metals which are easily fusible may be safely exposed to a very fierce fire provided they contain water; for the water remains at the temperature 100° C., and by its contact with the vessel prevents the metal from rising to a dangerous temperature.

626. The fact that the temperature continues stationary while water is boiling suggests to us, in analogy with what we have seen in considering fusion, that heat must be absorbed and become *latent* in the transformation of water to steam; and this suggestion is readily confirmed by experiment. It is found that the number which represents the amount of this latent heat is 540° C., or roughly about 1000° F. That is, the heat which is absorbed in converting a pound of water to steam at the temperature of 100° C. would be sufficient to raise the temperature of 540 pounds of water 1° C.; and on the other hand, if the steam were condensed into water of the same temperature it would give out this quantity of heat. Or the fact may be stated thus: if steam be condensed into water it gives out as much heat as would raise the temperature of a thousand times its own weight of steam 1° C.: we shall see in the Chapter on *Specific Heat* that the two statements as to the amount of heat are equivalent

627. One mode of performing the experiment may be noticed, because it gives a very clear idea of the fact involved, although it is not susceptible of great accuracy. Some water was placed in a vessel over a fire, and it was observed that in four minutes the temperature was raised from 50° F. to 212° F., that is through 162° F.; then in twenty minutes more the whole water boiled away, that is was converted into steam at the temperature of 212° F. Let us assume that during the twenty minutes five times as much heat was supplied as during the preceding four minutes; then heat measured by $162^{\circ} \times 5$ F., that is by 810° F., has been supplied to the water and become latent in the steam. The amount here differs much from the standard 1000° F. which we have stated to be approximately the proper result; but this is not strange considering the rough nature of the process. We may also conduct the experiment in the same way as in Art. 614 with respect to fusion. Let eleven ounces of water be taken at the temperature 0° C., and put into a vessel; and let steam arising from water boiling at 100° C. be allowed to enter by a pipe into this vessel: the steam becomes condensed and parts with heat, while the temperature of the water rises until it becomes 100° C. Then let the supply of steam be stopped,

and the water in the vessel weighed; it will be found to be about thirteen ounces. Hence we have the following fact: steam weighing two ounces at 100° C. has been condensed into water at the same temperature, and this has raised the temperature of eleven ounces of water from 0° C. to 100° C. Thus the number which expresses the amount of the latent heat of steam is $\frac{11}{2} \times 100°$ C., that is 550° C.

628. Water in passing into the gaseous form and becoming steam receives an enormous increase of volume. A cubic inch of water becomes about 1700 cubic inches of steam, that is, nearly a cubic foot.

629. We have hitherto supposed that water boils under the ordinary pressure of the atmosphere; and it is found that so long as this pressure remains unchanged the *boiling point* is a fixed temperature. But if the pressure alters the boiling point alters, rising if the pressure is increased, and falling if the pressure is diminished. This statement is easily verified by experiment, for by means of an air pump the pressure of the atmosphere may be diminished, or almost removed. By ascending a high mountain the pressure of the atmosphere is much diminished, and consequently water boils at a lower temperature. It is found that if the pressure is about half the ordinary pressure of the atmosphere water boils at 180° F.; and if the pressure is double that of the atmosphere water boils at 250° F. A rough idea of the height of a mountain may be obtained by observing the temperature at which water boils; if the height be a moderate quantity it may be found roughly in feet by multiplying 600 by the number of degrees of Fahrenheit *below* 212 at which water boils.

630. The power of altering the boiling point of water by altering the pressure on the water is one of great importance in the arts. Thus in the preparation of various vegetable extracts some of the chemical compounds would be partially or wholly decomposed at the temperature of 100° C.; and the operation is therefore carried on under a diminished pressure, and therefore at a much lower temperature. The process of sugar refining is an example

of this. On the other hand, a higher temperature than 100° C. is sometimes required in water, as for instance in extracting gelatine from bones. This is obtained by boiling the water in a close vessel, in which the pressure, and consequently the temperature, may be raised to a high pitch. Such a vessel is called a *Papin's Digester:* it has a *safety valve,* like that of the boiler of a steam engine, which opens spontaneously when the pressure reaches a dangerous point, and allows steam to escape.

631. Steam itself is invisible like common air; apparently we see the steam issuing from a steam-engine, or from the spout of a boiling kettle, in the form of a light cloud: but this cloud arises from the condensation of steam as it comes into contact with the colder atmosphere. What we see then is not steam, but particles of water; if steam be kept in a heated glass vessel it remains invisible.

632. The number 540° C., which expresses the latent heat of steam, supposes the water to boil under the ordinary pressure of the atmosphere. If the steam is produced at a temperature differing from 100° C. the quantity of latent heat changes. Watt came to the conclusion that the sum of the latent heat and the sensible heat is always the same. Thus if the sensible heat is denoted by 100° C. the latent heat is 540°, so that the sum is 640° C. Hence, according to this law, it follows that if water boils at 50° C. the latent heat must be 590° C., so that the sum may be 640° C. But Watt's conclusion has not been confirmed by more recent investigators. Regnault finds that the sum of the latent heat and the sensible heat is not constant, but is equal to the sum of two terms; one term is the constant 606°·5 C., and the other is the product of ·305 into the Centigrade temperature of the water from which the steam is produced.

633. Other liquids exhibit the same phenomena as water under the continued application of heat; namely, the temperature rises to a certain point, and then remains stationary while the liquid is converted into a vapour. The following Table gives in Centigrade degrees the boiling points of some liquids, and their latent heats when converted into vapour.

	Boiling Point.	Latent Heat.
Ether	37	90
Alcohol	78	208
Water	100	540
Turpentine	160	74

634. By continually imparting heat to a liquid it is finally transformed to a state of vapour; so on the other hand, if the temperature of any vapour which has been raised from a liquid is continually reduced that vapour will return to the liquid state. In returning to the liquid state heat is given out equal in amount to that which becomes latent in passing from the liquid state to that of vapour.

LX. THE THREE STATES.

635. There is ground for believing that any body may pass through the three states, solid, liquid and gaseous; though there are bodies which at present have not been put into all these states. Thus there are solids which have not been fused; and there is one liquid, namely alcohol, which has not been frozen. There are also a few gases which as yet experimenters have not been able to transform into the other states, namely, oxygen, hydrogen, nitrogen, nitric oxide, carbonic oxide, and marsh gas; hence they are sometimes called *permanent gases.*

636. Bodies in the aeriform state are divided into two classes, vapours and gases. Vapours are those which are known to have been raised from liquids by the application of heat, and which can be reduced again to liquids by the abstraction of heat. Gases, strictly so called, are not known to have been raised from liquids, and though we may anticipate that they are reducible to the liquid form, yet, as we have stated in the preceding Article, there are some which up to the present time have not been so reduced. On the other hand, various gases, not originally known to have arisen from liquids, have been reduced to the liquid form by enclosing them in strong vessels and subjecting them to enormous pressure.

637. Speaking generally we may consider that the state, out of the three, in which a body appears, is mainly dependent on the temperature. Thus an inhabitant of a tropical climate might pass his life without even becoming acquainted with the fact that water can become solid. Again, ether boils at a temperature below 100° F.; and hence at some seasons it would not exist as a liquid in hot countries, unless it were confined in a close vessel.

638. If the distance of the earth from the sun were very much greater or very much less than it really is the condition of things on its surface would be extremely different from that which really exists. It is conceivable that the heat received from the sun might be so great as to convert the most stubborn metals into vapours; it is conceivable, on the other hand, that the heat might be so little that every liquid would be frozen, and the ocean form one solid mass: and it is scarcely extravagant to imagine that the atmosphere itself should become a thin hard crust.

639. Metals are sometimes combined to form a compound which is called an *alloy;* and it is found that the point of fusion of an alloy is apparently quite independent of the points of fusion of the combined metals. Thus, take for example the compound called *Rose's fusible metal*, which we have already noticed in Art. 577: this compound is formed of lead, bismuth and tin, and fuses at about 200° F.: lead alone fuses at 626° F., bismuth at 508° F., and tin at 442° F.

640. It belongs to chemistry to exhibit the influence of heat in separating a compound body into the elements which compose it: we will merely illustrate the subject by two examples. Suppose salt to be dissolved in water; we thus obtain a mixture the composition of which is known: we may verify the nature of the composition, or we might discover it if unknown, by the following process. Let the mixture be put into a vessel and subjected to the action of heat; it will at length boil, and the vapour given off can be collected and condensed in another vessel. The process may be continued until no liquid remains in the vessel

exposed to heat; then it will be found that the condensed vapour is pure water, and nothing is left in the vessel which originally contained the mixture, except crystals of salt: the weight of these will be equal to the weight of the salt originally put into the water. Again, let alcohol be boiled until it is converted into a vapour; then if the vapour is made to pass through a tube of porcelain heated red hot, it is found that *carbon* is deposited in the tube: thus carbon is shewn to be a constituent of alcohol vapour, and therefore of alcohol itself.

641. Some bodies pass from the solid to the gaseous state without going through the intermediate state of a liquid. Thus iodine when solid is a ruby-coloured crystal, and if heat is applied it becomes a gas of the same colour.

642. Some experiments and reasoning seem to shew that there is not a sharp distinction between the state of a liquid and the state of a vapour. In 1822 Cagniard de la Tour enclosed liquid in a glass tube so as almost to fill the tube. The tube was then exposed to a great heat; up to a certain point of temperature the substance was partly liquid and partly vapour, and then suddenly became uniform in appearance, where there was no evidence of any distinction of state. More recently Dr Andrews has examined the behaviour of carbonic acid under various pressures and temperatures; and from his experiments and those of Cagniard de la Tour he infers that "the gaseous and liquid states are only widely separated forms of the same condition of matter, and may be made to pass from one into the other without any interruption or breach of continuity": see Professor Maxwell's *Theory of Heat*.

LXI. VAPOURS.

643. We have hitherto supposed that vapour is obtained by *boiling* a liquid, but it is a fact that vapour is produced from the surface of a liquid at *any temperature*. Suppose that a small quantity of liquid is introduced into a vessel from which the air has been withdrawn, as the exhausted receiver of an air-pump; then the vessel is filled almost

instantaneously with vapour, and the pressure and the density of this vapour are found to depend only on the temperature, so long as the whole of the liquid is not converted into vapour. If the space in which the liquid is placed is increased more vapour is formed; if the space is diminished some of the vapour already formed is condensed into liquid: but the pressure and the density remain the same in both cases so long as the temperature remains unchanged. If the temperature be raised more of the liquid becomes vapour, and the pressure and the density are increased, if the temperature be lowered some of the vapour returns to the liquid state, and the pressure and the density of the rest are diminished. Vapour may also be derived from solid bodies; thus ice gives forth aqueous vapour.

644. The pressures exerted at different temperatures by the vapours of various liquids in contact with the liquids from which they proceed have been determined by experiments and recorded in Tables; but no simple law has yet been discovered by which the pressure can be immediately determined as soon as the temperature is known. An important fact which is obtained from the experiments may be thus stated: *let two vessels which are partly filled with the same liquid at different temperatures be connected; then the pressure of the vapour will become the same throughout, namely, that which corresponds to the lower temperature.* For example, let one vessel contain water at 100^0 C. together with corresponding vapour; then the pressure of this vapour will be denoted by 30 inches of mercury. Let another vessel contain water at 0^0 C. together with corresponding vapour; then the pressure of this vapour is found by experiment to be about one-fifth of an inch of mercury. Now let a communication be opened between the parts of the two vessels which contain vapour; then the hotter vapour passes towards the colder vessel, and is there condensed, and the whole vapour takes the temperature 0^0 C. and the corresponding pressure.

645. Experiments in order to form such Tables as those mentioned in the preceding Article were first made on a large scale by Dalton, in a manner we will now describe. Take a glass tube a yard long, as in the process

for constructing a barometer, explained in Vol. I. Art. 489; fill it nearly with mercury, and the rest of it with any liquid: put the finger at the open end just above the liquid, invert the whole, and put the open end under the surface of mercury in a vessel. Then the liquid by reason of its inferior density rises to the top. Thus instead of a true barometer we have the pressure of the atmosphere supporting a column of mercury and the liquid above it; so that the height of the column of mercury alone will be less than that of a true barometer at the same time and place. If the liquid did not become vapour the difference of heights would be just equal to the height of a column of mercury equivalent to that of the liquid; this can be calculated if we know the quantity of the liquid and its specific gravity, and will be almost insensible if that quantity be so small that the liquid is a mere film. But practically the liquid becomes wholly or partially vapour, and then its pressure exerts a sensible influence on the height of the mercury; and thus this pressure becomes known. For example, if the mercury is depressed 2 inches, this shews that the pressure of the vapour is measured by $\frac{2}{30}$ of the ordinary pressure of the atmosphere. The part of the tube which is above the surface of the mercury, together with that which contains the upper portion of the mercury, may be surrounded by hot water, so as to raise the temperature to any desired point, and thus the pressure of the vapour may be determined for various temperatures. In this form of the experiment the pressure of the vapour will always be *less* than the pressure of the atmosphere; but it is easy to arrange so as to meet the case of a pressure greater than this: see Vol. I. Art. 494.

646. When liquid is introduced into a vessel which contains air the same process of producing vapour from the liquid goes on, but more slowly: the quantity of liquid converted into vapour is finally the same as if there had been no air in the vessel. When as much vapour has been formed as can be raised from the liquid at that temperature, the air is said to be *saturated* with the vapour; or if there be no air the vapour may be said to be in a saturated condition.

647. Vapour in the *saturated* condition differs much from a permanent gas; for, as we have seen in Art. 643, the pressure and the density of the vapour remain constant so long as the temperature does. But vapours in the unsaturated condition resemble the gases in this respect, that they obey the laws of Boyle and Charles. Hence we may say with respect to any vapour, so long as it remains in that state without condensing wholly or partially to a liquid, that the *connexion between the pressure the density and the temperature is the same as for air or any other permanent gas.* For example, vapour raised from water at a temperature of 180° F. exerts a pressure measured by 15 inches of mercury: find what pressure it will exert if the temperature be raised to 200° F. without any increase of volume. By Art. 593 the pressure will be obtained if we multiply 15 inches by $1+\frac{168}{180}\times\cdot 3665$, and divide by $1+\frac{148}{180}\times\cdot 3665$; hence it is about $15\times\frac{1\cdot 34}{1\cdot 30}$, that is 15·5 nearly. We know however that neither Boyle's Law nor Charles's Law is absolutely correct: see Art. 594, and Vol. I. Art. 497; and it is found that steam and other vapours conform more strictly to those Laws at high temperatures than at low temperatures.

648. The density of aqueous vapour when in a saturated condition is found to be ·6225 of the density of air at the same temperature and pressure: this decimal may be taken equal to $\frac{5}{8}$ approximately.

649. Suppose that a vapour is mixed with air or a gas, then the whole pressure is the sum of the pressures of the air and the vapour. The pressure of the air is the same as that of the atmosphere if the density is the same, and for any other density is proportional to that density. The pressure of the vapour can be found by the aid of such Tables as are mentioned in Art. 644. But although this is the *whole* pressure, yet the pressure which limits the amount of vapour formed is simply the pressure of the vapour itself, as we have stated in Art. 646, and is quite

independent of the amount of air in the vessel. Thus it is only the pressure of its *own* atmosphere so to speak, which prevents a volatile substance from dissipation. If we put camphor, musk, or spirits in an *uncorked* bottle, they soon disappear in the form of vapour, as the pressure of the air has no influence in retaining them; but if the bottle be corked, the vapour which arises from the liquid exerts a pressure that preserves the rest of the substance in the liquid form.

650. The fact involved in the preceding Article respecting the mixture of *air* and a *vapour* is sometimes expressed by saying that each behaves *as a vacuum* with respect to the other. The fact is one which obviously holds with respect to a mixture of one portion of air with another. Suppose a certain amount of air in a closed vessel; then the air will exert a certain pressure; add just as much air then the *whole* pressure is doubled, but *each portion* may be said to exert just as much pressure as before, and retains the same bulk. The statement respecting the *whole* pressure when air or gas is mixed with vapour does not hold for the mixture of two vapours produced from a mixture of two liquids: the pressure is then in general intermediate between those which correspond to the two vapours singly.

651. Suppose that a solid surrounded by a mixture of air and aqueous vapour has its temperature gradually lowered; then the stratum of air in contact with the solid at last becomes so cool that the vapour in the air will not all remain in this state, but some of it is condensed in the form of a *dew* on the body. Then if the temperature of the body is gradually raised this dew returns again to the form of vapour. The lowest temperature at which the whole of the vapour in a mixture of air and vapour will remain in the form of a vapour is called the *dew point:* it may be determined in the way just suggested. Let a body be cooled down gradually until dew begins to be deposited on its surface; next let the body be warmed until the dew is displaced: the temperature in the former case will be a little lower than the dew point, and the temperature in the latter case a little higher, so that

the temperature midway between them may be taken for the dew point. The air is saturated with aqueous vapour when the temperature of the dew point is the same as that of the air; for then the air will hold no more of the vapour at that temperature.

652. Various instruments have been contrived for the purpose of determining the amount of vapour present in the atmosphere at a given time; such instruments are called *Hygrometers*. They mostly depend on the same principle; by the aid of them we ascertain the dew point, and then by consulting Tables which have been drawn up we learn the pressure of the aqueous vapour which corresponds to this point. We can then determine by calculation the amount of aqueous vapour which is present in a given space of the atmosphere. We can also determine by the aid of the Tables the *proportion* which this amount bears to the amount which could exist in the atmosphere at the existing state of temperature. It is on this *proportion*, and not on the *absolute* amount of aqueous vapour, that our sensation of the state of the atmosphere depends: we find the atmosphere moist or dry according as this proportion is high or low.

LXII. EVAPORATION.

653. When a liquid is exposed to the atmosphere vapour is continually formed, and passes away; if there be a current of wind over the surface of the liquid the vapour is removed more quickly and a fresh supply raised. Also the larger the surface of the liquid exposed the greater is the quantity of vapour produced; so that liquid disappears more rapidly if placed in a broad shallow vessel, than if placed in a narrow deep vessel. The process goes on over the large masses of water which are found on the globe in the form of oceans, lakes and rivers. The influence thus exerted on animal and vegetable life is very considerable; and it is usual to discuss the circumstances under the title of *Evaporation*.

654. The subject of evaporation was much studied by Dalton, and he obtained by experiment some interesting results. He began by investigating the rate of evaporation from water kept at a temperature much above that of the surrounding atmosphere. A small vessel containing water was hung from one end of a balance, and a lamp, placed under the vessel, maintained the temperature of the water at an assigned point; the rate of evaporation was determined by the aid of the balance, for that indicated at any instant the weight of the water remaining in the vessel. The nature of the results obtained will be seen from the following selections from them:

212	30	30
164	10·4	10
138	5·4	5

The first column gives the temperature of the water in degrees Fahrenheit, the second column gives the elastic force or pressure of the vapour in inches of mercury, and the third column gives the amount of vapour as measured by the weight of water in grains lost from the vessel. By comparing the second and third columns it follows that the rate of evaporation is, at least nearly, proportional to the elastic force of the vapour produced.

655. The atmosphere always contains vapour mixed with it, but the elastic force of such vapour is small when compared with that of the vapour raised at temperatures like those of the preceding Article; hence the vapour in the atmosphere exerts very little influence in obstructing evaporation at *high* temperatures, and with respect to such evaporation the air may be considered as quite dry. But when evaporation takes place at *low* temperatures the extent of it is much checked by the presence of the vapour already in the atmosphere, and Dalton obtained the general result that the rate of evaporation is then proportional to the difference between the elastic force of the vapour produced and the elastic force of the vapour contained in the atmosphere. This result may be held to include that of the preceding Article, because the elastic force of the vapour contained in the atmosphere is usually practically insensible compared with that of the

vapour formed at high temperatures. The elastic force of the vapour contained in the atmosphere is found in the manner suggested in the preceding Chapter; the dew point is ascertained by observation, and then the elastic force inferred by the aid of tables.

656. Evaporation is promoted by the passage of a current of air over the surface of the liquid which is producing the vapour; for by the aid of this current the vapour is removed to a great extent as fast as it is produced, and the space above the liquid is made to resemble dry air. Dalton found that under the influence of a current of air 45 grains of water were evaporated in the same time as 30 grains were evaporated when the air was still.

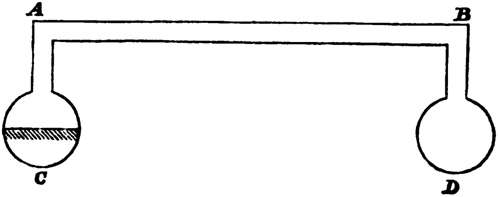
657. The general result obtained by Dalton, and given in Art. 655, was found by him to hold with respect to other vapours as well as to the vapour of water. It follows that for liquids which boil at a very high temperature the amount of evaporation for ordinary low temperatures is very small. For example, water boils at 212° F., and we see by Art. 654 that when the temperature is lowered to 138° the evaporation is only about one-sixth of what it was at first: mercury boils at 660° F., and hence it is that at ordinary temperatures of the atmosphere the evaporation from the surface of mercury is practically insensible. Various exemplifications of the principles involved in evaporation occur in nature and the arts, and we will proceed to notice some of them.

658. Suppose two liquids are mixed together; they may form a compound in which the two components are feebly united, and then on the application of heat each liquid is converted into vapour, independently as it were of the other: the distillation of spirits depends on this principle. Suppose, for instance, that equal weights of water and alcohol are mixed together, and let the mixture be raised very nearly to the boiling point of alcohol: this is about 180° F. The density of the vapour of alcohol is about seven times that of the vapour of water at the same temperature; and thus in the vapour which arises from the mixture there is seven times as much alcohol as water by weight: therefore when the vapour is condensed a very *strong* spirit is produced.

659. In some processes of distillation it is necessary to keep the temperature as low as possible, because one of the elements which occur in the compound is injured if it is exposed to too great heat; in such a case the process may with advantage be carried on in a vacuum: see Art. 630. For example, it is found that in the distillation of vinegar it is impossible to avoid the introduction of a burnt flavour unless the process is effected at a low temperature; and this is accomplished by conducting it in a vacuum. In a similar manner, it is found in sugar refining that if the syrup is raised to too high a temperature in boiling it the sugar is spoiled; accordingly the process is generally carried on in a vacuum, and a large fortune was made by the person who first adopted this method.

660. We have seen that in the process of boiling a liquid a large quantity of heat becomes *latent* in the vapour which is formed; and so also when vapour is raised from a liquid at temperatures below that of the boiling point of the liquid much heat is carried away in a latent state. This lowering of the temperature of a liquid produced by the evaporation of part of it is applied to various useful purposes. The following experiment contrived by Leslie shews how we may procure ice by the aid of evaporation. A shallow vessel containing water is placed under the receiver of an air-pump; and also another containing strong sulphuric acid. The air is now exhausted, and in consequence vapour rises copiously from the water. If there were no sulphuric acid this vapour would soon have such an elastic force as to prevent the formation of more vapour. But the sulphuric acid has the property of absorbing the vapour of water, so that the vapour rapidly disappears and fresh vapour is continually formed. This formation of vapour requires much heat which must be obtained from the water, and finally the temperature is reduced so low that the water freezes. In India ice is obtained by putting water into pots of unglazed earthenware; these pots are thus porous and allow water to penetrate so as to keep the outside always moist: then if there be a copious evaporation from the surface the temperature is so much lowered that the water within the pot gets a crust of ice on its surface.

661. Dr Wollaston contrived a little instrument for exhibiting the cold produced by evaporation: this he called a *cryophorus*, a word which signifies *frost-bearer*. *AB* is a glass tube; and *C* and *D* are bulbs which are



placed on short branches attached to the tube. Some water is contained in one bulb, as *C*, and the rest of the inside of the instrument is, as nearly as possible, a vacuum. Let the bulb *D* be placed in a freezing mixture; then the vapour arising from the water in *C* is condensed in *D* as fast as it is formed: thus the temperature of the water in *C* is continually lowered, until at last the water freezes. This instrument is originally furnished with water in a manner like that in which mercury is introduced into the bulb and tube of a thermometer.

LXIII. SPECIFIC HEAT.

662. We have drawn attention to the change of volume which is produced when heat is applied to a body; and also to the change of state from the solid to the liquid, and from the liquid to the gaseous form, under the same agency: we now propose to consider how we may obtain an estimate of the quantity of heat required to effect such changes in various bodies.

663. In attempting to compare one quantity of heat with another it is obvious that we must endeavour to apply them so as to produce effects of the same kind, and make a numerical estimate of such effects, One of the most

convenient applications which can be made, is that of using the heat to *melt ice*. Take a vessel containing ice, and place in it the body on which we have to experiment, say a bar of iron at the temperature of 60° F., and observe how much ice has been melted when the temperature of the iron has fallen to 32° F. Then the amount of water thus obtained by the melting of the ice may be taken as a numerical estimate of the heat which the iron lost in sinking from 60° to 32° F. If instead of a bar of iron we take a bar of lead, of the same weight, it will be found that the amount of water melted will be scarcely a quarter of that melted by the iron. This simple example furnishes an easy illustration of the kind of information which can be derived from experiments on the subject of the present Chapter.

664. The remarks made in the preceding Article amount to an illustration of the general principle which applies to every measurable thing, namely, that such a thing must be measured by a *unit* of its own kind: see Vol. I. Art. 30. If then we wish to measure heat we must select a unit of heat, which is also termed a *thermal unit.* Thus we may take for this unit the amount of heat necessary to raise the temperature of one pound of water from 0° C. to 1° C.; or we may take for this unit the amount of heat necessary to melt one pound of ice at 0° C.: but having adopted our unit we must keep to it throughout any numerical calculation we may be making. It follows from Art. 614 that the second unit just mentioned is about 80 times as great as the first, for we may take it as a fact, obtained by experiment, that to raise the temperature of a mass of water from 0° C. to 80° C., about 80 times as much heat is necessary as would raise the temperature from 0° C. to 1° C. It must be observed that although rise in temperature is an effect of the application of heat, yet temperature and heat are distinct things: we see from an example given in the preceding Article that the same amount of heat may produce very different changes of temperature in two different bodies of the same weight.

665. We have spoken of applying heat to melt ice, but it is plain that some precautions will be necessary in

order to secure accurate results in estimating the amount of heat in this way. If we put a hot body into a vessel of ice, on the one hand some of the heat of the body may be spent in warming the vessel containing the ice, and the surrounding air; and on the other hand, if the temperature of the surrounding air is high some of the melting may be due to the heat from this source: we must therefore arrange so as to avoid the action of these disturbing influences. To effect this, the vessel containing the ice is put inside a larger vessel, and the space between the two is filled with pounded ice; the large vessel is carefully closed. Thus no heat penetrates from the surrounding air to the inner vessel; for such heat is intercepted by the pounded ice between the vessels, and expends itself in melting that: the water thus produced is allowed to escape by means of a tube. The water arising from the ice melted in the inner vessel also escapes by a tube which is carried through the space between the vessels and beyond the outer vessel: thus this water can be carefully collected and its amount determined. The whole apparatus is called a *calorimeter;* it was invented and used by Lavoisier and Laplace, and for its design and its results may justly be held worthy even of these great names: see Professor Maxwell's *Theory of Heat.*

666. The calorimeter then will give us immediately a measure of the amount of heat lost by any solid body while the temperature sinks from an assigned point down to 0^0 C. Thus, for example, by one experiment we can determine the amount of heat lost by an iron globe in sinking from 30^0 C to 0^0 C; and by another experiment we can determine the amount of heat lost by the same globe in sinking from 20^0 C to 0^0 C. Thus the difference of the preceding two results will give the amount of heat lost by the globe in sinking from 30^0 C to 20^0 C.

667. Next suppose we wish to make experiments on a liquid. The liquid must be enclosed in a flask or bottle, and then by the calorimeter we can determine how much heat is lost by the *compound* body, that is by the flask and the liquid, as the temperature sinks from an assigned

point down to 0^0 C. The flask must now be emptied, and then by experiment we can determine how much heat is lost while it sinks through the same range of temperature as in the former experiment for the compound body. The difference of the two results gives us the loss of heat which the liquid alone experiences.

668. In the case of a liquid we must remember that if the liquid freezes at some temperature above 0^0 C., the amount of ice melted in cooling the liquid down from above its freezing point to 0^0 C., is partly due to the influence of the latent heat which is liberated when the liquid freezes. Thus, for example, olive oil freezes at about 2^0 C., so that if we cool the oil from 5^0 C. to 0^0 C. the ice has been melted partly by the sensible heat lost, and partly by the latent heat liberated from the oil in freezing. If we first determine the amount of heat lost as the oil cools from 5^0 C. to 0^0 C., and then the amount of heat lost as the oil cools from a higher temperature, say 10^0 C., to 0^0 C., the difference will be the correct amount of heat lost as the oil cools from 10^0 C. to 5^0 C.: for the liberation of the latent heat produces the same effect in the two cases, and therefore this effect disappears in taking the difference of the two results.

669. The amount of heat absorbed by a solid in the process of melting, or liberated by a liquid in the process of freezing, may be determined by the aid of the calorimeter. Take for example the liquid just considered, namely, olive oil. By two experiments we may determine accurately the heat which the oil gives out in sinking from the temperature 4^0 C. to 3^0 C.; and we may assume that double the amount of heat would be given out in sinking from 4^0 C. to 2^0 C. Again, take oil just frozen, and by the calorimeter determine the heat given out as the oil sinks to 0^0 C. Finally, put oil at the temperature 4^0 C. in the calorimeter, and determine the whole heat given out as the oil sinks to 0^0 C.; this consists of three parts, the heat given out as the temperature sinks from 4^0 C. to 2^0 C., the heat liberated as the oil freezes, and the heat given out as the temperature sinks from 2^0 C. to 0^0 C.: the first and the third of these parts have been separately

determined, and so by subtraction the second part becomes known.

670. We see from Art. 663 that to produce the same change of temperature in the same weight of different substances very different quantities of heat may be required. The quantity of heat required to produce an assigned change of temperature in an assigned weight of any substance is called the *specific heat* of that substance; this supposes that we fix on some substance as the standard, and denote the specific heat of that substance by unity: it is convenient to take water for the standard substance. We can thus form Tables of the specific heats of various bodies, solid and liquid. Thus, for example, we find recorded in such Tables that the specific heat of mercury is ·033, that is about $\frac{1}{30}$: this means that if a certain weight of water in sinking through one degree of temperature will melt an ounce of ice, then an equal weight of mercury in sinking through one degree of temperature will melt one-thirtieth of an ounce of ice. Instead of *specific heat* the term *specific capacity* is sometimes used; we might then define the *capacity* of a body for heat as the amount of heat necessary to raise its temperature one degree, and the *specific capacity* as the proportion of the capacity of the body to the capacity of an equal weight of a standard substance, as water. Some writers however use the single word *capacity* as equivalent to *specific heat.*

671. We see from the preceding example that mercury has much greater *sensibility* with respect to heat than water has; it requires thirty times as much heat to raise water as to raise an equal weight of mercury through one degree of temperature. Water has a greater specific heat, and consequently a less sensibility to heat, than almost any other substance, solid or liquid. Thus the quantity of heat which would raise the temperature of water from 0^0 C. to 100^0 C. would raise the temperature of an equal weight of iron to about 900^0 C., that is, would bring the iron to a white heat. And similarly, water in sinking through a certain range of temperature gives out more

heat than almost any other body of the same weight. This property of water is usefully applied in various ways, as in the hot water pipes for warming buildings, and in the hot water cans used in railway travelling for keeping the feet warm. The specific heat of ice is about half that of water; so that half as much heat is required to raise the temperature of ice through one degree as to raise the temperature of an equal weight of water through one degree: the ice must be supposed to be at a temperature of more than a degree below 0° C., so as to avoid any error arising from the absorption of latent heat in melting. For other substances also the specific heat is different according as the substance is in the solid or liquid state: thus for tin the specific heat in the former state is ·0562 and in the latter ·0637; the corresponding numbers for lead are ·0314 and ·0402.

672. We have next to consider whether it requires the same amount of heat to raise the temperature of a body through one degree whatever the temperature of the body at starting may be. Practically within ordinary limits of temperature it does: but when extreme accuracy is regarded this is not true, for the specific heat generally increases slightly as the temperature rises. Thus the specific heat of mercury between 0° C. and 100° C. is on an average ·033, and between 100° C. and 240° C. is on an average ·035.

673. The fact that the specific heat of water is practically constant may be verified by a simple experiment. Let a pound of water at any temperature be mixed with a pound of water at any other temperature; then it is found that the temperature of the mixture is half the sum of the original temperatures: for instance, if one pound of water is at the temperature 80° F., and the other at the temperature 120° F., the temperature of the mixture will be 100° F. This shews that the heat lost by the pound of water which sinks from 120° F. to 100° F. is equal to that gained by the pound of water which rises from 80° F. to 100° F.; or, in other words, that the same quantity of heat which would raise a pound of water from 80° F. to 100° F.

would just be sufficient to raise a pound of water from 100° F. to 120° F.

674. But now consider the case in which *different* liquids at different temperatures are mixed; suppose, for example, that a pound of water at 45° F. is mixed with a pound of mercury at 200° F.: then it will be found that the temperature of the mixture is about 50° F. Thus the mercury has lost 150° of temperature, while the water has gained only 5°. Hence the quantity of heat which will raise water through 5° of temperature would raise an equal weight of mercury through 150° of temperature; and so the specific heat of mercury is to the specific heat of water in the same proportion as 5 is to 150, that is as 1 to 30. This method may be used to find the specific heat of any liquid: it is called the *method of mixtures*.

675. If we consider the specific heat of water to be unity, that of mercury is about $\frac{1}{30}$; this means, according to Art. 670, that to raise mercury through a certain temperature requires only one-thirtieth of the heat which would be necessary to raise the same weight of water through the same temperature. Or we may say, as we see by Art. 674, that a given amount of heat will raise mercury through thirty times the extent of temperature which it would raise the same weight of water. This is expressed in technical language by saying that the specific heats of substances are *inversely* proportional to the changes of temperature produced by the application of the same amount of heat to the same weights of the substances.

676. The specific heat of gases is a subject of more difficulty than the specific heats of solids or liquids, and may be omitted by a beginner. It becomes necessary to distinguish between two cases, namely, that in which the *pressure* on the gas is kept constant, and that in which the *volume* of the gas is kept constant. Suppose a mass of air put into a vessel and kept under a *constant pressure;* the vessel must not be rigidly closed so as to retain the same volume, but must be furnished with a piston, or some other contrivance, so as to leave to the air the power of

expanding. Let heat be applied, so that the temperature rises; then the air expands, and the new temperature is connected with the new volume by the law given in Art. 593. Next let the air be compressed until it has its original volume; then it is found by experiment that a further rise of temperature takes place. The aggregate of the two rises in temperature will be to the first rise in the same proportion as the specific heat of the air for constant pressure is to the specific heat for constant volume. This follows from the result stated in Art. 675, that the specific heats are *inversely* proportional to the changes of temperature caused by the same application of heat.

677. It is found by experiments that this proportion is expressed by the number 1·41 for air, and that it is the same whatever be the temperature and the pressure. The same number also expresses the proportion for various gases, as oxygen, hydrogen, and nitrogen; but for some gases the number is smaller.

678. The following Table gives the specific heats of some bodies; the gases are supposed to be under constant *pressure*, and by the definition of specific heat equal *weights* of the various substances are taken. The numbers may be considered as the average values for temperatures between 0° C. and 100° C.: see Art. 672.

Water	1·000	Iron	·114
Hydrogen	3·410	Zinc	·096
Alcohol	·548	Copper	·095
Ice	·504	Silver	·057
Steam	·480	Tin	·056
Nitrogen	·243	Mercury	·033
Air	·237	Gold	·032
Oxygen	·217	Lead	·031
Glass	·177		

679. Chemistry teaches that each of the various substances presented to us in nature may be considered to be made up of an excessively large number of very small portions which are called *atoms* or *molecules*; see Vol. I. Chapter LXI. The weight of an atom of one substance is found to be always in a constant proportion to the weight

of an atom of a second substance; and thus every substance has its own *atomic* weight. For example in the cases of iron, silver, and gold these numbers are 56, 108, and 196 respectively. Now it is found on trial that the product *of the atomic weight of a solid* into the specific heat gives always nearly the same result. Thus in the cases of iron, silver, and gold these products are about 6·37, 6·16 and 6·35 respectively.

LXIV. RADIATION.

680. We know that when light proceeds from a luminous point it is sent forth in all directions in straight lines; in like manner a hot body sends forth heat in all directions, whether the body be luminous like a lamp, or obscure like a kettle of boiling water: the heat is sent forth with apparently the same velocity as light. This mode of communicating heat is called *radiation*, and the term *radiant heat* is applied to the heat so communicated; we must not however suppose that radiant heat is any new kind of heat; it merely denotes heat considered with respect to its passage from one body to another not in contact with the body which sends out the heat. Take a hot body, as a ball of iron raised to any temperature, for example that of 300° F., and place it in the middle of a room; then heat is sent forth in every direction, and apparently instantaneously. The presence of air is not necessary, as is shewn by the fact that if a thermometer be placed in the exhausted receiver of an air pump which is exposed to the influence of the hot body, the mercury in the thermometer rises. The proof that air is strictly speaking *radiated*, that is propagated in straight lines, is like the proof of the corresponding property in Optics.

681. There is an analogy in various respects between light and heat; it will be sufficient here to recall some of the facts stated in Chapter XXXI. We have seen in that Chapter that in the solar beam there are some rays which are both luminous and hot, some which are luminous but almost destitute of heat, and some which are hot but not

luminous. Experiment also shews that rays of heat both luminous and non-luminous undergo refraction. The non-luminous rays of heat are often called *obscure heat* or *dark heat.*

682. It is easily shewn by experiments that rays of heat undergo reflection from smooth polished surfaces after the same law as rays of light. Let two spherical reflectors be placed so as to have a common axis, at some distance apart, and with their concavities turned towards each other. Put an origin of light at the principal focus of one reflector; the rays of light falling on this reflector are reflected parallel to the common axis, and after falling on the second reflector are brought to a point at its principal focus: all this is known from the principles of Optics. But the rays of heat are also brought to this focus as well as the rays of light: for if a thermometer be placed at the point, the rise of the mercury shews that the heat is concentrated there. This concentration occurs not only for luminous heat but also for dark heat; instead of a bright point at the principal focus of the first reflector we may put a hollow unpolished ball of metal filled with hot water: then there will be no light brought to the principal focus of the second reflector, but the thermometer, as before, will shew that heat is brought to that point. Instead of spherical reflectors parabolic reflectors may be used with advantage: see Art. 211.

683. The thermometer which is best suited for shewing the concentration of heat is the *differential thermometer:* see Art. 606. The heat diffused through the room in which the experiment is conducted will in general affect each bulb of the instrument in the same way; then when one bulb is put at the principal focus of the second reflector, and we find the reading immediately affected, we know that there is a marked difference between the temperature at that spot and the temperature of the adjoining spot which is occupied by the other bulb.

684. When heat falls on a body part of it is reflected according to the same law as light is reflected; part of it is scattered in all directions; part of it enters the body and either passes through it or is absorbed by

it: the establishment of the facts connected with this distribution of heat constitutes a large field of experimental investigation which was first systematically cultivated by Sir John Leslie. For sources of heat he used cubical boxes filled with hot water; the sides of the boxes were of various materials, so as to shew the difference in radiating power of different substances. The source of heat was placed before a reflector, which brought the rays to a focus; and by varying the nature of the reflector the differences of the reflecting power of different substances could be observed. Finally, at the focus was placed the bulb of a differential thermometer; and as the bulb could be surrounded by various envelopes the differences of the absorbing powers of different substances could be observed.

685. The experiments shewed that when the radiating body changes its distance from the reflector the effect changes; diminishing when this distance is increased, and increasing when this distance is diminished. Also the effect increases or diminishes in the same proportion as the size of the radiating surface is increased or diminished. These two results might naturally have been expected. A more important law is that the intensity of radiation is proportional to the excess of the temperature of the radiating body over the temperature of the surrounding air. Thus suppose that the temperature of the surrounding air is 60^0 F., and that the temperature of the radiating body is 100^0 F.; and let the thermometer placed at the focus of the reflector shew a rise of 15^0 F. in temperature: then if the temperature of the radiating body is increased to 140^0 F. the thermometer at the focus will shew a rise of 30^0 F. in the temperature. In the former case the difference between the temperatures of the radiating body and the air is 40^0 F., and in the latter case it is 80^0 F., which is double the former; and the effect produced in the latter case is double that produced in the former.

686. The difference in the radiating power of various substances is considerable. Three cases may be taken as examples: first let the vessel containing the hot water be of polished tin, next let the vessel be coated with isinglass, and finally let it be coated with lamp black; then the

effects produced are represented by 12, 80, and 100 respectively, so that the radiating power of isinglass is nearly seven times that of polished tin, and the radiating power of lamp black is more than eight times. The following is a selection from some of the results obtained by Leslie with respect to the radiating power of various substances; that of lamp black is denoted by 100, and the others are expressed relatively to this: the numbers must be considered as only rough approximations.

Lamp black	...100	Indian ink	88
Paper	98	Tarnished lead ...	45
Sealing wax ...	95	Mercury	20
Crown glass ...	90	Gold, silver, copper	12

687. The following conclusions are drawn from the experiments. Metallic bodies are in general the most feeble radiators; good radiators are bad reflectors, and good reflectors are bad radiators; if the surface of a body is rendered smooth and polished the reflecting power is increased, and the radiating power is diminished; if the surface is rendered rough and tarnished the radiating power is increased, and the reflecting power is diminished. The power of absorption is generally proportional to that of radiation; so that a body which absorbs well radiates well and reflects badly, and a body which absorbs badly radiates badly and reflects well. This statement is easily seen to be what we might naturally expect; a good reflector throws off at once what heat it receives, while a bad reflector allows the heat to enter easily, that is, it absorbs the heat readily.

688. In Optics we distinguish between *transparent* bodies which allow light to pass through them, and *opaque* bodies which do not. In like manner heat passes through some bodies, which are therefore called *diathermanous*, and will not pass through other bodies, which are therefore called *athermanous*. Rock salt and dry air are examples of the former class of bodies, as they allow heat to pass through them with scarcely any loss. The metals are examples of the latter class of bodies; for even when reduced to a state of extreme thinness they seem to intercept the passage of heat completely: thus tin foil and gold leaf exert this power. In general bad radiators obstruct the

passage of heat most, and good radiators least; this is natural, for a bad radiator is a good reflector, and so sends back the rays of heat instead of allowing them to proceed, while a good radiator is a bad reflector, and so allows the rays of heat to proceed. It is found that the power of rays of heat to penetrate a body increases with the temperature of the body which sent forth the rays; thus rays proceeding from a body at a certain temperature fail almost entirely to penetrate glass, but if the temperature of the body be raised considerably the rays will penetrate the glass readily.

689. The fact just noticed is analogous to what is known with respect to light. Thus a blue glass is a glass which allows the blue rays to pass through it, but will not allow the red rays to do so; that is, in optical language, the more refrangible rays pass through the blue glass, while the less refrangible rays do not. In like manner some substances will allow luminous heat to pass through them while they intercept dark heat: luminous heat in general proceeds from sources at high temperatures, though such sources send out dark heat also. Some substances, as black glass, allow heat to pass, but intercept light.

690. We will now give examples of substances which exhibit in a high degree the powers of absorption, reflection, and transmission respectively. Lamp black is remarkable for its power of absorption; it transmits none of the radiant heat which falls on it, and reflects very little. Polished silver is a very good reflector; it transmits none of the radiant heat which falls on it, and absorbs only about a fortieth part. Rock salt transmits radiant heat exceedingly well; it absorbs scarcely any and reflects less than a twelfth part.

691. Suppose a person in front of a fire to use a fire-screen to screen himself from the heat, then we may ask what becomes of the heat which is intercepted. This heat is absorbed by the screen, and raises the temperature of it; thus eventually the screen becomes a source of heat, but it sends the heat out in *all directions*, and therefore far less may fall on the person who holds the screen than if he were unprotected. The example shews how a substance which *intercepts* radiant heat may itself become a

source of heat to surrounding objects, though it is itself *athermanous.*

692. Professor Tyndall has made numerous experiments relative to the absorption of dark heat. Dry air, oxygen, hydrogen, and nitrogen, have scarcely any absorbing power: vapour of water, to the extent in which it occurs in the atmosphere on an ordinary day, absorbs more than sixty times as much as dry air. Olefiant gas has an extraordinary absorbing power; when it is so rarefied as to exert only one-thirtieth of the pressure of the atmosphere it absorbs ninety times as much as air under the ordinary pressure.

693. The pressure of vapour in the atmosphere exercises an important influence in maintaining the temperature of the earth, as it obstructs the escape of the heat into space. "Remove for a single summer night the aqueous vapour from the air which overspreads this country, and you would assuredly destroy every plant capable of being destroyed by a freezing temperature. The warmth of our fields and gardens would pour itself unrequited into space, and the sun would rise on an island held fast in the iron grip of frost. The aqueous vapour constitutes a local dam, by which the temperature at the earth's surface is deepened; the dam, however, finally overflows, and we give to space all that we receive from the sun." Professor Tyndall's *Heat, a Mode of Motion.*

694. Newton supposed that if a hot body be placed in a medium of lower temperature, the loss of heat in an assigned time is proportional to the excess of the temperature of the body above that of the medium: this supposition is called *Newton's Law of Cooling.* When the difference of temperature does not exceed 10° F. this is sufficiently accurate; but it does not hold when the difference of temperature is great. The true law of cooling has been investigated by Dulong and Petit, but it is not sufficiently simple for exposition here. The specific heats of two bodies are sometimes compared by observing the speed with which they cool; the method is called the *method of cooling.* It is assumed that if we take two bodies of equal weight at the same temperature, and put them in the same circumstances, the one which has the smallest specific heat will cool fastest.

LXV. THEORY OF EXCHANGES.

695. The results of experiments like those of the preceding Chapter are embodied in what is called the *Theory of Exchanges*, first proposed by Prevost of Geneva. According to this theory every point at or near the surface of a body radiates heat in all directions, and receives heat from other bodies which also radiate; the heat received is partly absorbed and partly dismissed by radiation. Thus there is a constant interchange of heat going on between every pair of bodies.

696. Suppose there are two bodies, one of higher temperature than the other, each radiating heat, but the hotter body radiating more copiously than the other. Then the hotter body loses more than it receives, and so its temperature falls; the temperature of the other body at the same time rises, until at last both bodies have the same temperature.

697. In such an apparatus as that mentioned in Art. 684 suppose that instead of a vessel of hot water we put a lump of ice before the reflector; then the thermometer in the focus will indicate a decline of temperature: thus apparently the ice radiates *cold*. This however is not the case, for even ice really contains some heat, and can be made to supply heat to a colder body. The fact is that both the ice and the thermometer radiate heat, but the ice much less copiously than the thermometer; thus the thermometer sends out more heat than it receives, and so its temperature falls.

698. Various examples illustrative of the principles established with respect to radiation and reflection naturally present themselves. Vessels which are intended to retain liquids as long as possible at a high temperature should be constructed of materials which radiate heat feebly; thus tea-pots are advantageously made of bright metal, because such a substance is a good reflector and a bad radiator: on the other hand, black earthenware is an unsuitable material for the purpose, as it radiates heat very copiously. In like manner the cylinders of steam engines are kept highly polished in order that heat may not be lost by radiation. The polished fire irons used in drawing

rooms being good reflectors absorb heat slowly, and so may remain for a long time without rising to any excessive temperature; but rough unpolished fire irons, being bad reflectors, absorb heat copiously, and soon become inconveniently hot.

699. If a body which is a good radiator is placed near other bodies which do not radiate freely its temperature may be considerably reduced, because it sends out heat copiously and receives but little in return. An experiment has been made which illustrates this point. Let a glass cup be placed in a silver basin, and exposed to the atmosphere on a cold clear night; it will be found in the morning that the glass is covered with moisture while the basin is dry. If a silver cup is placed in a glass basin, and exposed as before, the glass is again covered with moisture while the silver is dry. Silver is a bad radiator of heat, and glass is a good radiator. Thus the silver sends away much less heat than the glass, and neither of them receives any appreciable quantity again during the cold clear night; therefore the glass becomes much colder than the silver, and accordingly the vapour of water which is contained in the air becomes condensed on the surface of the glass.

700. The preceding experiment suggests the explanation of all the circumstances connected with the formation of *dew*. On a clear cold night things which are good radiators dismiss their heat rapidly and receive little or none in return; hence their temperature must fall. Grass, and other forms of vegetation in general radiate well; and hence we may expect that they will undergo a fall of temperature during a clear cold night, and this has been verified by experiment: for a thermometer placed on the grass has been found to be at 20° F. below one placed a few feet above the ground. Then the vapour which is contained in the atmosphere becomes condensed on the surfaces of cold bodies, and the name *dew* is given to the moisture so produced. Dew is therefore deposited most copiously on those substances which radiate well, and scarcely at all on those substances which radiate badly. In close sheltered places, like the streets of towns, dew is not deposited: for there the heat sent forth by radiation is restored by radiation from neighbouring objects. Again, on cloudy nights dew is not deposited; for although the

good radiators send forth heat as usual, yet the clouds radiate heat back again, and so prevent the fall of temperature which leads to the deposition of dew. If the temperature sinks below the freezing-point of water after dew has been deposited the dew takes the form of *hoar frost.*

701. If a body instead of being exposed to the open sky is screened in any way the deposition of dew is prevented, because the escape of heat by radiation is checked. This explains the advantage of the precaution taken by gardeners to cover tender plants during cold nights by mats; the warmth acquired by the plants during the day is thus prevented from escaping by radiation during the night.

702. We have stated that a thermometer placed a few feet *above* the ground may indicate a temperature much higher than one *on* the ground: the reason appears to be this. The air has free access to the thermometer above the ground, and when cooled by contact with it the air becomes heavier and so sinks; then its place is supplied by warmer air, so that the temperature of the thermometer is kept from falling much below that of the air. In a similar way if there be a current of air over any object, whose temperature is below that of the air, the temperature of the object is preserved higher than it would otherwise be, and dew is not deposited.

703. In India ice is sometimes obtained in the following manner. Water is put into shallow pans which are placed on dry straw, and left exposed to the sky on a clear cold night: then in the morning a crust of ice is found to have been formed on the surface of the water. During the night the water loses heat rapidly by radiation, and consequently the temperature sinks; and then the ice is formed. It has been pointed out by Professor Tyndall that the success of the process is greatly promoted by the absence of aqueous vapour from the atmosphere; for if vapour is present the radiant heat is much obstructed in its attempt to escape: see Art. 693.

LXVI. CONDUCTION.

704. We are now about to consider the circumstances which accompany the transmission of heat between par-

ticles of the same body in contact. Let a bar of metal be heated at one end, for instance, by exposing that end to the flame of a lamp; let thermometers be placed at various points along the rod, so as to determine the temperatures of the rod adjacent to these points. The thermometers at first will not be affected by the heat applied to the end of the bar, but after some time that nearest to the end will begin to rise in temperature; after a further interval the next thermometer will begin to rise; and so on. If the end which is exposed to heat be maintained at a constant temperature then every thermometer will at last be stationary; that nearest the heated end will be highest, and that most remote will be lowest, in temperature, while the intermediate thermometers will be at intermediate temperatures.

705. We may notice the most accurate way of determining the temperatures at different parts of the bar in the preceding experiment. Small cavities are made in the bar at those points, and filled with mercury; the bulbs of the thermometers are inserted in the cavities, and so surrounded by the mercury. The thermometer readily takes the temperature of the surrounding mercury, while the mercury readily takes the temperature of the adjacent parts of the bar.

706. This mode of conveying heat through a body is called *conduction;* the process is much more rapid in some bodies than in others, and the more rapid it is the *better conductor* the body is said to be, or it is said to have greater conductive power, or greater conductivity. But the process is in all cases excessively slow compared with radiation; for radiant heat flies through space with the swiftness of light. We are familiar with various examples which illustrate the difference as to conducting power between different substances. Thus, if we put one end of a poker into a fire and make it white hot, the other end may also be too warm to handle; but if we put one end of a stick of the same length in the fire the other end may be held without inconvenience: we infer that iron is a much better conductor of heat than wood. Let a brass cylinder be covered tightly with thin paper; then if this be held over a flame for a short time the paper will not be scorched, for the metal being a good

conductor carries away the heat rapidly: but if a cylinder of wood be used instead of that of brass the paper becomes scorched almost immediately. An experiment was suggested by Franklin for the purpose of illustrating the difference in the conductive power of various substances. Bars of various substances were coated with wax; one end of each bar was immersed in a hot liquid, and the extent to which the wax was melted was observed. Then, other things being the same, the further the wax was melted the better conductor the substance was. It is found that the conductive power of wood is much smaller in a direction transverse to the fibre, than it is along the fibre; moreover the conductive power is much smaller for the bark which surrounds the timber of a tree than for the timber itself: thus a tree is protected against the injuries which it might receive from sudden changes of temperature.

707. In making experiments to compare the conductivity of different substances precautions must be taken to prevent the intrusion of heat by other processes besides that of conduction. For instance, if various bodies in the neighbourhood radiate heat, and the substances we are comparing differ in their powers of absorption, we must allow for this circumstance. Again, take the experiment which was suggested by Franklin, as mentioned in the preceding Article; Professor Tyndall has pointed out an important consideration which must be regarded. Suppose we take two bars, one of bismuth and one of iron; then the wax melts from the bismuth *much sooner* than from the iron, but still iron conducts heat much better than bismuth. The explanation is this: the specific heat of iron is more than three times as great as that of bismuth, and thus when a certain amount of heat is given both to the iron and to the bismuth the temperature of the iron rises much less quickly than that of the bismuth.

708. The accurate determination of the conductive powers of substances is a matter of considerable difficulty, and is best effected by a combination of experiments with theory; the method is not suitable for elementary exposition. Results obtained by Wiedemann and Franz

are adopted by most modern writers; they are contained in the following Table, in which the conductive power of silver is denoted by 100:

Silver	100	Lead	9
Copper	74	Platinum	8
Gold	53	German silver	6
Brass	23	Rose's metal	3
Tin	15	Bismuth	2
Iron	12		

709. Air so long as it does not move is a bad conductor of heat. To this property may be referred the fact that feathers conduct heat badly; for they contain air both inside the quill part, and also adhering to the other part: a similar remark applies to wool and fur. Hence, by the aid of feathers, wool, and fur, the temperature of animals is maintained at the high standard which is necessary for life: the heat cannot readily escape into the surrounding atmosphere. Man by his artificial clothing attains the same end: flannel is found especially advantageous, for it prevents the escape of heat in cold weather, and the intrusion of heat in hot weather. If ice is to be preserved from melting it would be serviceable to wrap it in blankets for the same reason. It is because air is a bad conductor of heat that the human body can endure contact with very hot or very cold air, so long as the air is still; but if there be any *current* the air will become intolerable, for then fresh portions of it are perpetually brought against the body and change its temperature.

710. Glass and porcelain are very bad conductors of heat; hence we see the object for which the handles of kettles are sometimes constructed of such materials. The same fact explains why a glass tumbler is readily cracked if hot water is poured into it; the bottom with which the water first comes into contact is suddenly heated and expands, but the heat does not pass readily enough into the other parts of the tumbler, so as to expand them in proportion: hence the bottom separates itself by a crack from the part which resists its expansion.

711. In a fluid heat is in general communicated from one point to another in a way which differs in an important

respect from that conduction in the case of a solid body of which we have spoken. Suppose a source of heat is applied to the bottom of a vessel of liquid; the particles of liquid near the source of heat become hot, expand, and rise to the top; colder particles descend, and in their turn receive heat and then rise to the top. Thus an upward and a downward current are established, and in this way heat becomes gradually diffused through the whole mass. The term *convection* is sometimes used to denote this process. In such a case the heat may finally pass from particle to particle by the same method as in conduction, but the motion greatly assists in the propagation of heat, by bringing particles of different temperatures near each other. That fluids in themselves have a very small conductive power may be shewn by experiment. On the surface of water in a vessel put a little spirits of wine, and set fire to it; then by the aid of a thermometer it can be shewn that the temperature of the water below is scarcely affected: the slight rise of temperature which really occurs may be attributed mainly to conduction through the sides of the vessel.

712. Examples of the diffusion of heat by currents of water present themselves both in the arts and in nature. It is in this way that buildings are warmed by the aid of hot water pipes; a current of hot water proceeds from a boiler through a system of pipes, and returns to the boiler again after discharging its heat. The *Gulf Stream* is a well known illustration of the same process on a large scale; this consists of a current of warm water, forming a sort of river in the ocean, which proceeds from the Gulf of Mexico in a North Easterly direction, and reaches the coasts of Great Britain and Norway: it exercises, as is well known, a favourable influence on the climate of the western shores of England and Ireland, rendering it much milder in winter than it would otherwise be. In the recent scientific voyage of the *Challenger* the surface temperature suddenly rose from $65\frac{1}{2}^{0}$ to $75\frac{1}{2}^{0}$ as the ship entered the Gulf Stream; the warm current was found to be 15 miles broad, and 600 feet deep, running at the rate of 3 miles an hour; this gives a volume of heated water of 108 cubic miles discharged in a day into the North Atlantic Ocean.

713. Our *sensation* of heat depends very much on the conductive power of substances, and we will therefore make a few remarks on the subject here. The organs of sense by which we are conscious of sound, and light, and heat, give us information which is sufficient for practical life, though it falls short of that exactness which we require in the pursuit of theoretical science. Thus we may be able to discern by the unassisted sight that one light is brighter than another, though we may not be able to state precisely the numerical proportion between them. In the case of heat we may pronounce one body to be hotter than another, judging by our senses, though we may be unable to make any accurate estimate of their relative temperatures; and indeed we may be wrong even in our supposition that one body is hotter than the other. Thus, for example, let one hand be plunged into ice-cold water and the other into water at as high a temperature as can be endured; then suddenly put both hands into vessels containing water at a moderate temperature, say 60° F.: the water will seem warm to the hand which was formerly exposed to the temperature of ice, and will seem cool to the hand which was formerly exposed to the high temperature. The reason is obvious; the cold hand receives heat from the water which is warmer than itself; and the hot hand gives up heat to the water which is colder than itself.

714. Our sensations then do not give us very exact evidence with respect to temperature; and with respect to the amount of heat in a body they supply no information. Thus take a vessel of water and one of mercury, both at the temperature of 32° F.; let the temperature of both be raised by some means not known to us to 100° F.; then our senses would not give us any hint of the fact that about thirty times as much heat has been given to the mercury as to the water, for equal weights of the two.

715. Suppose that a thermometer shews the air and all the objects in an apartment to be at the same moderately high temperature, yet the sensations experienced on touching these objects will be different. Carpets and curtains will seem to be warm, while marble and metals will seem to be cold. The temperature of the human body is about 99° F., so that in the moderately cold apartment the

hand imparts heat to every object which it touches. In metals the conducting power is good, so that they draw heat rapidly from the hand; in carpets and curtains the conducting power is feeble, so that they draw heat slowly from the hand; hence the former appear cold, and the latter do not. So also sheets in cold weather appear colder than blankets, because they are better conductors of heat, and therefore carry off heat more rapidly from the body.

LXVII. TERRESTRIAL HEAT.

716. Suppose we observe the temperature at an assigned place at various times throughout a day; we shall then be able to ascertain what we may call the *mean* or *average* temperature for that day. For example, we might observe the temperature at the beginning of each hour for 24 hours in succession; then by adding the temperatures together and dividing the sum by 24 we obtain the average result. But in order to save trouble fewer observations are made to suffice; and it is found that if we take half the sum of the temperatures at nine in the morning and at nine in the evening, we obtain a result which in general does not differ much from that of the more elaborate process.

717. Suppose we take the mean temperatures as found on 365 consecutive days, add them together and divide by 365; thus we obtain what may be called the *mean annual temperature*. If a line be drawn on a map or a globe passing through all the places which have the same mean annual temperature; such a line is called an *isothermal line*. Generally speaking, the further a place is from the Equator the less is the mean annual temperature, but this is not universally true. Thus it is found that in England the mean annual temperature of a place is the same as for a place in Asia or America which is ten degrees of latitude nearer to the Equator. The difference between the summer and winter temperatures is greatest in the interior of large continents and least in small islands; this arises from the fact that the specific heat of water is very great, and so a large mass of water, like a sea or ocean, changes its temperature slowly; and this prevents extreme variations of temperature from occurring in the neighbourhood. It is

well known that the winters in some parts of Europe much to the south of England are more severe than with us.

718. Results of great interest are obtained by observing the temperature at different depths below the surface of the earth. It is found that a cellar, or well, or pit of any moderate depth has always very nearly the same temperature, namely, the mean annual temperature of the place. The facts can be stated more precisely with respect to any assigned place. Thus for a place in the southern part of England at a depth of 3 feet the variation of the temperature throughout a day is scarcely sensible, that is the temperature is practically the same throughout a day of 24 hours. At the depth of 50 feet the temperature remains the same all the year round, namely about 49° F.; for the same depth at the equator the temperature would always be about 84° F.

719. It is found that the temperature in all parts of the world increases considerably, as we descend below the point at which it first remains the same all the year round. In the deepest mine in England, which is near Wigan, the temperature of the rock at the depth of 2440 feet is 94° F. The rate of increase of temperature as the depth increases is different at different places, but may be taken on the average as about 1° F. for 55 feet of depth.

720. It might appear to follow from the preceding fact that at no great depth below the surface of the earth everything must be in fusion, owing to the immense heat; but there are considerations which shew that this inference cannot be certainly drawn. In the first place, it seems that in very deep borings the temperature does not increase so rapidly in the lower part as in the upper part. Thus at Speremberg, near Berlin, a boring has been carried to the depth of 4172 English feet; for the lower half the increase of temperature seems to be at the rate of about 1° F. for 95 feet of depth, while for the upper half of the cutting it seems to be about 1° F. for 50 feet. In the second place, it is known that the temperature of the point of fusion of various bodies is *raised* when the pressure is increased; and if the pressure is enormous this rise of the point of fusion may be considerable. Hence, even if the temperature in the interior of the earth is much higher than the point at which rocks would fuse at the

surface, it does not follow that the matter there is in a liquid state. We have already drawn attention to the fact that ice possesses the contrary property; the melting point is lowered by increase of pressure: see Art. 619.

721. Experiments and calculations have been made by Sir J. Herschel with the view of determining the amount of heat received by the earth from the sun. He came to the conclusion that the heat thrown on a square mile exposed at noon under the Equator, supposing all the heat to pass through the atmosphere, would be sufficient to melt about 60,000,000 pounds of ice. This result would have to be multiplied by about 50,000,000 to give the effect produced on a diametral section of the earth. It is deduced from this that the heat received by the earth in one year from the sun, would be sufficient to melt a layer of ice 100 feet thick covering the entire surface of the earth; or it would be sufficient to raise the temperature of a layer of water 80 feet thick from 0° C. to 100° C.

722. We may conveniently place in connection with the subject of terrestrial temperature a few words on clouds and rain. When vapour is condensed in the air it is usually called *mist* if near the ground, and *vapour* if at some distance from the ground. There is a difference of opinion as to the nature of the particles of vapour, and the way in which they are supported in the atmosphere. Some persons hold that they are hollow, and in consequence use the word *vesicles* for them: others hold that they are solid globules like rain-drops, only very much smaller.

723. Observation shews that there is a great variety in the appearance of clouds, but four marked classes have been named by Mr Howard, one of the first who paid attention to the subject: he called his classes *cirrus*, *cumulus*, *stratus*, and *nimbus*, respectively. *Cirrus* consists of small white clouds, which have a feathery appearance; they occupy the higher regions of the atmosphere, and are supposed to be made up of particles of ice: sailors call them *mare's tails*. Their appearance is often followed by a change of weather. *Cumulus* consists of clouds of a rounded spherical form, which look like mountains piled one above the other: sailors call them *wool-packs*. They are more frequent in summer than in winter, are usually

formed in the morning and disappear towards sunset. *Stratus* consists of long horizontal layers; they appear chiefly at sunset and disappear at sunrise. These are lower than the other classes and are probably formed by the cooling of the earth and the air near it by reason of radiation. *Nimbus* denotes the *rain* clouds: these have no special form, but are distinguished by their uniform grey tint, and by their fringed edges.

724. Clouds consist of condensed vapour, and the condensation is produced in the same way as in the case of vapours on which we can make experiments. The main cause of condensation is the lowering of temperature, and this may occur in various ways; as by the radiation of heat to the sky, or by the expansion which takes place as a mass of air pushes its way to the higher regions of the atmosphere, or by contact with a stratum of air which has in some way been cooled. Thus when there are clouds in the higher region of the atmosphere rain commences there as the particles of moisture yield to the attraction of the earth: the particles fall slowly, and it may happen that in falling they traverse moist air, and so condense more vapour and increase in size; or, on the other hand, they may traverse very dry air, and so become converted into vapour again.

725. The amount of rain which falls at an assigned place on the earth is estimated by the depth in inches of the pond which would be formed by the rain falling over a certain area, as an acre for example, if there were no loss or waste. Observations are made by exposing a funnel with its mouth upwards to the sky, and carefully collecting the rain which falls on it. Thus suppose we collect in a single day as much rain in the instrument as would rise to the height of half an inch in a cylindrical vessel of the same area as the mouth of the funnel; then we say that *half an inch of rain* fell during the day. The instrument is called a *rain-gauge*, or a *pluviometer*. Care must be taken to select a suitable place for the rain-gauge, where there are no currents of air arising from neighbouring obstacles, since these would obviously affect the amount of rain collected. It may be shewn that the rain which would cover an acre of surface to the depth of one inch would weigh about 100 tons.

726. Local circumstances affect the quantity of rain which falls on an average in the course of a year; speaking generally most rain falls in hot climates, for there vapour is formed most abundantly; also more rain falls in mountainous countries than in low countries, for the cold tops of the mountains condense the vapour. The average rain-fall during a year at Paris is about 22 inches; at London about 24 inches; at Lincoln, which is almost the driest part of England, 20 inches; at Glasgow 40 inches; and at the wettest place hitherto noticed in England, near Seathwaite in Cumberland, 165 inches. It is said that at a place about 300 miles North East of Calcutta the annual rain-fall is about 600 inches.

LXVIII. THERMODYNAMICS.

727. We are familiar with the fact that heat may be employed to perform mechanical work. An important example is supplied by a steam engine: the source of the power of the engine is the heat obtained by the combustion of coal, and by the aid of the power weights may be lifted from one place to another, thus overcoming the influence of gravity; or weights may be drawn from one place to another on the same level, thus overcoming the friction which opposes such motion; or various other operations may be effected which involve the exertion and prevalence of force against resistance. In modern times a science has grown up which seeks to trace the connection between heat and the mechanical work it can perform, and especially to determine the numerical relation between the *quantity* of heat supplied and the *quantity* of work executed: this science is called *Thermodynamics*, and we shall now make a few remarks on it; but to treat it fully would require much more mathematical knowledge than we can assume in the reader of this elementary treatise. It would be advantageous for the student to look over the Chapters on *Work* and *Energy* in the first volume before proceeding with this Chapter.

728. The foundation of the subject is in fact a part of the general principle of the *Conservation of Energy*, and is often called the ***First Law of Thermodynamics;*** it may be stated thus: *when heat is transformed into work, or work into heat, the quantity of heat is equivalent to the quantity of work.* This law is explained and established by the experiments of Dr Joule and others, to which we have referred in the Chapter on *Energy* in the first volume. Let us take as our unit of heat the quantity of heat required to raise the temperature 1° F. of a pound of water at 60° F., and let us take as our unit of work the *foot-pound*, that is, the work done in raising a pound weight vertically through a foot: *then a unit of heat is equivalent to* 772 *units of work.* The experiments which lead to this result are numerous and varied; heat has been transformed into work, and work into heat: and when every attention is paid to accuracy the same numerical result is always obtained. This result is quite independent of the source from which the heat is derived, whether from a body at a high temperature, or from a body at a low temperature: so much heat as will raise the pound of water one degree will always perform 772 units of work.

729. The number 772, which is thus obtained, is called *Joule's Equivalent;* it is of the same importance in Thermodynamics as the number 32, which expresses the value of the force of gravity, is in Mechanics. In mathematical treatises on the subject the letter J is commonly used to denote *Joule's Equivalent.* The number will not change if we change the unit of weight; for instance, if instead of a pound we take an ounce, or take the French *kilogramme* which is equivalent to 15432 English grains. But the number will change if we change the scale of the thermometer, or the unit of length. For example, suppose we take one degree of the Centigrade thermometer, instead of one degree of Fahrenheit's thermometer, as the unit of temperature; then the number 772 must be multiplied by $\frac{9}{5}$: and if instead of a foot we take as unit of length a *metre*, that is, 39·3707 inches, the number must

be multiplied by $\frac{12}{39\cdot3707}$. Now $772 \times \frac{9}{5} \times \frac{12}{39\cdot3707} = 423\cdot5$ approximately; and this is the value of *Joule's Equivalent* used in foreign books, and in those English books which adopt the French system of weights and measures; the unit of heat here is the heat required to raise the temperature of a kilogramme of water one degree of the Centigrade scale; and the unit of work is the *kilogrammetre,* that is, the work done in raising a kilogramme vertically through a metre.

730. Since then heat is equivalent to mechanical work we are led to consider the kind of work which heat performs when it enters a body. One obvious kind of work is connected with the visible expansion; for the body is usually under the pressure of the atmosphere, and thus when it expands a force equal to this pressure is exerted over the whole surface through a space corresponding to the expansion: this may be called *external* work. But besides this there is also what we may call *internal* work performed. As we have explained in Chapter LXI. of the first volume, the particles of a body at a certain temperature are believed to be in vibration about certain mean positions; now when the temperature rises the mutual distances of these mean positions are increased, and also the range of vibration of each particle about its mean position is increased. Thus the internal work consists in an increase of what we may call the *energy of position* and the *energy of motion* of the particles of the body into which the heat enters: see Vol. I. Chapter LIII. A further internal work is effected when heat causes a body to pass from the solid state to the liquid, or from the liquid state to the gaseous.

731. An important experiment first made by Gay Lussac, and repeated by Dr Joule, deserves notice. Dr Joule took two vessels; in the one he compressed air until its density was twenty-two times that of the ordinary atmosphere; the other vessel was exhausted: the vessels were surrounded with water and then a communication was opened between them by turning the stop-cock of a tube which joined the vessels. Air rushed from one vessel into the other, and so became equally diffused between the vessels; but *no change of temperature* took place. The

inference from this is that no appreciable change of temperature takes place when air is allowed to expand in such a manner as *not to do any work.* It was formerly supposed that air by its mere expansion became colder, and by its mere contraction became warmer; but, by this experiment and by theoretical reasoning, we are now taught that the diminution of temperature arises from the work done by the air as it expands, and the elevation of temperature from the work done by the force which compresses the air.

732. In 1824 Sadi Carnot published a work entitled *Réflexions sur la Puissance motrice du Feu;* in this work he describes an engine which is now called *Carnot's Reversible Engine:* the meaning of the term *reversible* will be explained hereafter. It must be observed that this engine is purely imaginary, and one which it is impossible to construct; moreover Carnot himself fell into a serious mistake regarding it, owing to his belief that heat was a material substance; nevertheless the conception of the engine is found to be of great value in the theory of Thermodynamics.

733. The engine itself consists of a substance included in a cylinder, which is furnished with a piston that can be moved up and down; the substance may be any thing which is affected by heat, but for simplicity may be taken to be air. The piston, and the cylinder with the exception of its base, are supposed to be perfect non-conductors of heat, but the base is supposed to be a perfect conductor: moreover the base must be so thin, or of so small a capacity for heat, that the heat required to raise its temperature through such a range as we shall have to consider may be neglected. There are three bodies, or stands, on which the cylinder can be placed when required, which may be denoted by A, B, and C; A is maintained at a fixed temperature which we may denote by H, and B is maintained at a fixed temperature which we may denote by L, lower than the former: C is a perfect non-conductor.

734. The engine is now used for four successive operations:

First operation. Suppose the temperature of the air to be L; put the cylinder on C, and force the piston down, then the volume of the air will diminish, and, as no heat can escape, the temperature will rise: let the process continue until the temperature rises to H.

Second operation. Put the cylinder on A, and allow the piston to rise; A supplies heat, so as to maintain the temperature at H, and the air will expand, and will do work by pressing against the piston: the process may continue as long as we please.

Third operation. Put the cylinder on C, and allow the piston to rise; the air will continue to expand and to do work, and, as no heat can enter, the temperature will fall: let the process continue until the temperature sinks to L.

Fourth operation. Put the cylinder on B, and force the piston down, then the volume of the air will diminish, but the temperature will remain at L; let the process continue until the volume of the air is the same as it was at the beginning.

735. The four operations described in the preceding Article are called a *cycle*, because at the end of them the engine is in precisely the same state as at the beginning of them. During the first and fourth operations work is done *on* the engine by some external agent, namely the pushing of the piston down; during the second and the third operations work is done *by* the engine. During the second operation heat enters the engine from A; denote the amount of it by h: during the fourth operation heat passes from the engine to B; denote the amount of it by l. Now the first Law of Thermodynamics tells us that the excess of the heat h which enters, over the heat l which leaves, is mechanically equivalent to the excess of the work done by the engine over that expended on it; for as the engine is finally in exactly the same state as at the beginning we have no *internal work* to take into account: see Art. 730. If we call this excess the *gain of work* we may say that h is equivalent to the sum of l and the gain of work.

736. We have followed Professor Maxwell's *Theory of Heat* as to the order of the four operations of Art. 734. Carnot himself began with that which we call the *second*, and passed on to the *third*, *fourth* and *first*. Carnot held that h and l are equal, but that the heat coming from a body of higher temperature has more energy than the same amount of heat coming from a body of lower temperature;

it is now however clearly settled by reasoning and by experiment that h is greater than l: see also Art. 728.

737. Some interesting results are obtained by mathematical investigations respecting the four operations of Art. 734. In stating them we shall for brevity use *temperature* for *absolute temperature*: see Art. 597.

(1) The volume of the air at the end of the first operation is to the volume at the end of the fourth operation, in the same proportion as the volume at the end of the second is to the volume at the end of the third.

(2) The external work done during the first operation is exactly equal to that done during the third.

(3) The heat h bears to the heat l the same proportion as the temperature H bears to the temperature L.

(4) The gain of work is a certain fraction of the heat h derived from A; the numerator of this fraction is the difference of H and L, and the denominator is H. Or we may express the gain of work as a fraction of the heat l transferred to B; and then the numerator is as before, but the denominator is L.

738. Thus we see that by a *cycle* of the supposed kind only a fraction of the heat taken from the warmer body is transformed into work; the fraction being that stated in the preceding Article. The rest of the heat derived from the warmer body is transferred to tho colder body.

739. In the second operation of Art. 734 heat passes from A to the engine; strictly speaking then the temperature of the engine must be below that of A, though the slightest difference will be sufficient: a similar remark applies to the fourth operation. It is found on investigation that these slight differences of temperature may be treated as infinitesimal, so that they exercise no practical influence on the result. This consideration leads us to the fact that the action of Carnot's engine may be exactly *reversed*, and from this fact the title *reversible* is derived. We have only to rely as before on the statement that the infinitesimal differences in temperature between the engine and the stands on which it is successively placed are of no consequence.

740. The operations of Carnot's engine in its reversed action are the following:

First operation. Suppose the temperature of the air to be *L*; put the cylinder on *B*, and allow the piston to rise; *B* supplies heat so as to maintain the temperature: let the process continue until a quantity *l* of heat has been received from *B*.

Second operation. Put the cylinder on *C*, and force the piston down until the air reaches the temperature *H*.

Third operation. Put the cylinder on *A*, and continue to force the piston down until a quantity *h* of heat has been given off by the air.

Fourth operation. Put the cylinder on *C*, and allow the piston to rise until the air sinks to the temperature *L*.

741. In the reversed action of the engine a quantity *l* of heat is taken from *B*, and a quantity *h* is given to *A*: the work done *by* the machine is now less than that done *on* it, the difference being mechanically equivalent to the excess of *h* above *l*.

742. By the aid of the reversible engine a very important practical result is obtained which is called *Carnot's Principle:* suppose that a given reversible engine works between two fixed temperatures, a higher and a lower, that it receives a certain quantity of heat at the higher temperature and does a certain mechanical work; then no other engine whatever working between the same temperatures, and supplied with the same quantity of heat, can do more work. We may put this in a popular form by saying that *no heat engine can be better than a reversible engine.* For suppose, if possible, that a certain engine *E* working between the two temperatures *H* and *L* does more work than the reversible engine *X* working between the same temperatures, and using the same amount of heat. Then connect the two engines so that *E* by its direct action drives *X* in the reversed direction. At each stroke of the compound engine *X* takes a quantity *l* of heat from *B*, and by the expenditure of work supplies the heat *h* to *A*; then *E* receives this heat and by working through its cycle does

more work than is necessary to carry X again through its cycle. Thus at every complete stroke there will be a *gain of work* by the compound engine. The result is that in this way the compound engine would ultimately convert all the heat of the colder body into mechanical work. *This is contrary to experience.*

743. The principle derived from experience which is involved at the close of the preceding Article has been expressed in various forms. It is given thus by Clausius: heat cannot pass *by itself* from a colder to a warmer body, that is, unless some corresponding change takes place at the same time. It is given thus by Professor Maxwell: "it is impossible by the unaided action of natural processes to transform any part of the heat of a body into mechanical work, except by allowing heat to pass from that body into another at a lower temperature."

744. The principle just noticed is called by some writers the *Second Law of Thermodynamics;* but this term is applied differently by other writers. Thus Professor Rankine in his *Manual of the Steam Engine* gives as the Second Law the following statement: *if the total actual heat of a homogeneous and uniformly hot substance be conceived to be divided into any number of equal parts, the effects of these parts in causing work to be performed are equal.* The Second Law has also been put in a form which may be expressed in popular language thus; *all reversible engines are equally efficient:* see Art. 742. Another statement which has been given of the Second Law amounts to a generalisation of the third result of Art. 737; that result may be put in this form, the quotient of h by H is equal to the quotient of l by L. Let us use the term *relative heat* to denote the quotient of the quantity of heat by the temperature at which it enters or leaves. Then it can be shewn that when a body passes through any number of operations of the kind considered in Art. 734, and returns to its initial state, the aggregate of the relative heat which enters is equal to the aggregate of that which leaves the body.

745. We have shewn that for a certain engine, which is conceivable though it could not really be constructed, only a certain fraction of the heat derived from the source can be transformed into mechanical work: see Art. 738. We have also shewn that no other engine can be better than this theoretical engine: see Art. 742. It will however be readily supposed that in practice no engine can be made which can reach the theoretical standard of efficiency, though of late years great improvements have been made in steam engines. For further information on Thermodynamics the reader may consult Professor Maxwell's *Theory of Heat* and Professor Tait's *Lectures on some recent Advances in Physical Science.*

EXAMPLES.

I. VELOCITY OF SOUND IN AIR.

1. FIVE seconds elapse between a flash of lightning and the corresponding peal of thunder: find the distance of the origin of the flash and the peal.

2. Supposing the atmosphere to extend 45 miles above the surface of the earth, find the time it takes for sound to pass through the whole atmosphere.

3. Find the velocity of sound in air at the temperature of 80 degrees of Fahrenheit's thermometer.

4. Determine the temperature of the air if the velocity of sound is 1150 feet per second.

5. The bell in the clock-tower at Westminster is 300 feet above the surface of the ground: find the time sound takes to pass from the bell to a point on the ground 400 feet from the foot of the tower.

6. Supposing at any particular place and time the pressure of the atmosphere to be $14\frac{1}{2}$ pounds to the square inch, and a cubic foot of it to weigh 536 grains, and gravity to generate a velocity of 32·2 feet per second in one second of time, find the velocity of sound in air, there and then, according to Newton's result.

II. CHANGE OF INTENSITY.

1. A point A is distant 50 yards from the origin of a sound, and a point B is distant 70 yards: shew that the intensity of the sound at B is about half the intensity at A.

2. Compare the intensities of sound at two places, one 521 feet, and the other 902 feet from the origin.

3. Wherein does the transmission of sound through a smooth tube differ from its transmission through the open air?

4. In a certain experiment it was found that the temperature at 1 foot above the grass was 38 degrees, and at 8 feet above the grass 47 degrees, and that the speed of the wind was 1 foot per second at 5 feet above the grass; and then a bell was heard at the distance of 440 yards against the wind, and at the distance of only 270 yards with the wind. Explain this.

III. VELOCITY OF SOUND IN OTHER MEDIA.

1. Taking the length of iron pipe in Biot's experiment to be 3120 feet, the velocity of sound in air to be about 1100 feet per second, and in iron to be about 11000 feet per second, find the difference in the times taken by the two sounds to pass over the distance.

2. Make a similar calculation when instead of the iron pipe an iron wire 13350 feet long is used.

3. The density of oxygen is about sixteen times that of hydrogen: shew that the velocity of sound in hydrogen ought to be about four times that in oxygen.

4. If the pressure and density of a gas are both increased or both diminished in the same proportion, shew that the velocity of sound in the gas is not changed.

IV. REFLECTION OF SOUND.

1. A shot is fired before a cliff and the echo is heard in six seconds: find the distance of the cliff.

2. A person stands before a wall and by observing the interval between a clap and its echo he finds that sound travels to the wall and back in three-eighths of a second: determine the distance of the wall.

3. Two parallel walls are at a distance of 330 feet; a person stands between them so that his distance from one of them is twice his distance from the other: find the intervals after he makes a sound at which the first three echoes are heard.

4. Shew that wherever the person stands the interval between the production of the sound and the third echo is the sum of the other two intervals.

V. NATURE OF A WAVE.

1. In a tube the particles of air make 1056 vibrations in a second: shew that the length of the wave is a little more than a foot.

2. Find how many vibrations per second are necessary for the formation of sound waves 4 feet long, supposing the temperature to be such that the velocity of sound is 1120 per second.

3. Taking the velocity of sound as 1122 feet per second, find the length of wave if there are 440 vibrations in a second.

4. The waves produced by a man's voice in common conversation are from 8 to 12 feet long: find the corresponding numbers of vibrations, taking the velocity of sound at 1128 feet per second.

5. In a tube containing hydrogen the particles make 512 vibrations in a second: find the length of the corresponding wave, supposing the velocity of sound in hydrogen to be 4200 feet per second.

VI. MUSICAL SOUNDS.

1. Distinguish between musical and non-musical sounds.

2. Explain what is meant by intensity, pitch, and quality.

3. Give examples in which musical sounds agree in only one of the three characteristics.

4. Give examples in which musical sounds differ in only one of the three characteristics.

VII. STRETCHED STRINGS.

1. A string is 6 inches long and is stretched by a weight 400 times as great as its own: find the number of vibrations in a second.

2. A certain string vibrates 100 times in a second: find the number of vibrations of another string which is twice as long and weighs four times as much per foot, and is stretched by the same force.

3. Shew that the time of an *oscillation* is the same as the time of moving through the length of the string with the velocity which a heavy body would acquire in falling freely through half the tension length.

4. A musical string vibrates 400 times in a second: state what takes place when the string is lengthened or shortened without altering the tension; and also what takes place when the tension is made greater or less without altering the length.

VIII. QUALITY.

1. Suppose that a fundamental sound may be accompanied by four harmonics: find how many degrees of quality can be procured by the use of *one* harmonic with the fundamental sound.

2. Also how many by the use of *two* harmonics.

3. Also how many by the use of *three* harmonics.

4. In the violin where 8 harmonics may occur, find how many degrees of quality can be produced by combining a fundamental sound with two harmonics.

IX. NOTES FROM TUBES.

1. A tube open at both ends is to give a note corresponding to 32 vibrations per second: taking the velocity of sound at 1120 feet per second, find the length of the tube.

2. Also if the number of vibrations is 3480 per second.

3. Find the length of an organ pipe open at both ends which produces sound waves four feet long.

4. If the pipe is stopped at one end find its length if the waves are as in Example 3.

5. If the fundamental notes of two pipes, one open at both ends and the other open at one end only, are the same, state what notes are common to both.

X. RESONANCE.

1. Determine the length of a tube open at both ends which can be used as a resonance box for a tuning fork making 528 vibrations in a second.

2. Determine the length of a tube closed at one end which can be used as a resonance box for a tuning fork making 1120 vibrations in a second.

3. A tuning fork vibrates over a jar 15 inches long and a strong resonance is produced: determine the number of vibrations in a second.

4. A jar containing air and another containing hydrogen resound to the same tuning fork: compare the lengths of the jars.

XI. INTERFERENCE BEATS.

1. If one note corresponds to 100 vibrations in a second, and another note to 102 vibrations, find the number of beats in a second.

2. A stretched wire makes 400 vibrations in a second. Another wire stretched parallel to it produces very nearly the same note. When both wires are struck simultaneously two beats are heard in a second. Find the number of vibrations made by the latter wire in a second.

3. Two open organ pipes are in perfect unison being of the same length; one is then slightly shortened: find the effect on the ear when both of them are sounded together.

4. If the number of vibrations for one note is 400, and for another 500, in a second, shew that a combination tone is produced two octaves below the lower of the two notes.

XII. MUSIC.

1. The number of vibrations corresponding to a certain note is 132; find the number in the note which is a major sixth above the octave of this.

2. If a note is produced by 240 vibrations per second, what are the number of vibrations corresponding to its *major third, fifth,* and *octave* respectively?

3. Shew that by the combination of notes whose vibration numbers are as 1, $\frac{5}{4}$ and $\frac{5}{3}$, a pleasing triad is obtained.

4. Shew that, according to Helmholtz, the range of human hearing extends over more than eleven octaves.

XIII. INTERVAL TEMPERAMENT.

1. Shew that by proceeding successively through three intervals of a major third, we fall somewhat short of an octave.

2. Shew that by proceeding successively through four intervals of a major sixth, we fall short of three octaves.

3. Shew that by proceeding successively through four intervals of a fifth we pass beyond the interval consisting of two octaves and a major third.

4. Shew that by proceeding successively through three intervals of a fifth, and three of a major sixth, we fall short of four octaves.

XIV. MISCELLANEOUS EXAMPLES.

1. On a day when the temperature was 15 degrees Centigrade the report of a gun was heard a quarter of a minute after the flash was seen: required the distance of the gun.

2. Taking the velocity of sound as 1120 feet per second, find the length of the wave when there are 3520 vibrations in a second.

3. Shew that in an atmosphere of hydrogen waves of a given length would produce a note nearly two octaves higher than waves of the same length in air.

4. A tube is open at both ends: find its length if the fundamental note corresponds to 60 vibrations per second.

5. Give the numbers which express the relative rates of vibration of the following musical intervals: the octave, the fifth, the fourth, the major third.

6. A note and its *major sixth* are sounded simultaneously: shew that the combination tone is a *fifth* below the lower note.

XV. MISCELLANEOUS EXAMPLES.

1. Determine the temperature by the Centigrade Thermometer at which the velocity of sound is 1200 feet per second.

2. Taking the velocity of sound as 1120 feet per second find how many vibrations a *middle C* tuning fork must make before its sound is audible at a distance of 280 feet: see Art. 73.

3. The numbers of vibrations in a second of four tuning forks are respectively 256, 320, 384, and 512: find the corresponding wave lengths, taking the velocity of sound at 1120 feet per second.

4. What is meant by the fundamental note of a tube open at both ends? Shew that the next three notes which it can produce are the octave, the twelfth, and the double octave.

5. A note and its *fourth* are sounded simultaneously: shew that the combination tone is a *twelfth* below the lower note.

6. Explain how the sounds are produced which are often heard proceeding from telegraph wires, and why they are heard so much louder when the listener comes close to the posts which support the wires.

7. Two musical sounds, one produced by 400 vibrations and the other by 410 vibrations in a second are excited at the same time: describe and explain what is heard.

8. Describe an experiment by which it is shewn that in some cases a part of a sound may be louder than the whole.

XVI. MATHEMATICAL PRELIMINARIES.

1. Determine the reciprocals of the following numbers: $1\frac{1}{3}$, $2\frac{1}{4}$, 2·4.

2. Determine the sines of the following angles: $22\frac{1}{2}^0$, $44\frac{1}{4}^0$, $63\frac{3}{4}^0$.

3. Determine the angles corresponding to the following sines: ·284, ·710, ·977.

4. It is shewn in works on Trigonometry that the square of the sine of an angle and the square of the sine of the complement of the angle when added together are equal to unity: verify this from the Table, as being at least approximately true; for example, let the angles be 30^0 and 60^0.

XVII. RECTILINEAR PROPAGATION.

1. A coin an inch in diameter is held up before a bright point: determine the form of the shadow thrown on a wall parallel to the coin, and also on a wall not parallel.

2. Suppose the coin in the preceding Example to be parallel to the wall; let the distance of the coin from the origin be 15 inches, and that of the wall from the origin 5 feet: shew that the area of the shadow is 16 times that of the coin.

3. Suppose in Art. 175 that the distance between A and b is 3 inches, and the distance between B and a is 8 inches, and that the shadows a and b are equally obscure: then shew that the light B is more than seven times as powerful as the light A.

4. A small plane area is placed at a certain distance from a bright point: shew that when the plane is placed so that the rays make an angle of 30^0 with it the brightness is *half* what it would be if the rays fell perpendicularly.

XVIII. VELOCITY OF LIGHT. ABERRATION.

1. Shew that if light takes three years to pass from a star to the earth, that star is at least two hundred thousand times more distant from the earth than the sun is.

2. If the earth moved round the sun twice as fast as it does, what would be the change produced in the aberration?

3. If the weight of a molecule of light amounted to but one grain, shew that its momentum would be about equal to that of a cannon ball weighing 150 pounds, moving with the velocity of 1000 feet in a second.

4. It is said that the idea of aberration occurred to Bradley as he was sailing on the Thames; for he observed that the direction of the wind, as determined by the flag of the vessel, seemed to shift when the course of the vessel was changed. Explain this.

XIX. REFLECTION AT PLANE SURFACES.

1. Shew at what angle a ray must be incident on a plane reflecting surface in order that the reflected ray may make a right angle with the incident ray.

2. Trace the course of a ray which is reflected by a plane surface, and is then reflected by a second plane surface parallel to the former.

3. A ray of light is reflected successively by two plane mirrors, the plane of incidence being perpendicular to the line of intersection of the mirrors: shew that when the mirrors are at right angles to each other the final direction of the ray is parallel to its original direction.

4. Find the angle between two plane reflectors so that a ray originally parallel to one of them may after two reflections be parallel to the other.

5. Having given the position of a luminous point, and also the positions of two points from which a pencil diverges successively at two plane reflectors, determine the positions of the reflectors.

XX. REFLECTION AT SPHERICAL SURFACES.

1. The radius of a concave reflector is 18 inches; a bright point is on the axis at the distance of 12 inches from the vertex: determine the position of the focus of the reflected rays, and draw a diagram.

2. In the diagram of Art. 203 suppose that AQ is a *quarter* of the radius; then take for the radius in succession 12, 16, 20 inches, and so on: shew that in all these cases Aq is *half* of the radius.

3. Rays fall parallel to the axis on a convex reflector: draw an accurate diagram in the manner of Art. 208 to shew the course of the reflected rays.

4. If the axis AO of a concave spherical reflector meet the surface produced at R, shew that a ray proceeding from R and making an angle of 30^0 with the axis will be reflected *exactly* to the principal focus of the reflector.

5. A luminous point is placed inside a reflecting sphere; the radius of the sphere is 20 inches, and the point is 10 inches from the centre. Trace the course of rays which proceed from the point to the furthest part of the reflector, are reflected there, and are again reflected by the opposite part of the reflector; the rays are supposed to keep near the axis.

6. Two concave mirrors of equal radii are placed with their axes and their principal foci coincident: shew by numerical examples that the focus of rays reflected successively once at each mirror coincides with the origin of light, the rays being supposed to keep near the axis.

XXI. IMAGES FORMED BY REFLECTION.

1. A man stands upright before a plane vertical reflector, and observes that he cannot see the image of his head or of his feet: shew that if he goes nearer to the reflector or further from it he can still see only the same portion of his image as before.

2. Shew in the preceding Example that the height of the visible portion of the image is equal to twice the vertical height of the reflector.

3. A man stands before a looking glass of his own height: shew that he can see his whole image, and determine how much of the looking glass is concerned in the formation of the image.

4. The sun is 30 degrees above the horizon and his image is seen in a tranquil pool: determine in this case the angle of incidence and reflection.

5. A picture covered with a glass hangs vertically on one side of a room, and there is a window in one of the adjacent sides: shew by a diagram within what limits a person may be placed so as to avoid seeing the reflection of any part of the window in the glass of the picture.

6. A man stands between two plane mirrors which are parallel and facing each other: shew that he will see himself repeated a great many times, the images appearing successively more and more distant and faint.

7. A man stands before a looking glass, with one eye shut, and covers its place on the glass with a wafer: shew that the same wafer will hide the other eye as soon as it is shut and the first is opened.

8. An object is placed before a convex reflector at the distance of the radius from it: draw the image, and shew that its height is one third of that of the object.

9. When an image is formed by a reflector it can be seen by an eye in certain positions, and cannot be seen by the eye in other positions; if however a sheet of thin paper be put at the place of the image it becomes visible to the eye in positions from which it was formerly invisible: explain this.

10. A small object is placed half-way between the centre and the principal focus of a concave reflector: draw the image, and shew in what proportion it is to the object.

XXII. LAWS OF REFRACTION.

1. A ray is incident at an angle of 40° on a surface of glass, find by the aid of the Table of sines the angle of refraction.

2. A ray is incident on a plane surface which separates a medium from vacuum at an angle of 30^0, and the angle of refraction is 90^0: find the index of refraction and trace the course of the ray.

3. A ray of light *AB* is at *B* partly reflected in the given direction *BC*, and partly refracted: what must be the direction of the reflecting surface at *B*, and in what plane must the refracted ray lie?

4. A ray falls on standing water, so that the angle of incidence is 30^0, and a point is taken on the refracted ray at the distance of a foot from the point of incidence: find the distance of the point so taken from the line drawn perpendicular to the surface of the water at the point of incidence.

5. A ray is incident on glass, and the difference between the angles of incidence and refraction is found to be 30^0: shew by the Table of sines that the angle of incidence must lie between 68^0 and 69^0.

XXIII. VARIOUS CASES OF REFRACTION.

1. Find the refractive index for a medium of which the critical angle is 30^0.

2. The refractive index for a certain crystal is 1·654; and for Canada Balsam it is 1·536: shew that in passing from the crystal to Canada Balsam the critical angle is about 68^0.

3. The refractive index of the diamond is very high: explain how this enhances the brilliance of the gem when cut in the usual manner; and shew that the advantage would be lost if the gem were cut in a spherical form.

4. A ray of light falling on glass at a given angle *A* is refracted at the given angle *B*; and a ray incident on water at the same angle *A* is refracted at the given angle *C*. A ray in water being incident on the surface of an immersed piece of glass at the given angle *D*, shew how the angle of refraction is to be determined.

5. *A* and *B* are two media separated by a plane bounding surface, and a ray passes from *A* to *B*: shew that the course of the ray in *B* is parallel to what it would be if the media *A* and *B* instead of being adjacent were separated by a vacuum bounded by planes parallel to the original plane of separation.

XXIV. REFRACTION AT CURVED SURFACES.

1. Suppose the radius of a concave refractor to be 12 inches, the distance *AQ* to be 30 inches, and the index of refraction to be $\frac{3}{2}$: find the distance *Aq*, and draw the diagram. See Art. 253.

2. A pencil of parallel rays is incident directly on a spherical refracting surface, and after refraction *diverges* from a point at a distance from the surface equal to three times the radius: determine the index of refraction, and draw the diagram.

3. A pencil of parallel rays is incident directly on a spherical refracting surface, and after refraction *converges* to a point at a distance from the surface equal to three times the radius: determine the index of refraction, and draw the diagram.

4. In a convex refractor the index of refraction is $\frac{3}{2}$, the radius of the surface 12 inches, and AQ is 60 inches: shew that Aq is also 60 inches.

5. Shew by other examples that if the index of refraction in a convex refractor is $\frac{3}{2}$, and AQ be *five* times the radius, then Aq is also *five* times the radius.

6. Verify by numerical examples the following statement with respect to a convex refractor, whatever may be the values of the index of refraction and the radius: let the distance AQ be equal to the product of the radius into the sum of the index of refraction and unity, divided by the difference between the index of refraction and unity; then Aq is equal to AQ.

7. In a convex refractor the index of refraction is $\frac{3}{2}$, the radius of the surface is 12 inches, and AQ is 48 inches: find Aq, and thus shew that Q and q are equally distant from O.

8. Shew by other examples that if the index of refraction in a convex refractor is $\frac{3}{2}$, and AQ be *four* times the radius, then Q and q are equally distant from O.

9. Verify by numerical examples the following statement with respect to a convex refractor, whatever may be the values of the index of refraction and of the radius: let the distance AQ be equal to the diameter divided by the index of deviation, then Aq is equal to the product of AQ into the index of refraction: and Q and q are equally distant from O.

10. Verify by numerical examples the following statement with respect to a concave refractor whatever may be the values of the index of refraction and of the radius: if AQ is half the radius then Aq is the product of the radius into the index of refraction divided by the sum of the index of refraction and unity.

XXV. IMAGES FORMED BY REFRACTION.

1. Explain how it is that to a person viewing the bottom of a river the depth seems less than it really is. If the real depth is 12 feet determine the apparent depth.

2. A stick is partly immersed in water and inclined to the surface: draw the image of the part immersed and trace the course of the pencil from any point of it to the eye.

3. State what would be the appearance of a man standing on the brink of a lake to an eye under the water.

4. A straight rod is immersed in water at an inclination of 45^0 to the horizon; the upper end of the rod is just at the surface of the water, and the lower end 4 feet below it: find the length of the image as seen by an eye above the water.

5. In the diagram of Art. 262 suppose AO to be 12 inches and AQ to be 24 inches: find the proportion of PQ to pq when the index of refraction is $\frac{3}{2}$; also when it is $\frac{4}{3}$.

6. Verify by numerical examples the following statement with respect to the diagram of Art. 262: if AQ is equal to the radius divided by the index of deviation, then PQ is twice pq.

7. Verify by numerical examples the following Rule for finding the number which expresses the proportion of PQ to pq in the diagram of Art. 262: multiply AQ by the index of deviation, and divide by AO; then increase the result by unity.

XXVI. SUCCESSIVE REFRACTION AT PLANE SURFACES.

1. Trace the course of a pencil by which an assigned point of the image is seen when an eye looks at an object through a transparent plate.

2. A rod inclined at any angle to a plate of glass is seen by an eye on the opposite side of the plate: shew that the length of the image of the rod is equal to the length of the rod. Is the image formed by refraction at the first surface of the same length?

3. The refracting angle of a prism of glass is 60°: find by the aid of the Table in Art. 161 the angle at which a ray is incident which passes through the prism with the least deviation.

4. The least deviation of a ray refracted through a prism with a refracting angle of 60° is 90°: shew that the refractive index of the prism for that ray is about 1·93.

5. A ray of light can just be made to pass through a prism whose refracting angle is 75°: find the refractive index of the prism.

6. Shew how a ray of light may be turned at right angles to its original direction by making it pass in and out of a rectangular glass prism, with intermediate total reflection.

7. Light from a bright point 6 inches above the surface of still water enters the water, is reflected from the bottom of the vessel which is 2 feet deep, and emerges: find the position of the final image formed.

8. If the refracting angle of a prism be greater than the critical angle, shew that rays incident from that side of the normal which is towards the edge will not pass through the prism.

9. If the refracting angle of a prism be greater than the critical angle, find what rays will pass through the prism.

10. A small pencil of light from an object Q passes through a triangular prism of glass, being once reflected at the inner surface: draw a diagram of the course of the rays, shewing the position of the focus after each refraction or reflection.

11. Find the least deviation when a ray of light passes through a prism of ice the refracting angle of which is 4°.

12. In the diagram of Art. 266 supposing the plate to be of glass, the index of refraction 1·5, and the angle of incidence 45°, find the angles RQK and QRH.

13. In the preceding Example if QK is a quarter of an inch, find QR: see Art. 159.

14. Hence find QH.

XXVII. LENSES.

1. Parallel rays falling on a convex lens are brought to a focus at a point 6 inches from the lens: find the focus of a pencil of rays proceeding from a point 18 inches from the lens.

2. The focal length of a convex lens is 2 feet: find the focus of a pencil of rays proceeding from a point 18 inches from the lens.

3. Give an elementary proof that the focal length is positive if the lens be thinnest in the middle.

4. The focal length of a convex lens is 8 inches: find the distance between Q and q when the distance of Q from the lens is 24 inches, 16 inches, 12 inches respectively.

5. Shew that the preceding Example and others of the same kind are consistent with the following general statement which may be demonstrated by mathematics: as Q moves from a very remote distance on the right hand up to G in the diagram of Art. 283 the distance between Q and q is least when Q is at twice the distance of G from the lens.

6. Shew how the focal length of a convex lens may be found by the aid of the example given in Art. 284.

7. In Art. 287 suppose that AQ is 10 inches, that BR is 12 inches, and the thickness half an inch: find AC and BC.

8. Find in the preceding Example the distances of C from the points where the axis meets the surfaces of the lens: and shew that the distances are in the same proportion as the radii.

9. A cylindrical pencil of light falls upon and envelopes a refracting sphere: find the refractive index in order that the extreme rays may emerge in a direction perpendicular to the axis of the cylinder.

XXVIII. VISION THROUGH A LENS. MAGNIFICATION.

1. Suppose the focal length of a lens to be 3 inches: find, as in Art. 302, the magnifying power.

2. Determine the focal length of a lens, so that the magnifying power, found as in Art. 302, may be 10.

3. Shew by examples, that if an object is placed before a concave lens, at the distance of twice the focal length, the height of the image is two-thirds of the height of the object.

4. A small object is 12 inches from a lens, and the image is 24 inches from the lens, on the other side of it: find the focal length of the lens.

5. A candle is placed 6 feet from a wall, and a distinct image of the flame is produced on the wall by a lens held 1 foot from the candle: shew that a distinct image will also be produced when the lens is 5 feet from the candle.

6. Compare the heights of the two images in the preceding Example.

7. It is required to determine the distance from a small luminous object at which a convex lens of 10 inches focal length must be placed, in order that a real image may be formed 6 feet from the object: shew that we require two numbers such that their sum is 72, and the sum of their reciprocals ·1; and by trial, or otherwise, find the numbers.

8. Compare the height of the image with that of the object in the preceding Example.

XXIX. COLOUR.

1. The refracting angle of a prism of crown glass of the same kind as in Art. 313 is 4^0; a ray of light is incident at the angle which corresponds to the least deviation: find the angle between the red and the violet emergent rays: see Art. 278.

2. The radius of each surface in an equi-convex lens is 10 inches; the lens is made of the crown glass of Art. 313; find the focal length for the mean red rays, and also for the mean violet rays.

3. Suppose, as Newton does, that in a glass lens the index of refraction for the most refrangible rays is $\frac{78}{50}$, and for the least refrangible rays is $\frac{77}{50}$: then shew by examples that the principal focus of the most refrangible rays will be nearer to the lens than the principal focus of the least refrangible rays by about $\frac{1}{27}$ or $\frac{1}{28}$ of the whole focal length.

4. In the experiment of Art. 306 let a second prism be placed with its edge *vertical* so as to receive the light after leaving the prism ACD: explain what will be seen on a screen placed so that the emergent light may fall on it.

5. The rays of the sun are received on a large converging lens, the focus being visible by the dust floating in the air; a screen placed a little in front of the focus shews a white circle surrounded by a red fringe, and placed a little behind the focus shews a white circle surrounded by a blue fringe; explain this.

XXX. ACHROMATISM.

1. In a diamond the index of refraction for the extreme violet ray is 2·467, for the mean ray 2·439, and for the extreme red ray 2·411: find the index of dispersion.

2. In a certain piece of crown glass the index of refraction for one ray is 1·530, and for another 1·536; the corresponding values in a certain piece of flint glass are 1·635 and 1·648 respectively: shew that the *dispersion* of the crown glass is about five-ninths of that of the flint glass.

3. A compound achromatic lens is to be formed from the pieces of glass mentioned in the preceding Example, so as to have a focal length of 6 feet: shew that the conditions are satisfied by taking the lens of crown glass with a focal length of 32 inches, and the lens of flint glass with a focal length of 57·6 inches.

4. The index of dispersion of a medium is ·04, and the index of refraction for the mean ray is 1·54: shew that if a lens is made of this medium the focal length for violet rays is about ·96 of the focal length for red rays.

XXXI. MISCELLANEOUS EXAMPLES.

1. A luminous point is placed three inches from the surface of a plane reflector, and five inches from the surface of a second plane reflector at right angles to the former: find how many images will be visible to a person standing in front of both reflectors.

2. A distant object subtends an angle of 2^0 at the centre of a concave reflector; the focal length of the reflector is 20 inches: shew that the height of the image in inches is about 20 times the sine of an angle of 2^0.

3. Shew that a concave air lens plunged into water produces an image like that produced by a convex lens of water in air.

4. State what differences present themselves in the spectrum when the prism is changed, and also when the nature of the light is altered.

5. Two thin convex lenses of equal focal length are placed close together, so as to have a common axis; a pencil of rays parallel to the axis falls on one lens: shew that it comes to a focus at the distance of half the focal length from the lens.

XXXII. MISCELLANEOUS EXAMPLES.

1. A person views his own image as formed in a looking glass inclined at an angle of 45^0 to the horizon: state the position and the magnitude of the image, and its inclination to the horizon.

2. In the preceding Example draw a diagram shewing the course of the rays by which the person sees the image of his feet.

3. The focal length of a concave reflector is 360 inches: determine the linear diameter of the image of the moon formed by this reflector, supposing the angular diameter of the moon to be half a degree.

4. If a real image five times as high as the object is to be thrown on a screen at a distance of 36 inches from the object, find the focal length of the lens to be employed.

5. A window bar is viewed through a prism, the edge of which is parallel to the bar: shew that the side of the bar which is nearer to the edge of the prism is fringed with red and orange, and the other side with violet and blue.

XXXIII. MISCELLANEOUS EXAMPLES.

1. A candle flame is placed at a distance of 3 feet from a concave mirror formed of a portion of a sphere of which the diameter is 3 feet: determine the nature and the position of the image of the candle flame produced by the mirror.

2. A man six feet high stands in front of a vertical plane mirror: find the least height of the mirror, and its exact position above the floor in order that the man may just see the image of his whole figure.

3. The plane surface of a glass hemisphere is silvered, and an eye is placed close to the curved part so that the radius passing through the eye is perpendicular to the plane base: find the position of the image which the eye sees of itself, and shew by examples, that it is at a distance equal to three times the radius behind the plane base.

4. State the nature of the changes which would occur in the Primary Rainbow if the refractive index of water were larger; for example, equal to that of glass.

5. Shew that the Secondary Rainbow may sometimes be seen when the Primary Rainbow is not.

XXXIV. MISCELLANEOUS EXAMPLES.

1. The sun is about 90000000 miles distant, and his diameter about 900000 miles: find the diameter of the image of the sun formed by a spherical mirror of 10 feet radius.

2 The focal length of a lens in air is 5 feet: find the focal length of the lens when placed in water.

3. A pencil of rays from a luminous point is brought to a real focus by a convex lens; at this focus is placed a plane mirror at right angles to the axis of the lens: indicate the course of the rays after reflection.

4. Shew that in the Astronomical Telescope if half the object glass is stopped the field of view suffers only in brightness; but that in Galileo's Telescope if half the object glass is stopped, part of the field of view is destroyed.

XXXV. MISCELLANEOUS EXAMPLES.

1. A small object is placed before a concave spherical reflector, at a distance from it equal to one-third of the radius: exhibit by a diagram the pencil by which any point of the image is visible to an eye in a given position.

2. Two cups of the same depth are filled with different liquids; both cups appear shallower than when they were empty, but one cup appears shallower than the other: explain this.

3. Trace the course of a pencil of rays from any point of the object to the eye in the case of Gregory's Telescope.

4. State the respective advantages of Newton's Telescope and Herschel's in making astronomical observations.

5. Newton's Telescope is in adjustment for viewing a very distant object: shew how it must be altered for viewing an object 200 yards off, the focal length of the reflector being 10 feet.

XXXVI. MISCELLANEOUS EXAMPLES.

1. A man standing before a vertical looking glass can always see the same identical portion of his person whatever may be his distance from the looking glass: illustrate this statement by a diagram.

2. A luminous point is placed between two plane mirrors which are inclined to each other, and are in assigned positions: find the limits between which an eye must be placed to see the second image.

3. The distance between an object and its image when they are equally distant from a plano-convex glass lens is 24 inches: find the radius of the spherical surface.

4. The focal length of a convex lens is 6 inches; a small object is placed 8 inches in front of the lens: shew that the image is 2 feet behind it.

5. The focal length of a convex lens is 6 inches; rays are converging to a point 3 inches behind the lens: shew that after passing through the lens they will converge to a point 2 inches behind it.

6. Shew how a convex lens of focal length 6 inches, held at a distance of 8 inches from the eye, can be used with the assistance of a looking glass as a compound microscope to view the eye itself: if the least distance of distinct vision is 6 inches find the magnifying power.

XXXVII. THE EYE.

1. A short-sighted person can see distinctly at the distance of nine inches: find the focal length of a lens which will enable him to see distinctly at the distance of a yard.

2. A person who reads small print at the distance of two feet, finds that with spectacles consisting of plano-convex lenses he can read it at the distance of one foot: find the radius of the curved surface of either lens.

3. A long-sighted person who can see most distinctly at the distance of two yards, uses glasses of two feet focal length: find at what distance from the glasses the object should be placed.

4. Explain why short-sighted persons are in the habit of partially closing their eye-lids in order to see more distinctly.

5. Explain for what purpose the eye of an ox is very full and convex.

6. Shew that if from any point of an object the light falls on the insensible part of one eye it will not fall also on the insensible part of the other eye.

7. Explain how it is that we estimate distances much less accurately with one eye than with both eyes.

8. Explain how it is that an unknown object in a mist seems greater than it really is.

XXXVIII. MISCELLANEOUS EXAMPLES.

1. Shew within what space an eye must be situated in order to see a given point by reflection at a plane mirror; and within what space a point must be situated to be seen by the eye in a given position.

2. A person can see distinctly objects at any distance exceeding 30 inches: find to what extent a convex lens of 30 inches focal length will enlarge his range of vision.

3. A person is able to see distinctly at a distance of not more than 10 inches: determine the focal length of the lens which will enable him to see at a distance of 12 inches.

4. In the preceding Example find the range of distinct vision with the lens if the person with his naked eye cannot see distinctly at a distance less than 4 inches.

5. The lens of a fish's eye is extremely convex, being almost a sphere: explain the advantage of this.

6. A man six feet high subtends at the eye of an observer an angle of half a degree: find the distance of the man from the observer; find also the diameter of the image on the retina of the observer.

XXXIX. MISCELLANEOUS EXAMPLES.

1. Shew that a person may see himself completely in a vertical mirror the height of which is half his own height.

2. A plane mirror is placed at right angles to the axis of a concave spherical reflector, nearer to the principal focus than to the face, and between them; rays from a very distant object fall directly on the spherical mirror: trace the pencil by which an eye will see the image after two reflections at each mirror.

3. A small object is placed between the point G of Art. 283 and the centre of the lens, the focal length of the lens being equal to the least distance of distinct vision; an eye retiring from the opposite side of the lens first sees a distinct image, when it has moved through half the focal length: shew by examples that the distance of the object from the lens is one-third of the focal length.

4. The object glass of an Astronomical Telescope is bisected, so as to form two semicircular lenses, and the two halves are placed at a small distance apart, but equally distant from the eye glass: state what will be seen on looking at a star.

XL. MISCELLANEOUS EXAMPLES.

1. If a circular arc be placed before a convex spherical reflector, and be concentric with it, shew that the image will also be a circular arc concentric with the object.

2. The faces of two walls of a room meeting at right angles are covered with plane mirrors: shew that a person will be able to see but one complete image of himself in either wall.

3. An eye is placed close to a sphere of glass, a portion of the surface of which most remote from the eye is silvered; shew by numerical instances that an image of the eye is formed in front of the silvered part at the distance of two-thirds of the radius from it.

4. Shew also in the preceding Example that as the rays after being reflected pass out through the first surface of the sphere they form an image at the distance of four-fifths of the diameter behind it.

5. An eye is placed close to a sphere of glass; a portion of the surface of which, most remote from the eye, is silvered; shew that, assuming 8 inches to be the least distance of distinct vision, the eye cannot see a distinct image of itself unless the diameter of the sphere be at least 10 inches in length.

XLI. MISCELLANEOUS EXAMPLES.

1. Given three points, determine in how many ways they may be the positions of an eye, a luminous origin, and an image formed of it by reflection: and construct in each case the position of the mirror.

2. A lighted candle is placed before a concave spherical mirror, the candle being at right angles to the axis of the mirror, and in the same plane with it: find how the image of the flame moves as the candle burns.

3. A convex lens of very short focal length is held at a distance from the eye, and objects are viewed through it: shew that they will be seen in miniature.

4. A luminous point is placed between two plane mirrors which are inclined at an angle of 30^0: determine the number of images formed.

5. Draw a diagram corresponding to that of Art. 448, supposing the plane mirrors inclined at an angle of 45^0.

XLII. MISCELLANEOUS EXAMPLES.

1. The North and the East sides of a room, and also the ceiling, are formed of plane mirrors; a man stands in the room facing North East: find how many images of himself he can see, partially or entirely, and the position of each of them.

2. A bright point moves in front of a concave mirror along the axis, starting from a remote point and going up to the mirror: shew by numerical examples that there are three positions of the bright point for which the distance between the point and its image is three quarters of the radius, namely, when the bright point is at a distance from the mirror equal to three-halves of the radius, or three-fourths of the radius, or one-fourth of the radius.

3. An eye is placed close to the surface of a sphere of glass which is silvered at the back: shew that the image which the eye sees of itself is three-fifths of the natural size.

4. Determine the velocity of light according to Fizeau's experiment: the distance between the origin and the plane reflector was 8663 metres; the wheel had 720 teeth, and it made 126 revolutions in 10 seconds when the first eclipse took place.

XLIII. THEORIES OF LIGHT.

1. Point out the phenomena which the Undulatory Theory of Light is most successful in explaining, and those in which difficulties still remain.

2. Describe some experiment by which it is shewn that the presence of two lights does actually in certain cases produce darkness.

3. If the rays of light of various colours did not all travel with the same velocity through the celestial spaces, shew that a satellite of Jupiter, undergoing eclipse, would become tinged with one colour just before disappearing, and with another just after reappearing.

4. Considering light as corpuscular we see from the fact of reflection that the velocity of a particle can be changed from 186000 miles per second in *one* direction to the same amount in the *opposite* direction, and that the change is made in an interval too brief to be appreciated: shew that the force of gravity if continued uniform would take about two years to destroy this velocity and reproduce it in the opposite direction.

XLIV. NEWTON'S RINGS.

1. Describe Newton's Rings as seen by homogeneous yellow light; and shew how the phenomena are explained by the principle of interference.

2. Describe the appearance which an iridescent soap-bubble would present in light of a single colour.

3. State the effect produced on Newton's Rings by putting water between the lenses which are used.

4. Taking the velocity of light at 186000 miles per second, and the wave length for a green colour to be one fifty-thousandth of an inch, find how many vibrations will be made in a second.

XLV. MISCELLANEOUS EXAMPLES.

1. A small concave spherical reflector, whose radius is 16 inches, is placed facing a large concave spherical reflector, whose radius is 20 feet, so as to have the same axis, and at a distance of 12 feet from it. A luminous point is placed on the axis at a distance of 80 feet in front of the large reflector, and behind the small reflector: find the position of the image after reflection at the small mirror.

2. When a lens is used for magnifying an object, as in a pair of spectacles, the object does not appear inverted; but when used for the purpose of throwing the image of an object on a screen the image appears inverted: explain this.

3. An object is seen through an Astronomical Telescope: if the object approach nearer should the eye glass be pushed in or out that the Telescope may be in the same state of adjustment for the eye as before?

4. What are the practical difficulties which limit the magnifying power of the telescope?

5. A small object is placed before a convex lens at the distance of a fourth of the focal length: shew that the image is virtual and at the distance of one-third of the focal length.

6. Two convex lenses have a common axis and equal focal lengths, and their distance is two-thirds of the focal length of either of them: find the point on the axis from which rays must diverge, in order that after refraction through both lenses the emergent pencil may consist of parallel rays.

XLVI. MISCELLANEOUS EXAMPLES.

1. A circular disc six inches in diameter is placed with its plane parallel to a vertical wall, and a person puts his eye any where in the plane of the disc: shew that a plane circular reflector, three inches in diameter, will, if placed in a proper position on the wall, just enable the person to see the whole image of the disc.

2. Objects seen across the top of a lime kiln in action appear unsteady and indistinct: explain the reason of this.

3. Explain what would be the effect in an Astronomical Telescope of covering a portion of the object glass; and also the effect of covering a portion of the eye glass.

4. A piece of Iceland spar is placed over a spot of ink: shew that an eye looking down through the spar sees two images, *one nearer to the eye than the other.*

XLVII. MISCELLANEOUS EXAMPLES.

1. Two spherical reflectors of equal radii are placed so that the centre of one is on the surface of the other: if a luminous point be situated at any point on their common radius find its image after any number of reflections.

2. A short-sighted person in order to see a small object distinctly finds that he has to hold it at the distance of six inches from his eye: determine the focal length of the lens which enables him to see the same object distinctly at the distance of nine inches.

3. The image of a lamp flame is to be formed on a screen at the distance of six feet by means of a convex lens: find the greatest focal length admissible, and the proportion which the height of the image bears to the height of the object.

4. In an Astronomical Telescope the focal length of the object glass is 3 feet, and that of the eye glass is 1 inch: determine the distance between the glasses when the Telescope is adjusted for a person who sees objects most distinctly at a distance of 6 inches.

XLVIII. MISCELLANEOUS EXAMPLES.

1. A person holding vertically in his hand a plane mirror sees reflected in it part of a wall behind him: shew that if when he changes his position he always holds the mirror at such a distance from him as to be himself half way between it and the wall he will always see reflected the same extent of the wall.

2. An Astronomical Telescope which has been adjusted by a long-sighted person is to be used by a short-sighted person: in what direction will the latter have to move the eye lens?

3. The focal length of an object glass is 10 feet: construct a Ramsden's eye piece so that the magnifying power may equal 200.

4. Define *polarised light;* and shew how the definition could be applied to determine if a given ray is polarised or not.

5. The focal length of the object glass of an Astronomical Telescope is 5 feet, and the Telescope is adjusted for viewing a very distant object: find how much the eye glass must be drawn out to view an object 605 feet from the object glass.

XLIX. POLARISATION BY REFLECTION.

1. Verify by the Table of Art. 161 that for water the polarising angle is about 53^0.

2. A ray of light issues from a point 3 feet above the surface of still water, and falling obliquely on the surface is divided into two parts, one of which is reflected and the other refracted: find the angle of incidence of the ray so that the reflected and the refracted rays may be at right angles to each other; find also the distance of the point of incidence from the origin.

3. Find the angle of polarisation by reflection when light passes from water to glass.

4. On a glass plate is placed a parallel stratum of water: shew that the ray reflected from the common surface will be polarised when the angle of incidence on the first surface of the water is about 90^0.

5. In the experiment of Art. 538 suppose that the ray *SR* is entirely extinguished; then by breathing on the reflector *CD* the ray becomes visible: explain this.

L. MISCELLANEOUS EXAMPLES.

1. On one wall of a room is placed a plane mirror, and opposite to it a convex mirror at the distance of 15 feet. A man stands with his back to the convex mirror, and looking into the plane mirror sees the back of his head in the reflection of the convex mirror. If the radius of the spherical mirror be 3 feet, and the man stand 3 feet from it, shew that the image of his head is one-third of the height it would be if both mirrors were plane, and its distance from the eye is 28 feet.

2. A bright point is placed in front of a sphere of glass: trace the course through the sphere of a ray which makes a small angle with the straight line which joins the bright point to the centre.

3. If the bright point in the preceding Example be at the distance of the radius from the sphere, find the distance of the images after the first and second refractions, taking in succession various numerical values for the radius.

4. A slender pencil of parallel rays falls on a refracting sphere, the axis of the pencil coinciding with a diameter of the sphere: trace the course of the rays through the sphere.

5. Shew by numerical instances that the rays in the preceding Example converge to a real focus, the distance of which from the centre of the sphere is a certain fraction of the radius; the numerator of the fraction being the index of refraction, and the denominator twice the index of deviation.

LI. MISCELLANEOUS EXAMPLES.

1. A convex lens, held 12 inches from a wall, forms on the wall a distinct image of a candle; when the lens is held 6 inches from the wall it is found that to produce a distinct image of the candle on the wall the distance of the candle from the lens must be doubled: find the focal length of the lens.

2. The focal length of the object glass of an Astronomical Telescope is one foot: find how far the eye glass must be moved from the standard position for viewing an object at the distance of 40 feet from the object glass.

3. What would be the effect in Galileo's Telescope of increasing the size of the object glass, and what would be the effect of increasing the focal length of it, the eye glass remaining unchanged?

4. What would be the effect in Galileo's Telescope of increasing the size of the eye glass, and what would be the effect of increasing the focal length of it, the object glass remaining unchanged?

5. The focal length of the object glass of an Astronomical Telescope is 50 inches; there is a Huygens's eye piece, the field glass of which has a focal length of 2 inches: find the distance between the object glass and the field glass of the eye piece when the Telescope is in proper adjustment for distant objects.

LII. MISCELLANEOUS EXAMPLES.

1. A Galileo's Telescope is adjusted so that a pencil from an object 289 feet from the object glass emerges as a parallel pencil; the focal length of the object glass is one foot, and that of the eye glass is one inch: shew that if the axis is directed to the sun, and a piece of paper held 25 inches from the eye glass, an image of the sun is formed on the paper.

2. In an Astronomical Telescope, adjusted as usual, the focal length of the eye glass is 3 inches: find how far a person must push the eye glass inwards if he sees distinctly an object 6 inches from his eye.

3. Determine the polarising angle when light is reflected from a diamond.

4. Between two plates of tourmaline cut as in Art. 544 and placed with their axes at right angles, a third such plate is interposed: state the result on looking through the plates.

5. Find the focal length of the object glass of a telescope which shall magnify 200 times, where the eye piece is Huygens's, the focal lengths of whose lenses are ·4 and 1·2 inches respectively.

LIII. INTRODUCTION TO HEAT.

1. Find the degree in the Centigrade scale which corresponds to 113° F., and that which corresponds to 140° F.

2. Find the degree in Fahrenheit's scale which corresponds to 15° C., and that which corresponds to 35° C.

3. Find the degree in the Centigrade scale which corresponds to 4° below zero in Fahrenheit's scale, and that which corresponds to 31° below zero.

4. Find the degree in Fahrenheit's scale which corresponds to 30° below zero in the Centigrade scale, and that which corresponds to 40° below zero.

5. Find the degree in the Centigrade scale which corresponds to 5° F., and that which corresponds to 14° F.

LIV. EXPANSION OF SOLIDS.

1. State the manner in which heat generally affects the size of bodies, and point out any exceptions to the general rule.

2. The volume of a glass vessel is 450 cubic inches at 32° F.: find the volume at 177° F., the coefficient of cubical expansion being taken at ·000014.

3. The railway from London to Edinburgh is about 400 miles long; the extreme variation of temperature between winter and summer is 90° F.: find the variation in the whole length of the rails of wrought iron.

4. A piano, which has been tuned in a drawing room in a morning, may produce discords in the evening when the room is heated by the presence of a large evening party: explain this.

5. If a *gridiron pendulum* were to be constructed of copper and platinum, having respectively the coefficients of expansion ·0000096 and ·0000048, how many bars of each metal would be required?

6. A metre denotes the length at 32° F. of a certain standard platinum bar; and a yard denotes the length at 62° F. of a certain standard brass bar; further it is known that one metre is equal to 3·2808992 feet: find the proportion between the lengths of the two bars at 80° F., the coefficients of expansion being as in Art. 578.

LV. EXPANSION OF LIQUIDS.

1. A flask with a long neck contains alcohol, which fills the flask and rises to some height in the neck; the flask is placed in hot water, and the liquid at first *falls* in the neck as if it were contracting: explain this.

2. Shew that the specific gravity of water at 84° C. will be about ·96, taking the specific gravity of water at its maximum density as unity, and the coefficient of expansion for water at ·0005 for 1° C.

3. Taking the coefficient of the absolute cubical expansion of mercury at ·00018, and the coefficient of the linear expansion of glass at ·0000086, find the coefficient of the apparent expansion of mercury in glass.

4. An English barometer is furnished with a brass scale which gives accurate inches when the temperature is 62^0 F.; the mercury stands at the apparent height of 30 inches when the temperature is 75^0 F.: find the true height expressed with respect to mercury at the temperature 32^0 F. The coefficient of the cubical expansion of mercury may be taken at ·0001, and that of the linear expansion of brass at ·00001.

LVI. EXPANSION OF GASES.

1. Shew that 30 cubic inches of air would expand to about 41 in passing from 0^0 C. to 100^0 C.

2. A tight bladder is carried from a very hot to a very cold room: state what will be the effect of the cold on the air in the bladder.

3. The specific gravity of oxygen at 87^0 F. is the same as that of air at 32^0 F.: denoting this by unity, determine the specific gravity of oxygen at 32^0 F., the pressure being constant.

4. A gas measures 98 cubic inches at 185^0 F.: find what it will measure at 10^0 C. under the same pressure.

5. A gallon of air, that is 277·25 cubic inches, is heated under constant pressure from 0^0 C. to 60^0 C.: calculate the volume of the air at the latter temperature.

6. A bottle is filled with air at the atmospheric pressure of 15 pounds to the square inch, and at the temperature 20^0 C.; the bottle is securely stopped, and the temperature reduced to 0^0 C.: find what will be the pressure of the air in the bottle.

7. A room is calculated to contain 3000 cubic feet of air at 10^0 C., and under the pressure of 30 cubic inches of mercury: find what would be the volume of the same quantity of air if it were measured at 0^0 C. and 31 inches pressure.

8. If 50 cubic inches of air at 5^0 C. *below* 0^0 C. are raised to 15^0 C. under the same pressure, find the volume.

9. The volume of a gas at 0^0 F. being known, it is required to find its volume at any other temperature, say 60^0 F.: shew that we must first multiply by $1+\frac{32}{491}$, and then the result by $1+\frac{28}{491}$; and similarly in other cases.

10. Shew in the preceding Example that almost exactly the same result is obtained if instead of the *two* multiplications we multiply *once* by $1+\frac{60}{491}$: and that in like manner having given the volume at 0^0 F. we may obtain the volume at 80^0 F. nearly by multiplying by $1+\frac{80}{491}$.

LVII. THERMOMETER AND PYROMETER.

1. Air which is known to have the volume 100 cubic inches at 0^0 C. is found to have expanded to 120 cubic inches without any change of pressure: determine the temperature.

2. Air which is known to have occupied 300 cubic inches at the temperature 0^0 C. and under the pressure of 28 inches, is found to have a volume of 320 cubic inches when the pressure becomes 30 inches, and the temperature is raised: determine the new temperature.

3. Mercury is put into a graduated glass tube at a certain temperature, say 30^0 C., and occupies a certain number of divisions, say 80: if the tube were cooled down to 0^0 C. without changing the temperature of the mercury, find how many divisions the mercury would occupy.

4. Shew from the Table of Art. 578 that if the volume of a glass vessel at 0^0 C. is unity its volume at 50^0 C. is about 1·0013.

5. Shew from Art. 589 that if the volume of a quantity of mercury at 0^0 C. is unity its volume at 50^0 C. is about 1·009.

6. The bulb of a thermometer and the tube up to the freezing point mark are immersed in water at the temperature 50^0 C., while the rest of the tube has the temperature 0^0 C.: if the volume of the mercury at the freezing point be denoted by unity, find the volume of the mercury which passes above the freezing point mark when the mercury is raised from 0^0 C. to 50^0 C.

7. Find the volume to which the preceding result is reduced when the temperature is lowered to 0° C.

8. Shew that this would occupy $\frac{50}{1+\cdot 009}$ divisions of the stem at 50° C.

9. Shew that this will occupy $\frac{50\,(1+\cdot 0013)}{1+\cdot 009}$ divisions of the stem at 0° C.

10. The bulb and the tube of a thermometer up to the freezing point mark are immersed in water at 50° C., while the rest of the tube has the temperature 0° C.: find what will be the reading of the thermometer.

LVIII. MELTING OR FUSING.

1. Find what weight of ice at 32° F. will be melted if put in a pound of water at 102° F.

2. One pound of ice at 0° C. and three pounds of water at 79° C. are mixed together in a close vessel the sides of which are supposed to be impervious to heat: find the temperature of the water after the melting of the ice.

3. A mixture is made of 3 pounds of water at 12° C. with 3 pounds of water at 16° C.: find the temperature of the mixture.

4. A mixture is made of 4 pounds of water at 7° C. with 6 pounds of water at 12° C.: find the temperature of the mixture.

5. A mixture is made of 9 pounds of water at 31° C. with 2 pounds of ice at 0° C.: find the temperature of the mixture.

6. The heat produced by the complete combustion of one pound of carbon is sufficient to convert 100 pounds of ice at 0° C. into water at 0° C.: find how many pounds of water would be raised by the same heat from 0° C. to 1° C.

7. Hot tea is slightly cooled by putting sugar into it, and hot soup by putting salt into it: explain this.

LIX. BOILING.

1. An ounce of steam at 100° C. is added to a pound of water at 0° C.; find the temperature of the mixture.

2. Find how much steam at 100° C. is required to raise the temperature of 540 ounces of water from 0° C. to 100° C.

3. Shew that a pound of steam at 100° C. in becoming water at 40° C. gives out as much heat as ten pounds of water in cooling down to the same temperature.

4. Find how much ice at 0° C. can be converted into water at 0° C. by an ounce of steam at 100° C.

LX. THE THREE STATES.

1. A piece of ice under the pressure of the atmosphere is exposed to a temperature below 0° C.; the temperature is gradually raised to 100° C.: what changes of volume and state take place?

2. Unglazed pottery is sometimes used to hold water and to keep it cool: explain this.

3. Carbonic acid may be reduced to the liquid form by strong pressure; when the pressure is removed the liquid returns to the state of gas, but some of it becomes *solid* carbonic acid: explain this.

4. Shew that heat is necessary for the production of snow.

LXI. VAPOURS.

1. Assuming that 1000 cubic inches of dry air at the temperature 0° C., and under the pressure of 30 inches of mercury weigh 328 grains, find the weight of 1000 cubic inches of dry air under the same pressure when the temperature of the air is 20° C.

2. The atmosphere is at the temperature 20° C., and the pressure is 30 inches of mercury; the dew point is 10° C.: find the weight in grains of the dry air in 1000 cubic inches of the mixture, the pressure of aqueous vapour at the temperature 10° C. being ·35 inches.

3. Find the weight of the vapour in 1000 cubic inches of the mixture in the preceding Example.

4. Find the extreme weight of vapour which could be supported in 1000 cubic inches of air at 20° C. under the pressure of 30 cubic inches of mercury; the pressure of aqueous vapour at 20° C. being ·66 inches.

5. Hence shew that the *proportion* which expresses the state of the atmosphere with respect to moisture in Example 2 is about $\frac{36}{66}$.

LXII. MISCELLANEOUS EXAMPLES.

1. The rails from London to Manchester are about 190 miles long: supposing these rails to form one continuous piece at the temperature 0° C., find the increase of length at 30° C., taking the coefficient of expansion of wrought iron for 1° C. at ·000012.

2. If a volatile liquid, even when warm, is dropped on the hand a sensation of cold is felt: explain this.

3. The elastic force of aqueous vapour at 60° C. is measured by 5·86 inches: if a cubic foot of dry air at 60° C., and 30 inches pressure, be saturated with moisture, find what volume it will occupy, the pressure and the temperature remaining the same.

4. A quantity of dry air measures 1000 cubic inches at 10° C. and 30 inches of pressure: if the air be heated to 27° C. and saturated with aqueous vapour at that temperature, find the volume of the mass of air in order that the pressure may remain unchanged; the elastic force of aqueous vapour at 27° C. being about 1 inch.

5. A cubic foot of air is confined over water at 58° F.: if the air be well dried, find its volume, the pressure being constant at 30·24 inches, and the elastic force of aqueous vapour at 58° F. being ·48 of an inch.

6. A cubic foot of oxygen is collected over water at 47° C. under a pressure of 735 millimetres of mercury: find the volume of the oxygen when dry at the standard temperature and pressure of 0° C. and 760 millimetres; the elastic force of aqueous vapour at 47° C. being 79 millimetres.

LXIII. SPECIFIC HEAT.

1. A pound of iron at 99° C. is immersed in a pound of water at 0° C.: find how many degrees the temperature of the water will be raised, taking the specific heat of iron at ·1.

2. A globe of copper weighing a pound is heated to the temperature of 320°, and immersed in a pound of water at 70°; it is found that the common temperature of the two becomes 92°; determine the specific heat of copper.

3. A pound of cold iron is placed in a pound of hot water; the water loses 5° of temperature, find what the iron gains: see Art. 677.

4. A pound of lead at 100° C. is immersed in a pound of water at 0° C.; find the common temperature of both: see Art. 677.

5. A pound of water at 40° C. is mixed with a pound of mercury at 90° C.: find the temperature of the mixture: see Art. 677.

6. An ounce of platinum is heated to the temperature of a furnace and then plunged into fourteen ounces of water at 0° C.; the platinum and the water together acquire a temperature of 2° C.: find the temperature of the furnace, taking the specific heat of platinum at ·035.

7. When 43 ounces of water are mixed with 100 ounces of turpentine the temperature is found to be midway between those of the two substances: find the specific heat of turpentine.

8. Find how many pounds of air at constant pressure can be raised 1° of temperature by a pound of water as it cools through 1° of temperature.

9. The weight of a volume of water is about 768 times that of an equal volume of air under the pressure of the atmosphere: shew that a cubic foot of water in cooling through 1° would raise through 1° about 3240 cubic feet of air under the pressure of the atmosphere.

10. Supposing so much heat as would raise the temperature of a cubic foot of water through 1° is applied to 100 cubic feet of air under the pressure of the atmosphere, find how much the temperature of the air will be raised.

11. A pound of lead is heated to 100° C. and then put into a pound of water at 0° C.; also at the same time a pound of iron is heated to 100° C. and put into a pound of water at 0° C.: find in which of the two cases the temperature of the water will rise most.

12. If equal weights of hydrogen at 0° C. and of oxygen at 100° C. be mixed, find the resulting temperature.

LXIV. RADIATION.

1. Explain the experimental method of determining the comparative radiating powers of substances.

2. The air on a high mountain may be intensely cold although the sun is shining, and no cloud exists: explain this.

3. The bulb of a mercurial thermometer is exposed to heat: will any difference be produced in the rate of rising of the mercury if the bulb is covered with silver foil?

4. By what experiments is it shewn that radiant heat obeys the same laws of reflection as light?

LXV. MISCELLANEOUS EXAMPLES.

1. A pound of ice at 32° F. mixed with five pounds of water at 72° F. gives six pounds of water at 42° F.: find how much the quantity of heat required to melt any quantity of ice at 32° F. would raise the temperature of the same quantity of water at 32° F.

2. A piece of iron is heated to 100° C. by immersion in boiling water: describe the method by which we ascertain the quantity of heat given off by the iron in cooling down to 0° C.

3. Two thermometers are taken having bulbs precisely alike; one bulb contains water and the other mercury, and they are raised to the same temperature; if they are put under the same circumstances it is found that the mercury thermometer cools twice as fast as the other: explain this.

4. Equal volumes of alcohol and water cool under the same circumstances through the same number of degrees in 2 minutes 20 seconds, and 5 minutes, respectively: find the specific heat of alcohol, taking its specific gravity at ·8.

5. Explain how it is that even a thin mat is found sufficient to shield a tender plant from cold at night.

LXVI. CONDUCTION.

1. Suppose we are provided with bars of copper, silver, gold, and platinum: explain how we must proceed to determine the conductive powers of these metals.

2. A piece of platinum may be held in the hand while one end is red hot, but a piece of copper of the same length under such circumstances will speedily burn the fingers: explain this.

3. Two bars *A* and *B* of different metals, of exactly the same dimensions, have each one extremity exposed to the same source of heat; a piece of phosphorus is placed on each bar at the same distance from the heated end, and it is found that the piece on *A* is first inflamed: can we infer that the conducting power of *A* is greater than that of *B*?

4. In the *Norwegian cooking stove* food to be cooked is boiled for some time in a close vessel; the vessel is then taken from the fire and placed in a box where it is surrounded with a thick layer of felt, and the cooking is finished without further application of heat: explain this.

5. A kettle which has been in use for some time usually becomes *furred*, and then water takes a long time to boil in it: explain this.

LXVII. TERRESTRIAL HEAT.

1. When we go into a cellar out of a summer sun, the cellar feels *cold*, but when we go into it out of a wintry frost it feels *warm*: explain this.

2. Explain how it is that the summer on an island is not so hot, and the winter not so cold, as on a continent in the same latitude.

3. Shew that an *inch* of rain expresses a fall of nearly 4·7 gallons on a square yard of surface.

4. Shew that an inch of rain expresses a fall of about 22600 gallons on an acre of surface.

LXVIII. THERMODYNAMICS.

1. A weight of a ton is lifted by a steam engine to the height of 386 feet: find how many units of heat are required for this work.

2. A 68 pound cannon ball strikes a target with a velocity of 1400 feet per second: supposing all the heat generated by the collision to be communicated to 68 pounds of water, find how many degrees the temperature of the water would be raised.

3. A cannon ball weighing 7 pounds strikes an iron target with a velocity of 400 feet per second. Suppose the whole of the motion to be converted into heat, and the heat to be uniformly distributed through 70 pounds of the target, find what change of temperature will be produced.

4. The specific heat of tin is ·056; the latent heat is 25^{0}·6 F.: find the mechanical equivalent of the heat required to raise 6 pounds of tin from 374^{0} F. to its melting point 442^{0} F., and to melt it.

5. A ball of lead strikes a target with a velocity of 1000 feet per second: shew that the heat generated would be sufficient to fuse the lead.

6. Shew that to raise the temperature of a pound of iron from 0^{0} C. to 100^{0} C. an amount of heat is required which would lift about 7 tons of iron a foot high.

7. A ball of iron moving with a velocity of 1340 feet per second strikes a target: supposing the heat generated to be all confined to the ball, find how many degrees the temperature of the ball would be raised.

ANSWERS.

I. 1. About 5500 feet. 2. About 216 seconds. 3. 1143 feet per second. 4. 87^0 F. 5. About five elevenths of a second. 6. About 937 feet per second.

II. 2. About 3 to 1.

III. 1. About 2·56 seconds. 2. About 10·9 seconds.

IV. 1. About 3300 feet. 2. About 206 feet. 3. ·2, ·4, ·6 of a second.

V. 2. 280. 3. 2·54 feet. 4. 141; 94. 5. About 8·2 feet.

VII. 1. 80. 2. 25.

VIII. 1. 4. 2. 6. 3. 4. 4. 28.

IX. 1. Seventeen feet and a half. 2. Nearly two inches. 3. Two feet. 4. One foot.

X. 1. About a foot. 2. About three inches. 3. About 224.

XI. 1. 2. 2. 398 or 402.

XII. 1. 440. 2. 300, 360, 480.

XIV. 1. 16800 feet. 2. Nearly four inches. 4. Rather more than nine feet.

XV. 1. 55^0. 2. 64. 3. $4\frac{3}{8}$, $3\frac{1}{2}$, $2\frac{11}{12}$, $2\frac{3}{16}$ feet.

XVI. 1. $\frac{3}{4}, \frac{4}{9}, \frac{5}{12}$. 2. ·383, ·698, ·897. 3. $16\frac{1}{2}^0$, $45\frac{1}{4}^0$, $77\frac{3}{4}^0$.

XIX. 1. 45^0. 4. 60^0.

XX. 1. Focus 36 inches distant. 5. First focus 15 inches from the furthest part of the reflector; second focus $16\frac{4}{5}$ inches from the opposite part.

XXII. 1. 25^0 24′. 2. Index of refraction 2. 3. Three-eighths of a foot.

XXIV. 1. *Aq* is 20 inches. 2. Index $\frac{3}{2}$; *concave* refractor. 3. Index $\frac{3}{2}$; *convex* refractor.

XXV. 1. 9 feet. 4. 5 feet. 5. 2; $1\frac{2}{3}$.

XXVI. 3. $48^0\ 35'$. 5. 1·64. 7. 42 inches below the surface. 11. $1^0{\cdot}24$. 12. $RQK=28^0$, $QRK=62^0$, $QRH=17^0$. 13. ·28 of an inch. 14. ·08 of an inch.

XXVII. 1. At a distance of 9 inches. 2. Virtual focus at a distance of 72 inches. 4. 36, 32, 36 inches. 7. *AC* is five-elevenths, and *BC* is six-elevenths of twenty-one inches and a half. 9. The reciprocal of the sine of 45^0.

XXVIII. 1. $\frac{13}{3}$. 2. $\frac{10}{9}$ of an inch. 4. 8 inches. 6. One is 25 times the other. 7. 12 inches, 60 inches. 8. Five times, or one-fifth.

XXIX. 1. About ·02 of 4^0. 2. 9·15; 9·05 inches.

XXX. 1. ·039 nearly.

XXXII. 3. About 3 inches: see Example XXXI. 2. 4. Six inches.

XXXIII. 1. Inverted, diminished, twelve inches from the reflector. 2. The mirror must be 3 feet high; the height above the ground of the bottom of the mirror must be half the height of the man's eye.

XXXIV. 1. ·05 of a foot. 2. 20 feet.

XXXV. 5. Pull out the eye glass about two inches.

XXXVI. 3. 3 inches. 6. 2.

XXXVII. 1. 12 inches. 2. 12 inches. 3. 18 inches.

XXXVIII. 2. To objects at a distance exceeding 15 inches. 3. 60 inches. 4. Between 12 inches and $4\frac{2}{7}$ inches. 6. 12×57 feet; $\frac{1}{228}$ of an inch.

XLII. 4. The time was $\frac{1}{2} \times \frac{1}{7{\cdot}20} \times \frac{1}{12{\cdot}6}$, that is $\frac{1}{18144}$, of a second; the velocity in metres per second was $18144 \times 2 \times 8663$, that is 314362944.

XLV. 1. Four feet behind the small mirror. 6. One-fourth of the focal length from the first lens.

XLVII. 1. See Example XX. 6. 2. 18 inches focal length. 3. 18 inches; the heights of the object and image are equal: see Example XXVII. 5. 4. 36⅗ inches.

XLVIII. 3. The focal length of each lens must be four-fifths of an inch. 5. Half an inch.

XLIX. 2. 53°; the distance in feet $= 3 \div \sin 37° = 5$ nearly. 3. About 49°.

L. 3. At the distance of three times the radius in front; at the distance of five times the radius behind.

LI. 1. 4 inches. 2. Four-thirteenths of an inch. 5. 49 inches.

LII. 2. One inch. 3. About 68°. 5. 120 inches.

LIII. 1. 45° C.; 60° C. 2. 59° F.; 95° F. 3. 20° C. below 0° C.; 35° C. below 0° C. 4. 22° F. below 0° F.; 40° F. below 0° F. 5. 15° C. below 0° C.; 10° C. below 0° C.

LIV. 2. 450·9135 cubic inches. 3. 1288 feet. 6. The number 3·2808992 must be multiplied by $1 + 48 \times \cdot 0000049$, and the result divided by $1 + 18 \times \cdot 0000104$.

LV. 4. Multiply 30 by 1·0013 and divide by 1·00043.

LVI. 3. About 1·112. 4. About 77 cubic inches. 5. About 338·2 cubic inches. 6. Nearly 14 pounds to the square inch. 7. About 2800 cubic feet. 8. About 53·7 cubic inches.

LVII. 1. One-fifth of 273° C., that is 54°·6 C. 2. One-seventh of 273° C., that is 39° C. 3. $80(1 + 30 \times \cdot 000026)$.

6. $1\cdot 009 - 1\cdot 0013$, that is ·0077. 7. $\frac{\cdot 0077}{1 + \cdot 009}$.

10. $\frac{50(1 + \cdot 0013)}{1 + \cdot 009}$.

LVIII. 1. Half a pound. 2. 39°·5 C. 3. 14° C. 4. 10° C. 5. 11° C. 6. 7900.

LIX. 1. About 38°C. 2. 100 ounces. 4. 8 ounces.

LXI. 1. About 305·6 grains. 2. The pressure of the aqueous vapour at 20° C. will be $\cdot 35(1 \times \cdot 03665)$, that is about ·36; hence the pressure of the dry air will be 29·64: therefore the required weight will be obtained by multiplying 305·6 by 29·64 and dividing the result by 30. The result is about 302 grains. 3. Multiply the result just found by $\frac{\cdot 36}{29\cdot 64} \times \frac{5}{8}$: see Art. 648. The result is about 2·3 grains. 4. $\frac{\cdot 66}{29\cdot 34} \times \frac{5}{8} \times 302$ grains.

LXII. 1. About 361 feet. 3. $\frac{30}{30-5\cdot 86}$ cubic feet. 4. $\frac{1000(1+27\times\cdot 003665)}{1+10\times\cdot 003665}\times\frac{30}{29}$. 5. $\frac{29\cdot 76}{30\cdot 24}$ of a cubic foot. 6. $\frac{656}{760}\times\frac{1}{1+47\times\cdot 003665}$.

LXIII. 1. 9° C. 2. About ·096. 3. About 44°. 4. $\frac{3100}{1031}$ degrees centigrade. 5. $40^0+\frac{33}{1033}\times 50^0$. 6. 802° C. 7. ·43. 8. About 4·2. 10. About 32°. 12. About 6° C.

LXV. 1. 140° F. 4. ·58.

LXVIII. 1. About 1120. 2. Nearly 40° F. 3. Rise of 2°·8 F. 4. About 24300. 7. About 320° F.

CORRECTIONS.

Art. 433. In the diagram the arrow denoted by *M* should be inverted.

Art. 520. Interchange the terms *optic axes* and *ray axes* in the fifth and ninth lines.

Art. 523. In the last line for *reflecting* read *refracting*.

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